

CHAPTER 6

TWO-WAY FLOOR DESIGN EXAMPLE



Post-Tensioned Concrete Frame under Construction
(California, P634)

FOREWORD

The following example represents a floor of a multi-story building with an irregular arrangement of supports. The slab is provided with both column drops for punching shear, and drop panels for additional strength in resisting the high negative moments over selected supports. Further, the optimum post-tensioning leads to different number of strands along the length of the design strip selected, as well as change in tendon profile from span to span.

The objective in selecting an irregular and complex structure is to expose you to the different design scenarios that are common in real life structures, but are not covered in traditional text books. Design conditions that are not directly covered in the following example, but are important to know of, are introduced and discussed as comments or added examples.

Steps that are common knowledge, such as the calculation of moments and shears, once the geometry of a structure, its material properties and loading are known, are not covered in detail. You are referred to your in-house frame programs for their computation.

The design example covers side by side both the unbonded and bonded (grouted) post-tensioning systems, thus providing a direct comparison between the design process of the two options. In addition, in parallel, the design uses both the current American building codes (ACI 318¹ and IBC²) and the European Code (EC2)³.

There are three methods commonly used for the design of a post-tensioned floor system – Simple Frame

¹ ACI 318-11

² IBC International Building Code 2012

³ EC2 EN 1992-1-1:2004(E)

Methods (SFM)⁴, Equivalent Frame Method (EFM); and Finite Element Method (FEM). Among the three, the EFM has been the primary method of design used by leading consulting firms for many years. However, due to its complexity, it does not lend itself to daily hand calculation of real structures in the environment of a consulting firm. Computer programs based on the EFM, such as ADAPT-PT are generally used. Recently, many consultants sacrifice the efficiency of designs based on EFM with respect to its capability to yield optimum designs, and opt for designs based on Finite Element Method (FEM), such as those based on Floor-Pro⁵. FEM-based designs can model the entire floor system and provide seamless integration of the design process from architectural drawings to fabrication documents.

Hand calculations, such as the one presented herein, use the SFM.

Two text fonts are used in the following. The numerical work that forms part of the actual calculations uses the font shown below:

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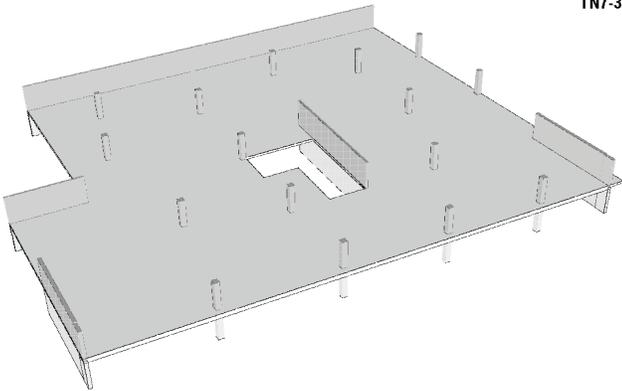
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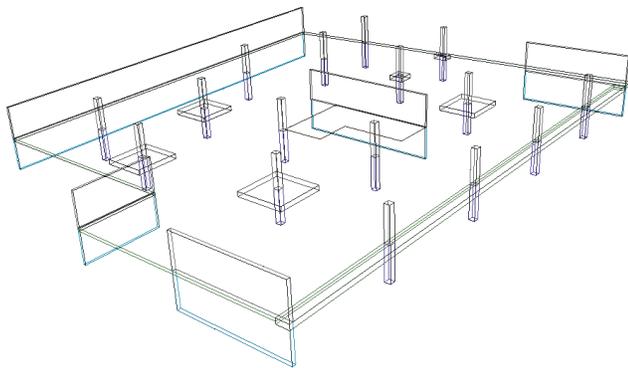
DESIGN STEPS

1. GEOMETRY AND STRUCTURAL SYSTEM
 - 1.1 Geometry of the Floor System
 - 1.2 Support Lines and Tributaries
 - 1.3 Idealized Design Strips
2. MATERIAL PROPERTIES
 - 2.1 Concrete
 - 2.2 Nonprestressed Reinforcement
 - 2.3 Prestressing
3. LOADS
 - 3.1 Selfweight
 - 3.2 Superimposed Dead Load
 - 3.3 Live Load
4. DESIGN PARAMETERS
 - 4.1 Applicable Code
 - 4.2 Cover to Rebar and Prestressing Tendons
 - 4.3 Allowable Stresses
 - 4.4 Crack Width Limitation
 - 4.5 Allowable Deflection
5. ACTIONS DUE TO DEAD AND LIVE LOADS
6. POST-TENSIONING
 - 6.1 Selection of Design Parameters
 - 6.2 Selection of Post-tensioning Values
 - 6.3 Selection of Number of Strands
 - 6.4 Calculation of Balanced Loads
 - 6.5 Determination of Actions due to Balanced Loads
7. CODE CHECK FOR SERVICEABILITY
 - 7.1 Load Combinations
 - 7.2 Stress Check
 - 7.3 Crack Width Control
 - 7.4 Minimum Reinforcement
 - 7.5 Deflection Check
8. CODE CHECK FOR STRENGTH
 - 8.1 Load Combinations
 - 8.2 Determination of Hyperstatic Actions

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(a) 3D Solid View of the Floor System (P471)



(b) See through View of the Floor System (P472)
0-3 3D Views of the Floor System

⁴ The Simple Frame Method (SFM) in UK and the literature based on UK practice is referred to as “Equivalent Frame Method.” It is based strictly on the cross-sectional geometry of the slab frame being designed. The term Equivalent Frame Method in the US literature is based on an approximation that is intended to simulate the two-way action of a floor slab.

⁵ www.adaptsoft.com

marked B, C, D and F in Fig.1.2-2. While it is practical to eliminate column drops at locations A and E through provision of punching shear reinforcement, the drop panels cannot be eliminated without causing congestion in top rebar.

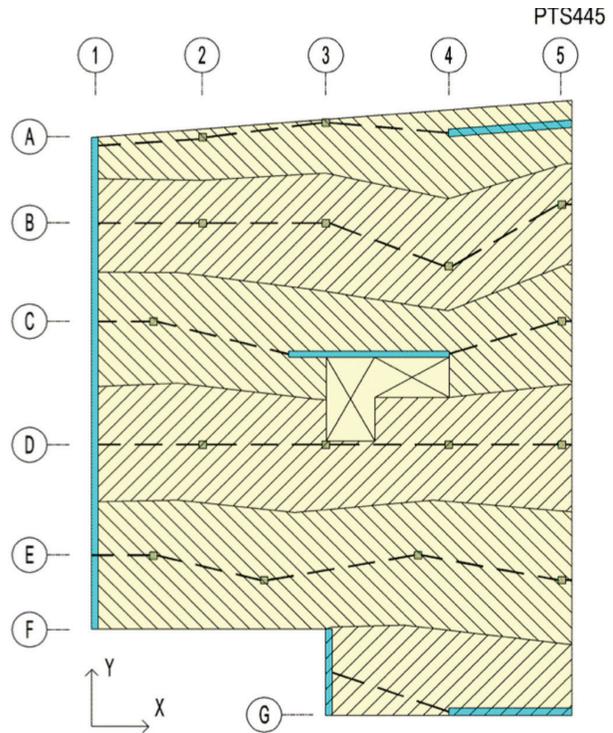
1.3 Support Lines, Tributaries and Design Strips

The breakdown of a floor into support lines, tributaries and design strips in two principal directions are explained in Chapter 3, as the first step in definition of load paths for design. The outcome is the subdivision of floor into design strips in each of the two orthogonal directions. In this example, we select and complete the design of one of the design strips in X-direction. The remainder of the design strips will be treated in a similar manner.

The design strips in X-direction are shown in Fig. 1.3-1. Each design strip is extracted from the floor system and modeled in isolation as an idealized single design strip, such as the design strip for support line B shown in Fig. 1.3-2a.

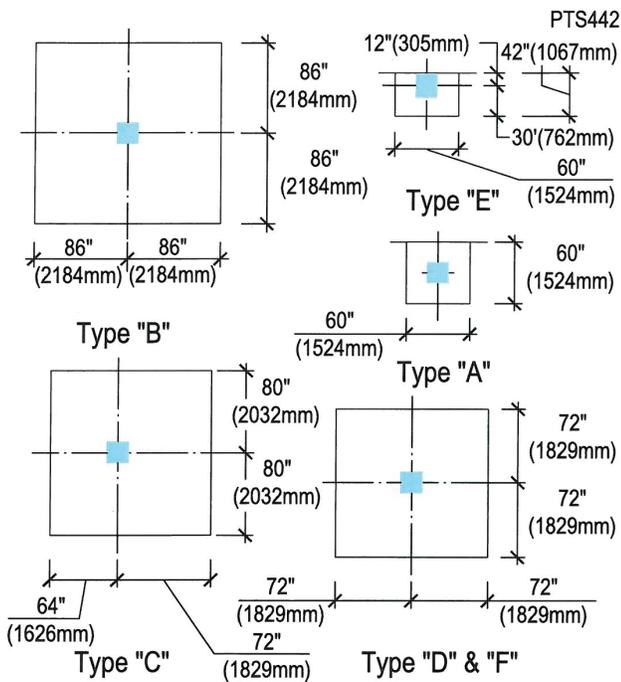
1.4 Idealized Design Strip

❖ Design strip dimensions:
The extracted design strip is "straightened" to simplify analysis (Fig. 1.3-2b). The tributaries of each span of of



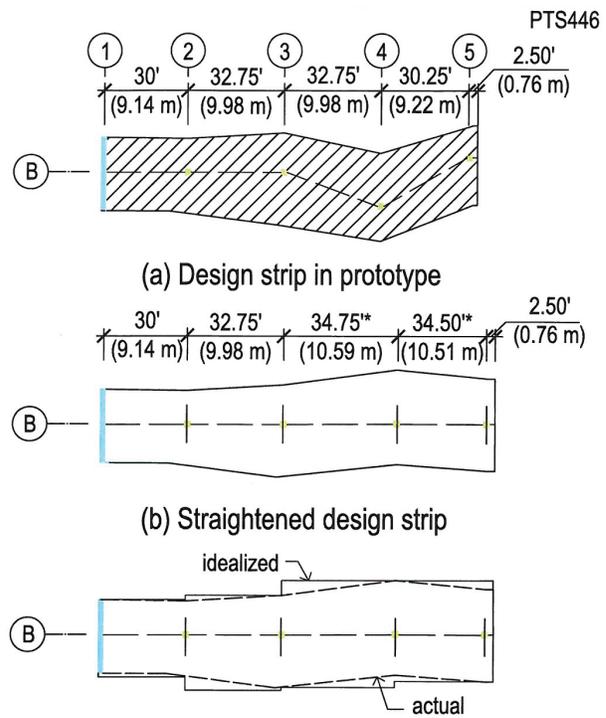
Tributaries for Design Strips in X - Direction

FIGURE 1.3-1



Plan Geometry of Drop Caps and Drop Panels

FIGURE 1.2-3



Construction of Design Strip in Plan

FIGURE 1.3-2

the extracted design strip are adjusted to the maximum width of the respective span on each side of the support line. The dimensions of the final design strip are shown in Figs. 1.4-1.

For gravity design of the structure, the practice in selection of boundary conditions of the extracted design strip is verbalized in ACI/IBC as follows. The strip is modeled with one level of supports immediately above and below the level under consideration. The far ends of the supports are assumed fixed against rotation.

The elevation of the idealized design strip and a three dimensional view of it are shown in Figs. 1.4-2 and 1.4-3

❖ Section properties:

The section properties of each span are calculated using the gross cross-sectional area of the idealized design strip as shown in Figs. 1.4-1 and 1.4-2. The values of the controlling locations are summarized in Table 1.4-1.

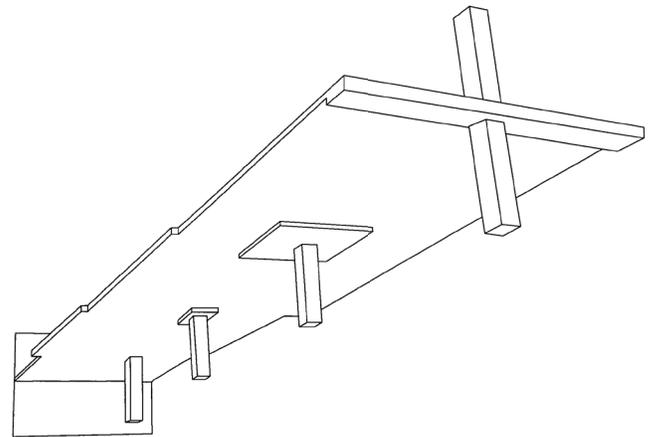
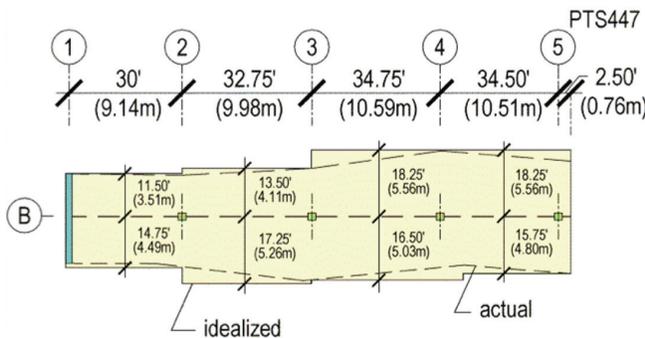
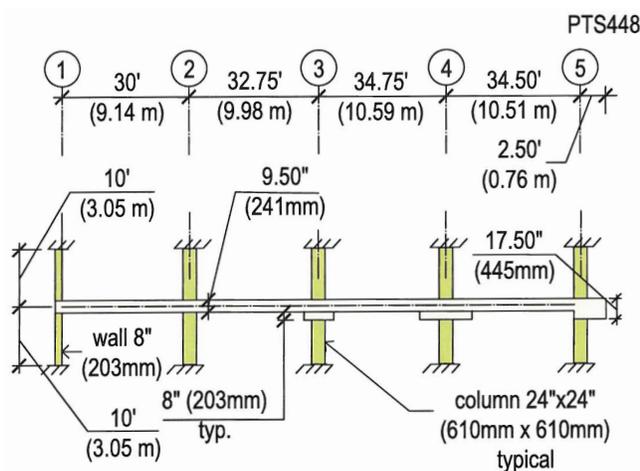


FIGURE 1.4-3 View of Design Strip (P473)



Plan of Design Strip B

FIGURE 1.4-1



Design Strip in Elevation

FIGURE 1.4-2

The stiffening of the slab due to the added thickness of the column drops and drop panels are accounted for in the calculation through their section properties. In SFM adopted in this example, the added stiffness in the slab immediately over the support is not included in the analysis. However, the EFM of analysis accounts for the aforementioned increase in stiffness.

2 - MATERIAL PROPERTIES

2.1 Concrete

f'_c, f_{ck} (28 day cylinder strength)⁶ = 5000 psi (34.47 MPa)

Weight = 150 pcf

$$\begin{aligned} \text{Elastic Modulus} &= 57000\sqrt{f'_c} = 4,030.50 \text{ ksi [ACI]} \\ &= 22 \cdot 10^3 \cdot [(f_{ck} + 8) / 10]^{0.3} \text{ [EC2]} \\ &= 33,950.59 \text{ MPa [4,924 ksi]} \end{aligned}$$

Creep/shrinkage coefficient = 2

Material factor, $\gamma_c = 1$ for ACI; 1.50 for EC2

The creep/shrinkage coefficient is used to estimate the long-term deflection of the slab.

2.2 Nonprestressed (Passive) Reinforcement:

$f_y = 60$ ksi

Elastic Modulus = 29,000 ksi

Material factor, $\gamma_c = 1$ for ACI; 1.15 for EC2

Strength reduction factor, $\phi = 0.9$ for ACI; 1 for EC2

⁶ f_{ck} is the European (EC2) symbol for f'_c

⁷ EN 1992-1-1:2004(E) Table 3.1

TABLE 1.4-1 Sectional Property of the Design Strip (T157)

Span	Tributary Width ft.	Depth in	I in ⁴	Y _b in.	Y _t in.
1 Left	26.25	9.50	22506	4.75	4.75
2 Middle	30.75	9.50	26364	4.75	4.75
2 Right	30.75	17.50	61248	11.70	5.80
3 Left	34.75	17.50	65132	11.80	5.70
3 Middle	34.75	9.50	29794	4.75	4.75
3 Right	34.75	17.50	104268	10.78	6.72
4 Left	34.00	17.50	103287	10.75	6.75
4 Middle	34.00	9.50	29151	4.75	4.75
4 Right	34.00	17.50	182219	8.75	8.75
Cantilever	34.00	17.50	182219	8.75	8.75

I = second moment of area (moment of inertia) of the tributary section; and
 Y_t, Y_b = top and bottom distances of the centroid of the section to the extreme fibers respectively

2.3 Prestressing: (see Figs 2.3-1 through 2.3-3)
 Material - low relaxation, seven wire ASTM 416 strand
 Strand diameter = 1/2 in (nominal).
 Strand area = 0.153 in.²
 Elastic Modulus = 28,000 ksi
 Ultimate strength of strand (f_{pu}) = 270 ksi
 Material factor, γ_c = 1 for ACI; 1.15 for EC2

System
Unbonded System

Angular coefficient of friction (μ) = 0.07
 Wobble coefficient of friction (K) = 0.001 rad/ft
 Anchor set (wedge draw-in) = 0.25 inch
 Jacking force = 80% of specified ultimate strength
 Effective stress after all losses = 175⁸ ksi

Bonded System

Use flat ducts 20x800 mm; 0.35 mm thick metal sheet housing up to five strands
 Angular coefficient of friction (μ) = 0.2
 Wobble coefficient of friction (K) = 0.001 rad/ft
 Anchor set (wedge draw-in) = 0.25 inch

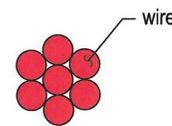
⁸ For hand calculation, an effective stress for tendon is used. The effective stress is the average stress along the length of a tendon after all immediate and long-term losses. The value selected for effective stresses is a conservative estimate. When “effective stress” is used in design, the stressed lengths of tendons are kept short, as it is described later in Section 6 of these calculations

Distance between centroids of strand and duct (z) = 1/8 inch
 Effective stress after all losses = 160 ksi

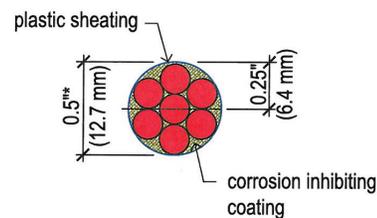
3 - LOADS

3.1 Selfweight

Slab = (9.5/12') * 150 pcf = 118.75 psf



(a) Seven wire strand



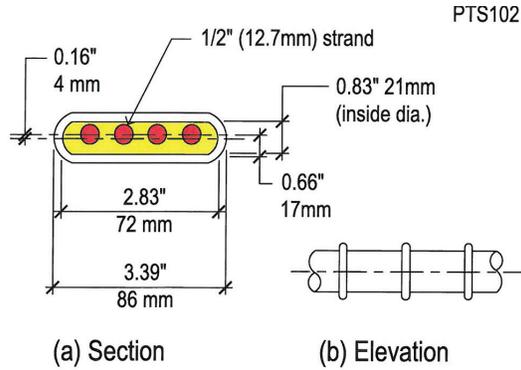
note: * nominal diameter

(b) View of tendon

Unbonded Tendon

FIGURE 2.3-1 Section View of an Unbonded Tendon

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Example of a Grouted Flat Duct Used in Building Construction

FIGURE 2.3-2 Plastic Flat Corrugated Duct Grouted Tendon

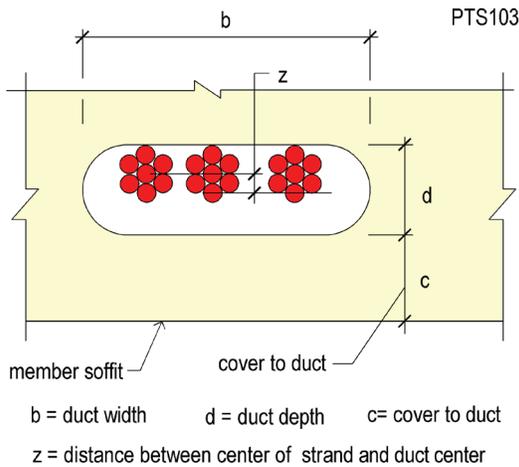


FIGURE 2.3-3 Section Through a Flat Duct at Low Point

3.2 Superimposed dead load

Fixtures and finishes = 7.00 psf
 Partitions = 20.00 psf
 Total Superimposed dead load = 27.00 psf
 Total dead load = SW+ SDL = 145.75 psf

Span 1 DL = 145.75*26.25/1000 = 3.826 klf
 Span 2 DL = 145.75*30.75/1000 = 4.482 klf
 Span 3 DL = 145.75*34.75/1000 = 5.065 klf
 Span 4 DL = 145.75*34.00/1000 = 4.956 klf
 Added dead load due to column drop, drop panel and thickened overhang

Column drop DL (support 3) = (60/12)*(8/12)*150 pcf/1000 = 0.500 klf

Load extends 2.5' on each side of the support 3

Drop panel DL (support 4) = (144/12)*(8/12)*150 pcf/1000 = 1.200 klf

Load extends 6' on each side of support 4

Added overhang depth (Cantilever.) = (34')*(8/12)*150 pcf/1000 = 3.400 klf

Load extends from 1' to left of support 5 to slab edge at overhang

3.3 Live Load 50 psf, reducible per IBC

The live load is reduced based on the area it covers. The effective area considered for the reduction of live load is the idealized span area shown in Fig. 1.4-1. This is somewhat larger than the actual area. It is used for the determination of total design load on each span, as well as for the reduction of the live load.

The following relationship from IBC⁹ is used:

$$R = 0.08(A - 150)$$

Where, R = reduction factor, not to exceed 40%; and A = tributary in square feet.

Nor more than

$$R = 23.1 (1 + D/L_0) = 23.1(1 + 145.75/50) = 90.43 \% > 40 \%$$

Where, D is the dead load and L₀ the applicable live load.

Span 1:

$$\text{Reduction} = 0.08 * (30 * 26.25 - 150) = 51\% > 40\% \text{ max}$$

$$\text{Live load} = (1.0 - 0.40) * 50.00 \text{ psf} = 30.00 \text{ psf}$$

$$= 30 \text{ psf} * 26.25 / 1000 = 0.788 \text{ klf}$$

Spans 2 to span 4:

By inspection the maximum reduction of 40% may be used

$$\text{Live load} = (1.0 - 0.40) * 50.00 \text{ psf} = 30.00 \text{ psf}$$

$$\text{Span 2 LL} = 30 \text{ psf} * 30.75 / 1000 = 0.923 \text{ klf}$$

$$\text{Span 3 LL} = 30 \text{ psf} * 34.75 / 1000 = 1.043 \text{ klf}$$

$$\text{Span 4 LL} = 30 \text{ psf} * 34.0 / 1000 = 1.020 \text{ klf}$$

Cantilever:

$$\text{Reduction} = 0.08 * (34 * 2.5 - 150) = 0\%$$

Live load = 50 psf

$$= 50 \text{ psf} * 34.0 / 1000 = 1.700 \text{ klf}$$

LL/DL ratio = 50/145.75 = 0.34 < 0.75 ∴ Do not skip Live Load

Live load is generally skipped (patterned), in order to maximize the design values. However, for two-way

⁹ IBC-12; Section 1607

floor systems, ACI 318-11 does not require live load skipping¹⁰, provided the ratio of live to dead load does not exceed 0.75. In this example, as in most concrete floor systems for residential and office buildings, the ratio of live to dead load is less than 0.75. Hence, the live load will not be skipped.

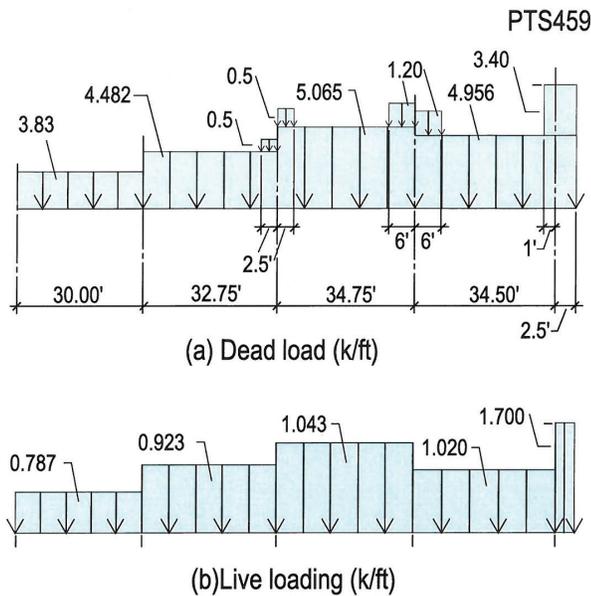
The loading diagrams are shown in Fig. 3-1.

4 DESIGN PARAMETERS

4.1 Applicable Codes

The design is carried out according to each of the following codes

- ❖ ACI 318-2011; IBC-2012
- ❖ EC2 (EN 1992-1-1:2004)



Loading On Design Strip

FIGURE 3.1

4.2 Cover to Rebar and Prestressing Tendons
Unbonded system

Minimum rebar cover = 0.75 in top and bottom

The slab is assumed to be in a non-corrosive environment. Cover to its reinforcement is based on a 2-hour fire rating with the exterior spans considered restrained. This requires a minimum cover of 0.75", using IBC-2012. Hence, the CGS (Center of Gravity of Strand) of 0.5" strand is 1" from top and bottom fibers of concrete outline. The existing concrete wall

¹⁰ ACI-318-11, Section 13.7.6

at one end of the design strip, and the thickened overhand at the other end of it are considered adequate to provide restraint against in plane expansion of the slab for fire resistivity. Hence, the end spans are considered "restrained."¹¹

Minimum strand cover = 0.75 in.

CGS, all spans = 1.00 in.

Bonded system (Fig. 4.2-1)

Minimum top and bottom rebar cover = 0.75 in

For post-tensioning tendons:

Cover to duct = 0.75in.

Distance to centroid of strand = 0.75 + 0.4 + 0.1 = 1.25 in.

Where, 0.4" is half duct diameter and z = 0.1 in

CGS. All spans = 1.25 in.

4.3 Allowable Stresses

A. Based on ACI 318-11/IBC 2012¹²

Allowable stresses in concrete are the same for bonded and unbonded PT systems

❖ For sustained load condition

Compression = $0.45 \cdot f'_c = 0.45 \cdot 5000 = 2,250 \text{ psi}$

Tension = $6 \cdot \sqrt{f'_c} = 424 \text{ psi}$

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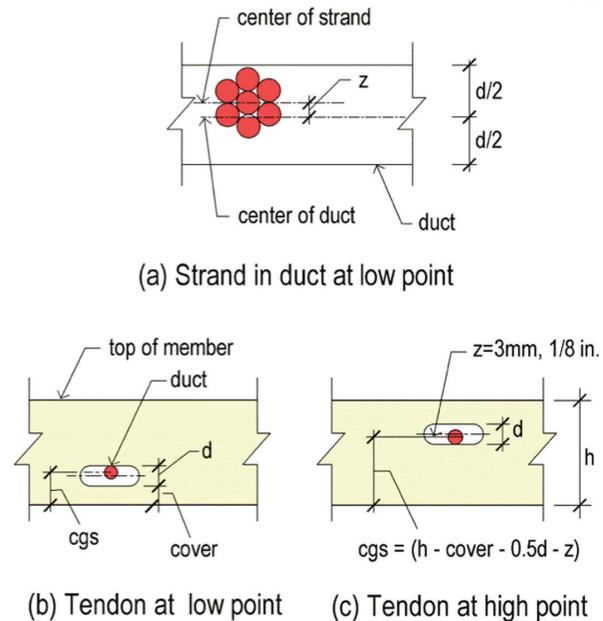


FIGURE 4.2-1

¹¹ In IBC-12, where a span is free to expand in its own plane, it is considered "unrestrained," and is required to have a larger cover for fire resistivity than a span that is not free to expand (restrained). IBC-12, Table 720.1

¹² ACI-318-11, Sections 18.3 & 18.4

❖ For total load condition

Compression = $0.60 \cdot f_c = 3,000$ psi

Tension = $6 \cdot \sqrt{f_c} = 424$ psi

❖ For initial condition:

Compression = $0.60 \cdot f_{ci} = 0.6 \cdot 3750 = 2,250$ psi

Tension = $3 \cdot \sqrt{f_c} = 184$ psi

In ACI 318/IBC the allowable stresses for two-way systems and one-way systems are different. The values stated are for two-way systems. These values may not be exceeded. Using ACI 318, two-way systems are deemed to be essentially crack-free when in service. In-service cracking, if any is not of design significance.

B. Based on EC2¹³

EC2 does not specify “limiting” allowable stresses in the strict sense of the word. There are stress thresholds that trigger crack control. These are the same for both bonded and unbonded systems.

❖ For “frequent” (total) load condition Concrete:

Compression = $0.60 \cdot f_{ck} = 20.68$ MPa (3,000 psi)

Tension (concrete) = $f_{ct,eff} = f_{ctm}^{14}$
 $= 0.30 \cdot f_{ck}^{(2/3)}$ (Table 3.1, EC2)

$= 0.30 \cdot 34.47^{(2/3)} = 3.18$ MPa (461 psi)

Tension (non-prestressed steel) = $0.80 \cdot f_{yk} = 330.95$ MPa (48,001 psi)

Tension (prestressing steel) = $0.75 \cdot f_{pk} = 1395$ MPa (202,331 psi)

❖ For “quasi-permanent” (sustained) load condition

Compression = $0.45 \cdot f_{ck} = 15.51$ MPa (2,250 psi)

Tension (concrete) = 3.18 MPa (461 psi) same as frequent load combination

Unlike ACI 318/IBC, provisions in EC2 permit¹⁵ overriding the allowable hypothetical tension stress in concrete, provided cracking is controlled not to exceed the selected “design crack width.”

❖ For “initial” load condition

Tension (unbonded) = $f_{ct,eff} = f_{ctm}$

$= 0.30 \cdot f_{ci}^{(2/3)}$ (Table 3.1, EC2) = $0.30 \cdot 25.85^{(2/3)} = 2.62$ MPa (380 psi)

Compression¹⁶ = $0.60 \cdot f_{ci} = 0.6 \cdot 25.85 = -15.51$ MPa (-2250 psi)

4.4 Crack Width Limitation:

¹³ EN 1992-1-1:2004(E), Section 7.2

¹⁴ EN 1992-1-1:2004(E), Section 7.3.2(4)

¹⁵ EN 1992-1-1:2004(E), Section 7.3.2(4)

¹⁶ EN 1992-1-1:2004(E), Section 5.10.2.2(5)

A. Based on ACI 318-11/IBC 2012

No explicit limit is imposed by the code for crack width calculation and or its control for two-way floor systems, since the designs are deemed to be essentially within the pre-cracking range of concrete.

B. Based on EC2

In EC2, the allowable crack width depends on whether the post-tensioning system used is “bonded,” or “unbonded,” and the load combination being considered.¹⁷

❖ Prestressed members with bonded tendons: 0.2 mm; to be checked for frequent load case

❖ Prestressed members with unbonded tendons: 0.3 mm; to be checked at quasi-permanent load case

4.5 Allowable Deflection**A. Based on ACI 318-11/IBC 2012¹⁸**

In major building codes, the allowable deflection is tied to (i) the impact of the vertical displacement on occupants; (ii) possible damage to the installed non-structural objects such as partitions, glass, or floor covering; and (iii) functional impairment, such as proper drainage. Details of the allowable values, their measurement and evaluation are given in Chapter 4. For perception of displacement by sensitive persons, consensus is the limit of $L/250$, where L is the deflection span. It is important to note that this limit is for deflections that can be observed by a viewer. Camber can be used to mitigate exceeding this limit.

Since in this design example carpet is assumed to be placed directly on the finished floor, the applicable vertical displacement for visual impact is the total deflection subsequent to the removal of forms.

Total allowable deflection based on ACI 318: $L/240$.

The second deflection check is for potential damage to non-structural brittle construction, such as partitions, resulting from displacement subsequent to installation of such members. The value recommended by ACI 318 is $L/480$. This is vertical displacement resulting from the full application of design live load together with the long-term deflection subsequent to the installation of the brittle members. Examples of such installations include application of plaster on partitions made of concrete masonry units, or installation of dry wall (gypsum boards). Raw framing or masonry units that are not finished are not considered to be subject to this deflection limitation.

¹⁷ EN1992-1-1-2004 (E) Table 7.1N

¹⁸ ACI 318-11, Sections 18.3.5

Total deflection subsequent to finish on partitions together with application of live load: $L/480$. Where, L is the length of deflection span. For this design example, the partitions are assumed to have been installed/finished 60 days after the floor is cast.

B. Based on EC2¹⁹

The interpretation and the magnitude of allowable deflections in EC2 are essentially the same as that of ACI 318. The impact of vertical displacement on the function of the installed members and the visual impact on occupants determine the allowable values. The following are suggested values:

Deflection subsequent to finishing of floors from quasi-permanent combination: $L/250$

Deflection subsequent to installation of construction that can be damaged from load combination quasi-permanent: $L/500$.

In summary, the allowable deflection from the two codes are essentially the same. Conservatively, it can be summarized as follows:

Total deflection from quasi permanent load combination - $L/250$

Where, L is the length of the span.

Deflection subsequent to installation of construction that can be damaged from load combination quasi-permanent $L/500$

5. ACTIONS DUE TO DEAD AND LIVE LOADS

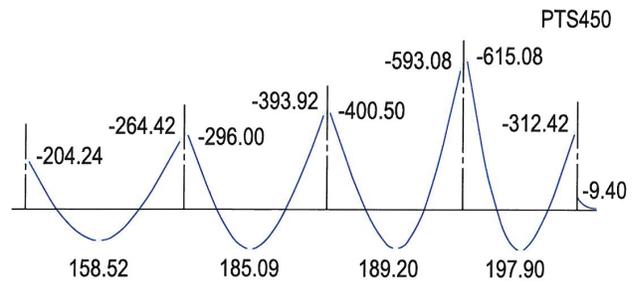
Actions due to dead and live loads are calculated by a generic frame program, using the frame dimensions shown in Fig. 1.4-2. The stiffness of each of the spans is based on the second moment of area given in Table 1.3-1. At locations of the column drop, drop panel, and transverse beam, the stiffness used includes the local thickening of the slab.

The moments calculated from the frame analysis refer to the center line of supports. These are reduced

to the face-of-support using the static equilibrium of each span. The computed moments from the analysis using Simple Frame Method (SFM) are shown in Fig. 5-1 and Fig.5-2. The values at each face-of-support and at midspan are summarized in Table 5-1.

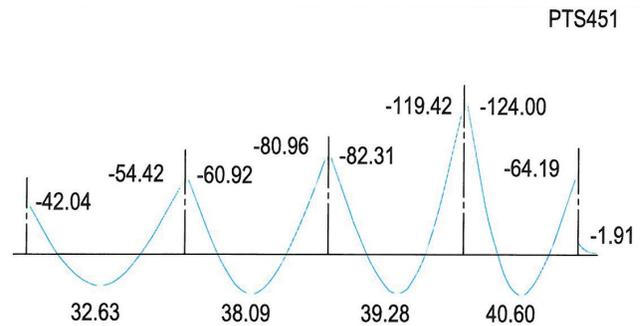
The critical design moments are not generally at midspan. But, for hand calculation, the midspan is selected. The approximation is acceptable when spans and loads are relatively uniform.

For hand calculations, a simple frame analysis is used (Simple Frame Method – SFM). The simple frame method of analysis lacks the specific features of the Equivalent Frame Method (EFM) that are listed below:



Moments Due to Dead Loading (k-ft)

FIGURE 5-1



Moments Due to Live Loading (k-ft)

FIGURE 5-2

(i) Increased stiffness of slab over slab/support interface is not accounted for. The stiffness of a slab over its support is assumed to be the same as that at the face of support;

(ii) Increased stiffness of the column within the slab, or within the column drop/panel is not accounted for. In other words, the stiffness of a column is assumed constant over its entire analysis length. Note that the analysis length of a column extends to the centroid of slab; and

(iii) The analysis does not account for the two-way action of the slab, as is implemented in the Equiva-

¹⁹ ACI 318-11, Sections 18.3.5

TABLE 5-1 Moments at Face-of-Support and Midspan (T158US)

Span	Location	M_D k-ft	M_L k-ft	$M_D + M_L$ k-ft
Span #1	Left FOS*	-204.25	-42.04	-246.29
	Midspan	158.52	32.63	190.50
	Right FOS	-264.42	-54.42	-318.84
Span #2	Left FOS	-296.00	-60.92	-356.92
	Midspan	185.09	38.09	223.18
	Right FOS	-393.92	-80.96	-474.88
Span #3	Left FOS	-400.50	-82.31	-482.81
	Midspan	189.20	39.28	228.48
	Right FOS	-593.08	-119.42	-712.50
Span #4	Left FOS	-615.08	-124.00	-739.08
	Midspan	197.90	40.60	238.50
	Right FOS	-312.42	-64.19	-376.61
Cantilever	Left FOS	-9.40	-1.91	-11.31

* FOS = face-of-support

lent Frame Method. The stiffness of the structure is strictly based on the cross-sectional geometry of the design strip.

The SFM is adequate when hand calculation is used for design. The EFM is more accurate, but it is too complex for hand calculation in the environment of a production oriented consulting office. It is important to note that the SFM provides a "safe" design, but not necessarily the most economical alternative within the scope of strip method. The EFM generally leads to smaller column moments, when compared to the SFM.

Examples of the EFM in the literature are generally limited to flat plates mostly without column drop or drop panel, and with uniform tributaries. The use of computer programs with EFM formulation is the practical way for design of complex floor systems with column drop, and/or drop panels, irregular tributaries and non-uniform loads.

6 - POST-TENSIONING

6.1 Selection of Design Parameters

Unlike conventionally reinforced slabs, where given geometry, boundary conditions, material properties and loads result in a unique design, for post-tensioned members, in addition to the above, a mini-

um of two other input assumptions are required, before a design can be concluded. Common practice is (i) to assume a value for precompression from prestressing, and (ii) target to balance a percentage of the structure's dead load. The procedure is explained in greater detail and outlined in a flow chart in Chapter 4, Section 4.8.2C. In this example, based on experience, the level of precompression suggested is larger than the minimum required by ACI 318 (125 psi). Other major building codes do not specify a minimum precompression. Rather, they specify a minimum reinforcement. Use the following assumptions to initiate the calculations.

1. Minimum average precompression = 150 psi
2. Maximum average precompression = 300 psi
3. Target balanced loading = 60% of total dead load, up to 80% where beneficial

The minimum precompression is used as the entry value (first trial) for design. The stipulation for a maximum precompression does not enter the hand calculation directly. It is stated as a guide for a not-to-exceed upper value. In many instances, floor slabs that require more than the maximum value stated can be re-designed more economically.

For deflection control the selfweight of the critical span is recommended to be balanced to a minimum

TABLE 5-1 Moments at Face-of-Support and Midspan (T158US)

Span	Location	M_D k-ft	M_L k-ft	$M_D + M_L$ k-ft
Span #1	Left FOS*	-204.25	-42.04	-246.29
	Midspan	158.52	32.63	190.50
	Right FOS	-264.42	-54.42	-318.84
Span #2	Left FOS	-296.00	-60.92	-356.92
	Midspan	185.09	38.09	223.18
	Right FOS	-393.92	-80.96	-474.88
Span #3	Left FOS	-400.50	-82.31	-482.81
	Midspan	189.20	39.28	228.48
	Right FOS	-593.08	-119.42	-712.50
Span #4	Left FOS	-615.08	-124.00	-739.08
	Midspan	197.90	40.60	238.50
	Right FOS	-312.42	-64.19	-376.61
Cantilever	Left FOS	-9.40	-1.91	-11.31

* FOS = face-of-support

of 60%. Non-critical spans need not be balanced to the same extent.

Effective stress in prestressing strand:

For unbonded tendons: $f_{se} = 175$ ksi

For bonded tendons: $f_{se} = 160$ ksi

The design of a post-tensioned member can be based either on the “effective force”, or the “tendon selection” procedure. In the effective force procedure, the average stress in a tendon after all losses is used in design. In this case, the design concludes with the total effective post-tensioning force required at each location. The total force arrived at the conclusion of design is then used to determine the number of strands required, with due allowances for friction and long-term losses. This provides an expeditious and simple design procedure for hand calculations. It is the common procedure in North America. In the “tendon selection” procedure, the design is based on the number of strands and recognition of local immediate and long-term stress losses. In the following, the “effective force” method is used to initiate the design. Once the design force is determined, it is converted to the number of strands required. A graphical presentation of the preceding assumptions is given in Fig 4.8.7.1-2

The effective stress assumed in a strand is based on the statistical analysis of common floor slab dimensions for the following conditions:

(i) Members have dimensions common in building construction;

(ii) Strands are 0.5 or 0.6 inch nominal diameter;

(iii) Tendons equal or less than 125ft long stressed at one end. Tendons longer than 125ft, but not exceeding 250ft are stressed at both ends. Tendons longer than 250ft are stressed at intermediate points to limit the unstressed lengths to 125ft for one-end stressing or 250ft for two-end stressing, whichever is applicable.

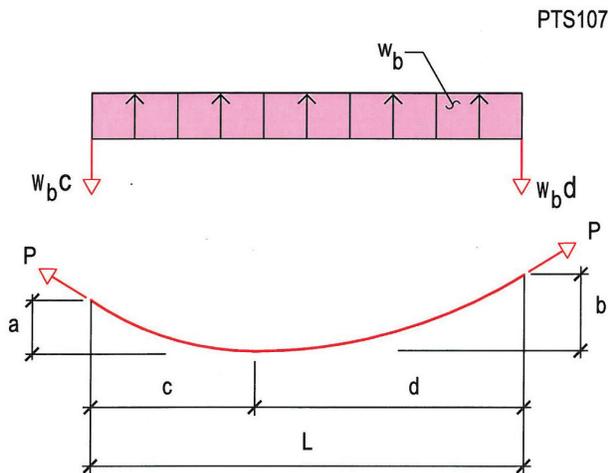
(iv) Tendons are coated with grease and plastic, with industry common friction coefficients as stated in material properties section of this example; and

(v) Tendons are stressed to 0.8 f_{pu} .

For other conditions, a lower effective stress is assumed. Alternatively, longer tendons are stressed at intermediate points. In the current design, the total length of the tendon is 136 ft. It is stressed at both ends. Detail calculations indicate that the effective tendon stress is 182 ksi for the unbonded system. Also, the computed stress is larger than the suggested average value for the grouted system.

6.2. Selection of Post-Tensioning Tendon Force and Profile

The prestressing force in each span will be chosen to match a whole number of prestressing strands. The following values are used:



$$c/L = [\sqrt{a/b} / (1 + \sqrt{a/b})]$$

$$w_b = 2aP/c^2$$

Geometry and Actions of a Parabolic Tendon

FIGURE C6.2-1

1. A constant effective force is assumed for each span. The design prestressing force is chosen as a multiple of the average force in each tendon.

For unbonded tendons

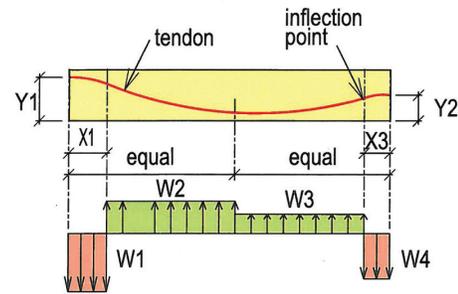
- ❖ Force per tendon = 175 ksi * 0.153 in² = 26.77 kips/tendon
- ❖ Use multiples of 26.77 kips when selecting the post-tensioning forces for design.

For bonded tendons

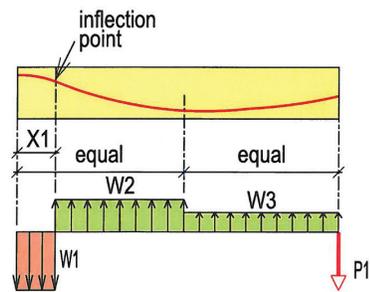
- ❖ Force per tendons = 160 ksi * 0.153 in² = 24.48 kips/tendon
- ❖ Use multiples of 24.48 kips when selecting the post-tensioning forces for design.

2. Tendon profiles are chosen to be simple parabola. These produce a uniform upward force in each span.

For ease of calculation the tendon profile in each span is chosen to be simple parabola from support centerline to support centerline (Fig. C6.2-1). The position of the low point is selected such as to generate a uniform upward force in each span. The relationship given in Fig. C6.2-1 defines the profile. For exterior spans, where the tendon high points are not generally at the same level, the resulting low point will not be at midspan. For interior spans, where tendon high points are the same, the low point will coincide with midspan. Evidently, the chosen profile is an approximation of the actual tendon layout used



(a) Reversed parabola with two inflection points



(b) Reversed parabola with one inflection point

FIGURE C6.2-2 Two Examples of Common Tendon Profiles

in construction. Sharp changes in curvature associated with the simple parabola profile at supports are impractical to achieve on site. The tendon profile at construction is likely to be closer to reversed parabola, for which the distribution of lateral tendon forces will be somewhat different as discussed henceforth. Tendon profiles in construction and the associated tendon forces are closer to the diagrams shown in Fig. C6.2-2 for two common cases.

For the cantilever at the right end, the profile selected is a straight line, due to short length of the overhang (Fig.C6.2-3).

6.3 Selection of Number of Strands

Determine the initial number of strands for each span based on the assumed average precompression, and the associated cross-sectional area of each span's tributary. Then, adjust the number of strands selected, based on the uplift they provide, followed by practicality of their layout.

Unbonded Tendon

- ❖ Span 1 area = 26.25 * 12 * 9.50 = 2992.50 in²
- ❖ Span 1 force = 150 psi * 2992.50 in² / 1000 = 448.88 kips

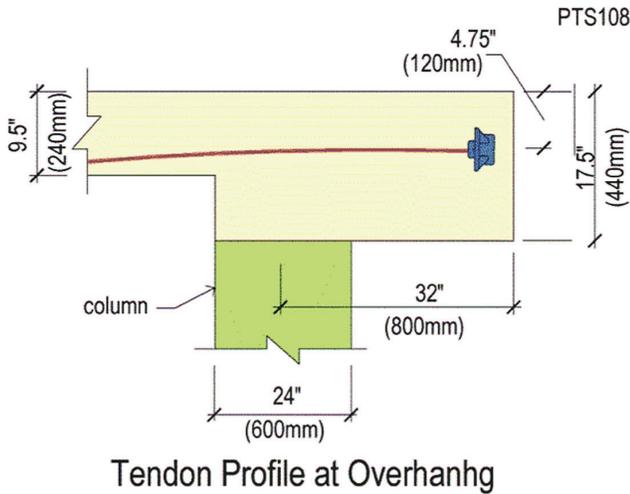


FIGURE C6.2-3

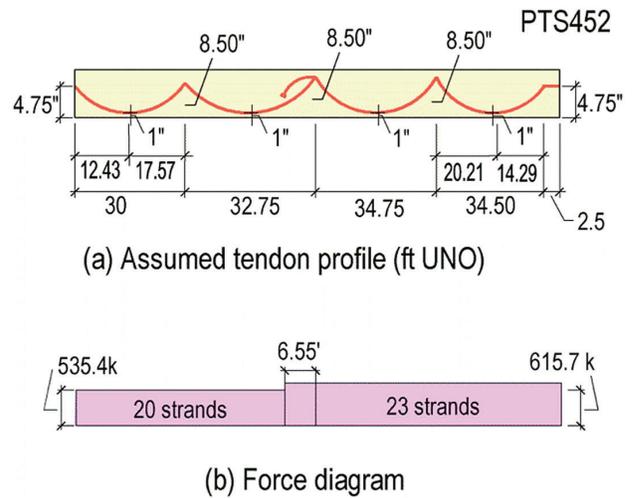


FIGURE 6.3-1 Tendon Profile and Selected Force

- ❖ Number of tendons = $448.88/26.77 = 16.77$; say 17
- ❖ Calculated values for other spans are shown in Table 6.3-1.

Bonded Tendon

- ❖ Span 1 area = $26.25 \times 12 \times 9.50 = 2992.50 \text{ in}^2$
- ❖ Span 1 force = $150 \text{ psi} \times 2992.50 \text{ in}^2 / 1000 = 448.88 \text{ kips}$
- ❖ Number of tendons = $448.88/24.48 = 18.34$; say 19
- ❖ Calculated values for other spans are shown in Table 6.3-1

It is noted that the number of strands required to satisfy the same criterion differs between the unbonded and bonded systems. Due to higher friction losses, when using bonded systems, more strands are generally needed to satisfy the in-service condition of design. For brevity, without compromising the process of calculation, in the following the same number of strands is selected for both systems. The strand numbers selected refer to that computed for unbonded system.

The number of strands in the above table is based on a minimum precompression of 150 psi at the mid-length of each span. The added cross-sectional area of column drops, drop panels and overhang are disregarded in the calculation of the force for minimum precompression. The selected number of tendons is chosen to avoid an overly complicated tendon layout. The precompression limit is disregarded for the short overhang.

The tendon profile and force selected for unbonded tendons is shown in Fig. 6.3-1

6.4 Calculation of Balanced Loads

Balanced loads are the forces that a tendon exerts to its concrete container. It is generally broken down to forces normal to the centerline of the member (causing bending) and axial to it (causing uniform precompression) and added moments at locations of change in position of member's centroidal axis. Fig. C6.2-2 shows two examples of balanced loading for members of uniform thickness.

TABLE 6.3-1 Tendon Selection Based on Minimum Precompression Using Unbonded Tendons (T159US)

Span	Tributary ft.	Thickness in.	Area in. ²	Force kips.	Tendons required	Tendons selected
1	26.25	9.50	2992.50	448.88	17	20
2	30.75	9.50	3505.50	525.83	20	20
3	34.75	9.50	3961.50	594.23	23	23
4	34.00	9.50	3876.00	581.40	22	23
Cant.	34.00	17.50	7140.00	1071.00	41	23

❖ Span 1:

Refer to Fig. C6.2-1 and Fig. C6.4-1

$$a = 4.75'' - 1.00'' = 3.75''$$

$$b = 8.50'' - 1.00'' = 7.50''$$

$$L = 30.00'$$

$$c = \{[3.75/7.50]^{0.5}/[1 + (3.75/7.50)^{0.5}]\} * 30' = 12.43'$$

$$W_b/\text{tendon} = 2 P * a/c^2 = 26.77 \text{ kips/tendon}$$

$$*(2 * 3.75/12)/12.43^2 = 0.108 \text{ klf/tendon}$$

$$\text{For } 20 \text{ tendons } W_b = 0.108 * 20 = 2.160 \text{ klf}$$

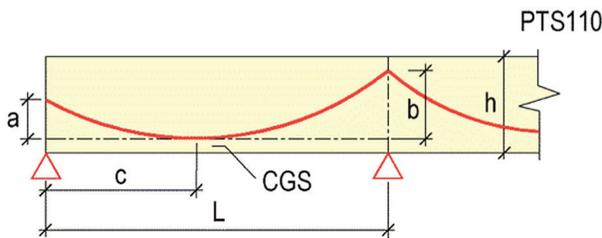
$$\% \text{ DL Balanced} = (2.160/3.826) * 100 = 56\%$$

OK (less than 60% target, but considered acceptable)

$$\text{Balanced load reaction, left} = 2.160 \text{ klf} * 12.43' = 26.85 \text{ k} \downarrow$$

$$\text{Balanced load reaction, right} = 2.160 \text{ klf} * 17.57' = 37.95 \text{ k} \downarrow$$

The tendon profiles of the first and last spans are selected to provide a uniform upward force on the respective spans (Fig. C6.2-1). Both spans appear to be critical and will be designed for maximum drupe to provide the maximum amount of balanced loading. If the low point of the tendon is not selected at the location determined by "c", two distinct parabolas result. Figure C6.2-2 illustrates the condition, where the low point is not at location determined by parameter c.



Tendon Elevation in First Span

FIGURE C6.4-1 View of Tendon Profile in First Span

❖ Span 2:

Span 2 has 20 continuous strands and three short strands (added tendons) that extend from span 3 to span 2 and terminate at its right end. The balanced load from each is calculated separately.

Continuous tendons

$$a = 7.5''$$

$$L = 32.75'$$

$$W_b/\text{tendon} = 8 * P * a / L^2$$

$$= (8 * 26.77 * 7.5/12) / 32.75^2 = 0.125 \text{ klf}$$

$$\text{For } 20 \text{ tendons } W_b = 0.125 * 20 = 2.500 \text{ klf}$$

$$\% \text{ DL Balanced} = 2.500 / 4.482 = 56\% \text{ OK}$$

$$\text{Balanced load reaction, left} = 2.500 \text{ klf} * 16.38' = 40.95 \text{ k} \downarrow$$

$$\text{Balanced load reaction, right} = 2.500 \text{ klf} * 16.38' = 40.95 \text{ k} \downarrow$$

Added Tendons:

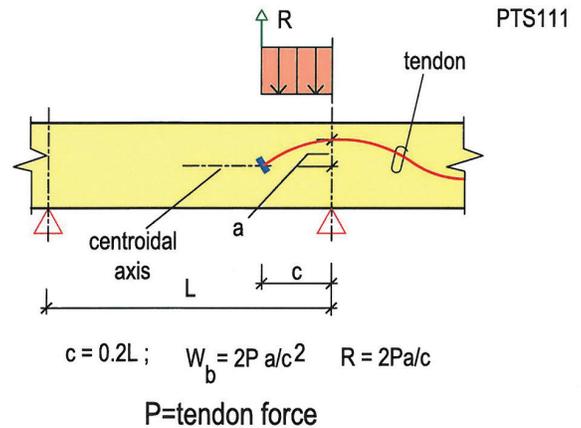
Increase in the number of strands from 20 to 23, from the third span results in 3 strands from the third span to terminate in the second span. The terminated three strands are dead-ended in the second span. The dead end is located at a distance $0.20 * L$ from the right support, at the centroid of the design strip (Fig. C6.4-2). The tendons are assumed horizontal over the support and concave downward toward the dead end. Hence the vertical balanced loads of these tendons will be downward, with a concentrated upward force at the dead end.

$$a = 3.75''$$

$$c = 0.20 * 32.75' = 6.55'$$

$$W_b = (3 * 26.77 * 2 * 3.75/12) / 6.55^2 = 1.170 \text{ klf} \downarrow$$

$$\text{Concentrated force at dead end} = 1.170 \text{ klf} * 6.55' = 7.66 \text{ k} \uparrow$$



Geometry and Actions of Tendons Terminated in Span

FIGURE C6.4-2

PT-induced moments due to shift in centroid:

Because the centroid of the design section at the face of the column drop, drop panel and the overhang strip is shifted from that of the uniform slab, there will be a moment due to axial force from prestressing at each of these locations. These moments must be included in the balanced loading to obtain a complete and correct solution. The moments are simply the post-tensioning force at each change in location of centroid multiplied by the shift in the section's centroid (see Fig. C6.4-3).

Moment at face of drop cap:

$$M = P * \text{shift in centroid} = P * (Y_{t-Left} - Y_{t-Right})$$

$$= 23 * 26.77 \text{ k} * (4.75'' - 5.80'') / 12 = -53.87 \text{ k-ft}$$

Moment at right support centerline:
 $M = 23 * 26.77 * (5.80 - 5.70) / 12 = 5.13 \text{ k-ft}$

❖ Span 3:

$a = 7.5''$

$L = 34.75'$

$W_b / \text{tendon} = 8 * P * a / L^2 = (8 * 26.77 * 7.5 / 12) / 34.75^2 = 0.111 \text{ klf}$

For 23 tendons, $W_b = 0.111 * 23 \text{ tendons} = 2.553 \text{ klf}$

% DL Balanced = $2.553 / 5.065 = 50\% \approx 60\% \text{ OK}$

Balanced load reaction, left = $2.553 \text{ klf} * 17.38' = 44.37 \text{ k} \downarrow$

Balanced load reaction, right = $2.553 \text{ klf} * 17.38' = 44.37 \text{ k} \downarrow$

PT-induced moments due to shift in centroid:

Moment at face of left column drop:

$M = P * \text{Shift in Centroid}$

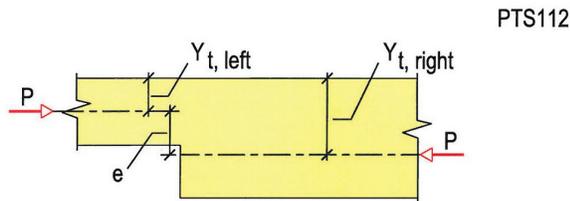
$= 23 * 26.77 \text{ k} * (5.70'' - 4.75'') / 12 = 48.74 \text{ k-ft}$

Moment at face of right drop panel:

$M = 23 * 26.77 * (4.75'' - 6.72'') / 12 = -101.08 \text{ k-ft}$

Moment at centerline of right support:

$M = 23 * 26.77 * (6.72'' - 6.75'') / 12 = -1.54 \text{ k-ft}$



Section at Change in Centroidal Axis

FIGURE C6.4-3

❖ Span 4:

Refer to Fig. C6.2-1

$a = 4.75'' - 1.00'' = 3.75''$

$b = 8.50'' - 1.00'' = 7.50''$

$L = 34.50'$

$c = \{ [3.75/7.50]^{0.5} / [1 + (3.75/7.50)^{0.5}] \} * 34.5' = 14.29'$

$W_b / \text{tendon} = 26.77 \text{ kips} * (2 * 3.75 / 12) / 14.29^2$

$= 0.082 \text{ klf / tendon}$

For 23 tendons, $W_b = 0.082 \text{ klf / tendon} * 23 \text{ tendons} = 1.886 \text{ klf}$

% DL Balanced = $(1.886 / 4.956) * 100 = 38\%$

The dead load in the fourth span tends to produce an upward “lift” on adjacent spans. Since the fourth span is next to a longer, more heavily loaded third span, it is advantageous to design the fourth span with a lower level of balanced loading and allow its non-prestressing load to counteract the actions in the adjoining longer span. For this reason, the level

of dead load balanced in the fourth span (38%) is acceptable, knowing that it is well below the target amount of 60% for the critical span. The above values will be assumed for a first try. If the stress check to follow will not be satisfactory the prestressing force will be adjusted.

Balanced load reaction, left = $1.886 \text{ klf} * 20.21' = 38.12 \text{ k} \downarrow$

Balanced load reaction, right = $1.886 \text{ klf} * 14.29' = 26.95 \text{ k} \downarrow$

Moment at left drop panel face:

$M = 23 * 26.77 * (6.75'' - 4.75'') / 12 = 102.62 \text{ k-ft}$

Moment at right beam face:

$M = 23 * 26.77 * (4.75'' - 8.75'') / 12 = -205.24 \text{ k-ft}$

❖ Cantilever:

Tendon is horizontal and straight. No dead load is balanced.

Moment due to dead end anchored away from centroid:

$M = 23 * 26.77 * (8.75'' - 4.75'') / 12 = 205.24 \text{ k-ft}$

There is no vertical force over the length of the cantilever from the tendon profile of this span. However, the eccentricity of the tendon at the edge of the slab results in a constant moment over the entire length of the cantilever.

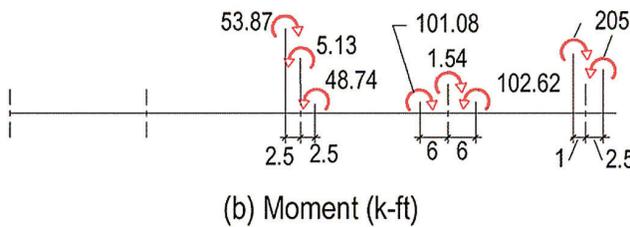
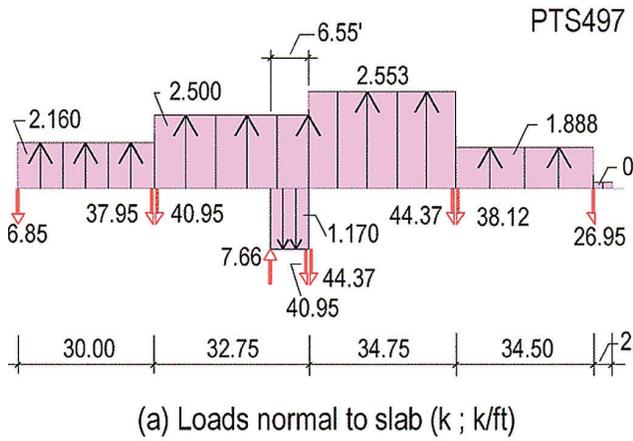
The complete balanced loading consisting of up and down forces (part “a” of the figure) and the associated moments (part “b” of the figure) are shown in Fig. 6.4-1. In addition to the forces shown in the figure, there is an axial compressive force that is shown in Fig. 6.3-1b.

The actions shown in Fig. 6.4-1 represent the forces from the simplified tendon profile assumed for hand calculation and shown in Fig. 6.4-2a. In construction where unbonded system is used, tendons in the design strip under consideration will be banded over the support line. In the perpendicular direction, the tendons will be distributed uniformly. The profile used for construction together with the one selected for hand calculation is shown in Fig. 6.4-2.

At this stage, before using the balanced loads to compute the post-tensioning moments, it is important to perform an equilibrium check, in order to verify the validity of the computed balanced loads. The balanced loads should satisfy static equilibrium. Otherwise, they are either incomplete, or incorrect.

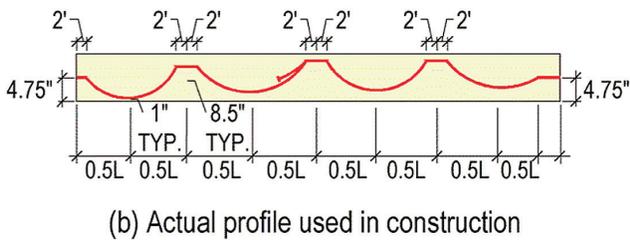
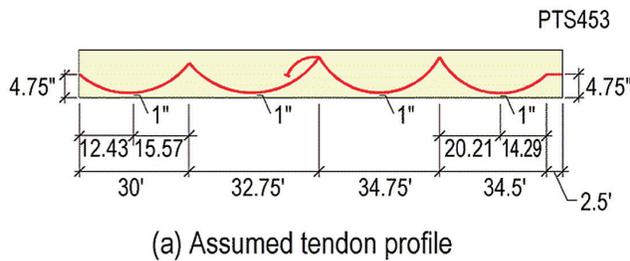
Equilibrium Check:

$\sum \text{Forces} \uparrow = -26.85 + (30 * 2.16) - 37.95 - 40.95 +$



Balanced Loading

FIGURE 6.4-1



Comparison of Simplified and Actual Tendon Profiles

FIGURE 6.4-2

$$(32.75 \times 2.50) - (6.55 \times 1.17) + 7.66 - 40.95 - 44.37 + (34.75 \times 2.553) - 44.37 - 38.12 + (34.5 \times 1.886) - 26.95 = -0.05 \text{ k} \approx 0 \text{ OK}$$

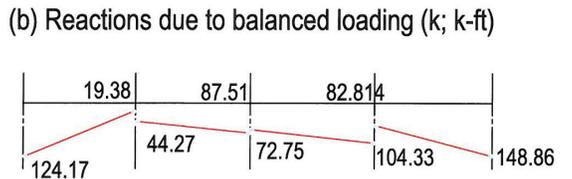
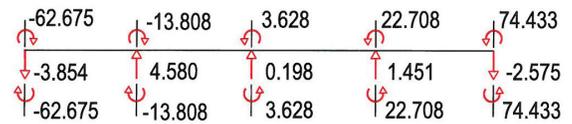
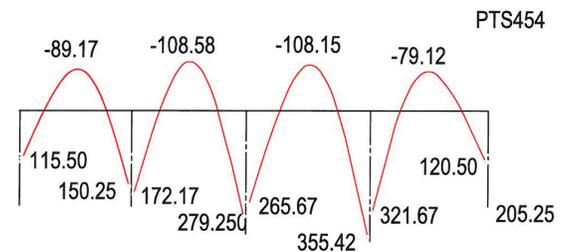
$$\sum M_{3rd \text{ Support}} = 26.85 \times 62.75 + (37.95 + 40.95) \times 32.75$$

$$- 7.66 \times 6.55 - 2.16 \times 30 \times 47.75 - (2.50 \times 32.75^2 / 2) + (2.553 \times 34.75^2 / 2) + (1.170 \times 6.55^2 / 2) + 1.886 \times 34.5 \times 52 - (44.37 + 38.12) \times 34.75 - 6.95 \times 69.25 - 53.87 + 5.13 + 48.74 - 101.08 - 1.54 + 102.62 - 205.24 + 205.24 = 0.96 \text{ k-ft} \approx 0 \text{ OK}$$

6.5 Determination of Actions due to Balanced (Post-Tensioning) Loads

The distribution of post-tensioning moments due to balanced loading, and the corresponding reactions at the slab/support connections, are shown in Fig. 6.5-1. These actions are obtained by applying the balanced loads shown in Fig. 6.4-1 to the frame shown in Fig. 1.4-2.

Actions due to post-tensioning are calculated using a standard frame program. The input geometry and boundary conditions of the standard frame program are the same as used for the dead and live loads.



Post-Tensioning Actions on Design Strip

FIGURE 6.5-1

7 - CODE CHECK FOR SERVICEABILITY

7.1 Load Combinations

The following lists the recommended load combinations of the building codes covered for serviceability limit state (SLS).

❖ [ACI, IBC]

Total load condition: $1*DL + 1*LL + 1*PT$

Sustained load condition: $1*DL + 0.3*LL + 1*PT$ ²⁰

❖ [EC2]

Frequent load condition: $1*DL + 0.5*LL + 1*PT$ ²¹

Quasi-permanent load condition: $1*DL + 0.3*LL + 1*PT$

For serviceability, the actions from the balanced loads due post-tensioning (PT) (Fig. 6.5-1a) are used. The background for this is explained in detail in reference [Aalami, 1990].

7.2 Stress Check

For hand calculation, the critical locations for stress check are selected using engineering judgment. The selected locations may or may not coincide with the locations of maximum stress levels. This will introduce a certain degree of approximation in design, which reflects the common practice for hand calculations. Computer-based solutions generally calculate stresses at multiple locations along a span, thus providing greater accuracy. For brevity, only three locations will be selected for this design example (Fig. 7.2-1). Point A is at the face-of-support; Point B is at the face of the drop panel; and point C is at the midspan.

Using the moment diagrams of Fig. 5-1 and 5-2 as guide, several critical locations are identified for the stress check. These are shown as sections A, B and C in Fig. 7.2-1.

Stresses:

$$\sigma = (M_D + M_L + M_{PT})/S + P/A$$

$$S = I/Y_c$$

Where, M_D , M_L , and M_{PT} are the moments across the entire tributary of the design strip. S is the section modulus of the entire tributary; A is the cross-sectional area of the entire tributary; I is the second moment of area of the entire tributary; and Y_c is the distance of the centroid of the entire tributary to the farthest tension fiber of the tributary section.

²⁰ ACI 318 specifies a “sustained” load case, but does not stipulate the fraction of live load to be considered “sustained.” It is left to the judgment of the design engineer to determine the applicable fraction. The fraction selected varies between 0.2 and 0.5. The most commonly used fraction is 0.3, as it is adopted in this design example.

²¹ The fraction of live loads in the combination depends on the occupancy of the floor system. The values selected are for common commercial and residential residencies

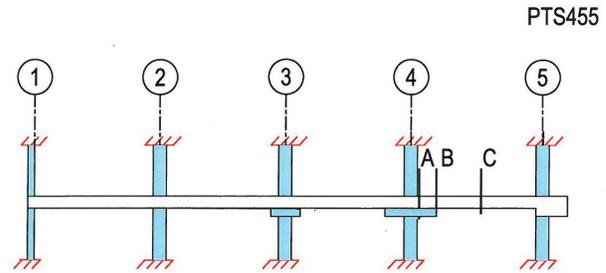


FIGURE 7.2-1

The parameters for stress check at point A are:

$$S_{top} = 103287/6.75 = 15302 \text{ in}^3 (2.508e+8 \text{ mm}^3)$$

$$S_{bot} = 103287/10.75 = 9608 \text{ in}^3 (1.574e+8 \text{ mm}^3)$$

$$A = 34*12*9.5 + 144 * 8 = 5028 \text{ in}^2 (3.244e+6 \text{ mm}^2)$$

$$P/A = -615.71*1000/5028 = -122 \text{ psi } (-0.84 \text{ MPa})$$

A. Based on ACI 318-11/IBC 2012:

Stress checks are performed for the two load conditions of total load and sustained load.

At point A:

❖ Total load combination:

$$\sigma = (M_D + M_L + M_{PT})/S + P/A$$

$$\text{Stress limit in compression: } 0.60*5000 = 3000 \text{ psi}$$

$$\text{Stress limit in tension: } 6*\sqrt{5000} = 424 \text{ psi}$$

$$M_D + M_L + M_{PT} = (-615.08 - 124.0 + 321.67) = -417.41 \text{ k-ft}$$

Bottom fiber:

$$\sigma = -417.41*12 *1000/9608 - 122 = -643 \text{ psi Compression} < -3000 \text{ psi OK}$$

Top fiber:

$$\sigma = 417.41 *12 *1000/15302 - 122 = 205 \text{ psi Tension} < 424 \text{ psi OK}$$

❖ Sustained load combination:

$$\text{Stress limit in compression: } 0.45*5000 = -2250 \text{ psi}$$

$$\text{Stress limit in tension: } 6*\sqrt{5000} = 424 \text{ psi}$$

$$M_D + 0.3 M_L + M_{PT} = (-615.08 - 0.3*124.0 + 321.67) = -330.61 \text{ k-ft}$$

Bottom fiber:

$$\sigma = -330.61 *12 *1000/9608 - 122 = -535 \text{ psi Compression} < -2250 \text{ psi OK}$$

Top fiber:

$$\sigma = 330.61 *12*1000/15302 - 122 = 137 \text{ psi tension} < 424 \text{ psi OK}$$

B. Based on EC2:

Stress checks are performed for the two load conditions of frequent load and quasi-permanent load.

At point A:

❖ Frequent load condition:

$$\sigma = (M_D + 0.5 M_L + M_{PT})/S + P/A$$

Stress thresholds:

$$\text{Compression} = 0.60 * 34.47 = -20.68 \text{ MPa (3000 psi)}$$

$$\text{Tension} = f_{ctm} = 3.18 \text{ MPa (461 psi)}$$

$$M_D + 0.5 M_L + M_{PT} = (-833.93 - 0.5 * 168.12 + 436.12) = -481.87 \text{ kN-m (-355.41 k-ft)}$$

Top:

$$\sigma = 481.87 * 1000^2 / 2.508e+8 - 0.84 \text{ MPa}$$

$$= 1.08 \text{ MPa (155 psi) Tension} < 3.18 \text{ MPa (461 psi) OK}$$

Bottom:

$$\sigma = -481.87 * 1000^2 / 1.574e+8 - 0.84$$

$$= -3.90 \text{ MPa (-566 psi) Compression} < -20.68 \text{ MPa (3000 psi) OK}$$

❖ Quasi-permanent load condition:

$$\sigma = (M_D + 0.3 M_L + M_{PT})/S + P/A$$

Stress thresholds:

$$\text{Compression} = 0.45 * 34.47 = -15.51 \text{ MPa (2250 psi)}$$

$$\text{Tension} = f_{ctm} = 3.18 \text{ MPa (461 psi)}$$

$$M_D + 0.3 M_L + M_{PT} = (-833.93 - 0.3 * 168.12 + 436.12) = -448.25 \text{ kN-m (-330.61 k-ft)}$$

Top:

$$\sigma = 448.25 * 1000^2 / 2.508e+8 - 0.84 \text{ MPa}$$

$$= 0.95 \text{ MPa (137 psi) Tension} < 3.18 \text{ MPa (461 psi) OK}$$

Bottom:

$$\sigma = -448.25 * 1000^2 / 1.574e+8 - 0.84$$

$$= -3.69 \text{ MPa (535 psi) Compression} < -15.51 \text{ MPa (2250 psi)}$$

Other points are evaluated in a similar manner. The outcome is listed in the following table (Table 7.2-1):

The following illustrates the calculation of moments at interior of a span, such as point B for span under consideration.

Centerline moments and shears for DL, LL and PT obtained from frame analysis, along with the externally applied loads are shown below for the fourth span. The calculation of the values at the face-of-support follows simple statics of the free-body diagram shown below. In the following the calculation of moment at the face of drop panel in the fourth span is detailed. Other locations follow a similar procedure (Fig. C7.2-1).

Moment due to DL at the face of drop panel distance 6ft from the fourth support

Moment due to DL

$$M_{DL} = 80.01 * 28.5 - 388.25 - 3.4 * 28 - 4.956 * 28.5^2 / 2 = -215.92 \text{ k-ft}$$

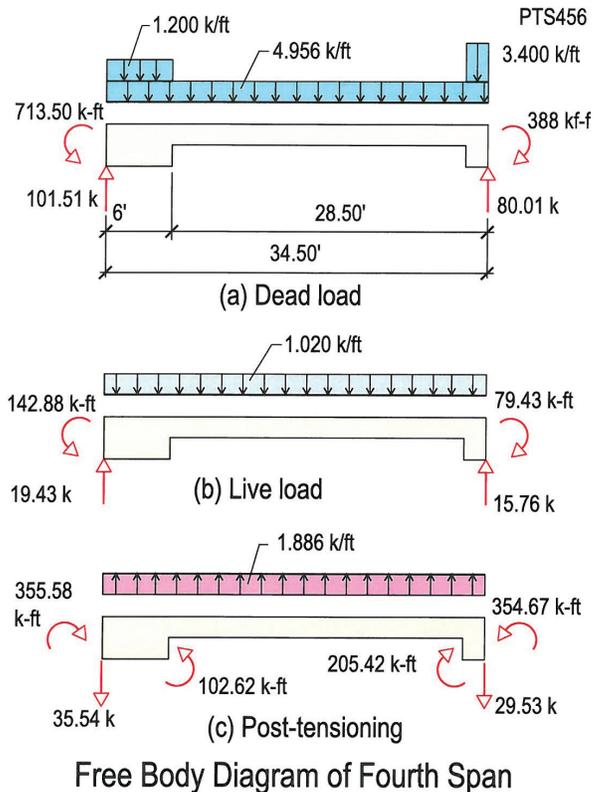


FIGURE C7.2-1

Moment due to LL

$$M_{LL} = 15.76 * 28.5 - 79.43 - 1.02 * 28.5^2 / 2 = -44.52 \text{ k-ft}$$

Moment due to PT

$$M_{PT} = -29.53 * 28.5 + 354.67 + 1.886 * 28.5^2 / 2 - 205.42 + 102.62 = 176.22 \text{ k-ft}$$

7.3 Crack Width Control

A. Based on ACI 318-11/IBC 2012:

ACI 318-11/IBC 2012 do not stipulate specific measures to follow for crack control of slabs designed as two-way systems. The limit imposed on tensile stresses keeps the slabs essentially crack free, when in service.

B. BASED ON EC2²²:

The allowable crack width for unbonded tendon (Quasi-permanent load combination) is 0.3 mm and for bonded tendon (Frequent load combination) is

0.2 mm. Since in this example the maximum computed tensile stress is within the threshold limit, crack width calculation is not required. If the computed tensile stress exceeds the threshold, EC2 rec-

²² EN 1992-1-1:2004(E), Section 7.3.3

TABLE 7.2-1 Summary of Service Stress Checks (T160US)

Span 4		Section A	Section B	Section C
Based on ACI 318-11/IBC 2012				
Sustained load	f_t (psi)	137	-54	-414
	f_b (psi)	-535	-262	98
	F_t (psi)	424	-2250	-2250
	F_b (psi)	-2250	-2250	424
		OK	OK	OK
Total load	f_t (psi)	205	7	-470
	f_b (psi)	-643	-323	153
	F_t (psi)	424	424	3000
	F_b (psi)	3000	3000	424
		OK	OK	OK
Based on EC2				
Frequent Load	f_t (psi) (MPa)	155 (1.08)	-38 (-0.26)	-429 (-2.96)
	f_b (psi) (MPa)	-566 (-3.90)	-278 (-1.92)	113 (0.78)
	F_t (psi) (MPa)	461 (3.18)	-3000 (-20.68)	-3000 (-20.68)
	F_b (psi) (MPa)	-3000 (-20.68)	-3000 (-20.68)	461 (3.18)
		OK	OK	OK
Quasi-Permanent Load	f_t (psi) (MPa)	137 (0.95)	-55 (-0.38)	-413 (-2.85)
	f_b (psi) (MPa)	-535 (-3.69)	-261 (-1.80)	97 (0.67)
	F_t (psi) (MPa)	461 (3.18)	-2250 (-15.51)	-2250 (-15.51)
	F_b (psi) (MPa)	-2250 (-15.51)	-2250 (-15.51)	461 (3.18)
		OK	OK	OK

Note: F_t and F_b are the respective top and bottom fiber allowable (threshold) stresses; F_c is allowable compressive stress.

ommends to limit the bar diameter and bar spacing to the values given in Table 7.2N or 7.3N of EC2 to control the width of probable cracks. The following example illustrates the point. An alternative to the selection of the amount and size of the bars from Tables 7.2N or 7.3N is to calculate the crack width and determine the bar area needed to bring the crack width to the design value. An example for this alternative is given in Chapter 7.

EXAMPLE 1

To illustrate the procedure for crack control recommended in EC2, as an example, let the maximum tensile stress exceed the threshold value by a large margin.

Given: computed hypothetical farthest fiber tensile stress in concrete $f = 30\text{MPa}$ (4,440 psi)

Required: reinforcement design for crack control

Calculate stress in steel at location of maximum concrete stress: $\sigma_s = (f/E_c) \cdot E_s$
Where, f is the hypothetical tensile stress in concrete under service condition.

$\sigma_s = (30/33951) \cdot 200000 = 177\text{ MPa}$ (262 psi) (this is a hypothetical value)

Crack spacing can be limited by either restricting the bar diameter and/or bar spacing.

Use the maximum bar spacing from Table 7.3 N for the σ_s of 177 MPa.

From Table, for 160 MPa - 300 mm (12in)
200 MPa - 250 mm (10 in)

By interpolation, maximum spacing for 177 MPa (262 psi) is 278 mm (11 in).

Limit the spacing of reinforcement to 275 mm (11 in) or less (270 mm (11 in)) in order to control cracking. Note that based on the magnitude of the computed tensile stress in concrete the minimum area of the required reinforcement is calculated separately under minimum requirements,

7.4 Minimum Reinforcement

There are several reasons why the building codes specify a minimum reinforcement for prestressed members. These are:

- ❖ Crack control, where potential of cracking exists: Bonded reinforcement contributes in controlling local cracks. The contribution of bonded reinforcement to crack control is gauged by the stress it develops under service load. Change of force in bonded reinforcement from applied strain is a function of its modulus of elasticity and its cross-sectional area. Hence, the area of reinforcement to be considered available for crack control is $(A_s + A_{ps})$, where A_{ps} is the area of bonded tendons. It is recognized that both bonded and unbonded prestressing provide precompression. While the physical presence of an unbonded tendon may not contribute to crack control, the contribution through the precompression it provides does. However, for code compliance and conformance with practice, the contribution of unbonded tendons is not included in the aforementioned sum.

- ❖ Ductility: One reason ACI 318 specifies a minimum bonded reinforcement over supports of members reinforced with unbonded tendons is to provide increased ductility at the location. Where unbonded tendons are used, the required minimum area is provided through A_s only. Current ACI 318/IBC do not specify a minimum for non-prestressed bonded reinforcement in post-tensioned members reinforced with bonded tendons.

- ❖ Cracking moment larger than moment capacity: Where cracking moment of a section is likely to exceed its capacity in flexure, reinforcement is added to raise the moment capacity. In such cases, the contribution of each reinforcement is based on the strength it provides. If the minimum value is expressed in terms of cross-sectional area of reinforcement, the applicable value is: $(A_s + A_{ps} \times f_{py} / f_y)$.

Use #5 (16 mm) bars [Area = 0.31 in² (201 mm²); Diameter = 0.625 in (16 mm)] for top

Use #6 (19mm) bars [Area = 0.44 in² (284 mm²); Diameter = 0.75 in (19 mm)] for bottom

A. Based on ACI 318-11/IBC 2012:

- ❖ Unbonded tendons
- ❖ Supports

ACI 318²³/IBC require a minimum area of passive (non-prestressed) reinforcement to be placed over the supports, where unbonded tendons are used. The minimum area is expressed in terms of the cross-sectional geometry of the design strip, and the strip orthogonal to it. A_{cf} is the larger gross cross-sectional area of the design strips in the two orthogonal directions for the support under consideration. Figure C7.4-1 illustrates the applicable locations to determine the cross-sectional areas. Line PP refers to the section in the design strip direction and FF to the section orthogonal to it.

Note that the enlargement of area due to drop panel is not considered in the minimum rebar calculations.

$$A_s = 0.00075 * A_{cf}$$

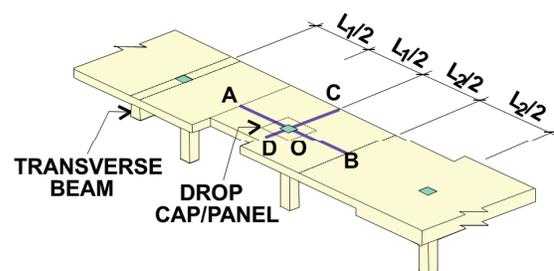
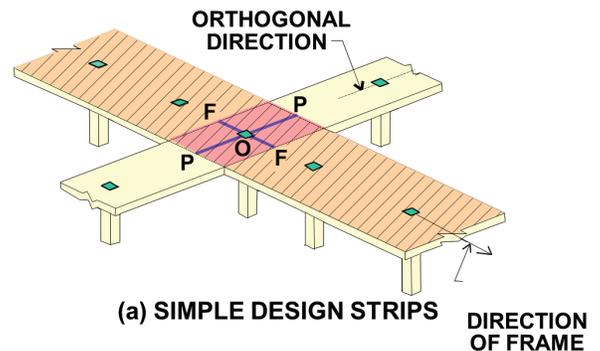
At section A (Fig. 7.2-1):

In direction of design strip:

$$A_s = 0.00075 * A_{cf}$$

$$= 0.00075 * 0.5 * (34.75 * 9.5 + 34.50 * 9.5) * 12 = 2.96 \text{ in}^2$$

In the orthogonal direction to the design strip the spans adjacent to the support under consideration are



(b) EXAMPLE OF A PRACTICAL DESIGN STRIP

FIGURE C7.4-1 Identification of Sections for Minimum Rebar Computation (P563)

²³ ACI 318-11, Section 18.9.3

34.75' and 34'. Hence,

$$A_s = 0.00075 * 0.5 * (34.75 * 9.5 + 34 * 9.5) * 12 = 2.94 \text{ in}^2$$

Use 10 - #5 bars = 10 * 0.31 in² = 3.1 in² > 2.96 in² provided top

❖ Spans

The minimum non-prestressed reinforcement at midspan for unbonded tendons depends on the value of computed (hypothetical) tension at the bottom fiber. If the hypothetical tension stress is less than $2\sqrt{f'_c}$, based on ACI 318²⁴, no minimum bottom rebar for span is required. It is reiterated that the computed tensile stress is not permitted to exceed $6\sqrt{f'_c}$.

At point C in span:

At midspan $A_s = N_c / (0.5 * f_y)$ if hypothetical tensile stress $> 2\sqrt{f'_c}$

Where, N_c is the total of tension force in the tensile zone of the section

Computed hypothetical tensile stress: $f_{ct} = 153 \text{ psi}$

Stress limit = $2\sqrt{f'_c} = 2\sqrt{5000} = 141 \text{ psi}$

$153 \text{ psi} > 141 \text{ psi} \therefore$ Minimum steel is required in positive moment region.

Compressive stress at top, $f_{cc} = -470 \text{ psi}$

Using $h = 9.5''$;

Distance of tension zone from bottom = $153 * 9.5 / (153 + 470) = 2.33''$

$N_c = 2.33'' * 153 \text{ psi} * 34 * 12 / (2 * 1000) = 72.72 \text{ kips}$

$A_s = 72.72 \text{ kips} / (0.5 * 60 \text{ ksi}) = 2.42 \text{ in}^2$

Number of bars = $2.42 / 0.44 = 5.51$

Use 6 - #6 bars = $6 * 0.44 \text{ in}^2 = 2.64 \text{ in}^2$ provided bottom.

A larger bar size is typically specified for bottom steel in the banded direction. The larger size is selected to reduce the number of bottom bars. In the banded direction, bottom bars are typically placed within the width of the tendon band.

❖ Bonded (grouted) Tendons

There is no requirement for minimum reinforcement based on either geometry of the design strip, nor its hypothetical tensile stresses. The minimum requirement is handled through the relationship between the cracking moment of a section and its nominal strength in bending. This is handled in the "strength" check of the member (section B of this example). The code check for strength adequacy after the initiation of first crack is carried out in the strength design (ULS) section.

B. Based on EC2²⁵:

EC2 specifies the same requirement for the minimum reinforcement at supports and spans, and also the same for both unbonded and bonded tendons. Two checks apply. One is based on the cross-sectional geometry of the design strip and its material properties; the other on computed stresses. In the former, the minimum reinforcement applies to the combined force contributions of stressed and non-prestressed reinforcement. Hence, the participation of each is based according to the strength it provides, the prestressing steel is accounted for with higher values. The reinforcement requirement for crack control is dealt with separately.

❖ Support

At section A (Fig. 7.2-1):

$$A_{smin} \geq (0.26 * f_{ctm} * b_t * d / f_{yk}) \geq 0.0013 * b_t * d$$

Since in EC2 the minimum reinforcement is a function of local ($b_t * d$) cross-sectional area, at the face-of-support the cross-sectional area including the drop panel is used.

Cross-sectional area;

$b_t = 34 \text{ ft} (10.36 \text{ m})$

Drop panel width = 144 in (3658 mm)

Drop panel depth below slab $g = 8 \text{ in} (203 \text{ mm})$

Tributary cross-sectional area = $34 * 12 * 9 + 144 * 8 = 4824 \text{ in}^2 (3.112 * 10^6 \text{ mm}^2)$

$f_{ctm} = 0.3 * 34.47^{(2/3)} = 3.18 \text{ MPa} (461 \text{ psi})$

(i) $A_{smin} = 0.26 * f_{ctm} * b_t * d / f_{yk} = 0.26 * 3.18 * 3.112 * 10^6 /$

PTS114

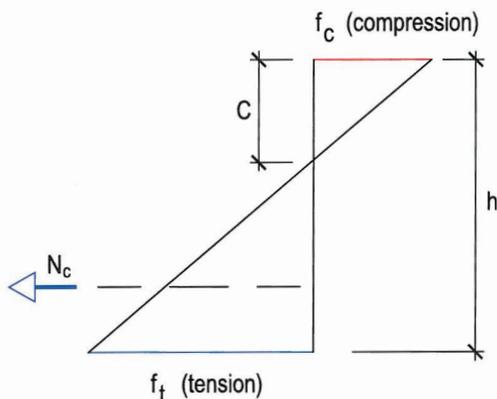


FIGURE 7.4-1 Distribution of Strain over Section of Member (PTS114)

²⁴ ACI 318-11 Section 18.9.3.2

²⁵ EN 1992-1-1:2004(E), Section 9.3.1 & 7.3.2

413.69 = 6,220 mm² (9.64 in²)
 (ii) $A_{smin} = 0.0013 * b_t * d = 0.0013 * 3.112 * 10^6 = 4046$
 mm² (6.27 in²)

Therefore, $A_{smin} = 6,220$ mm² (9.64 in²)
 Contribution of reinforcement from bonded prestressing:
 $A_{ps} * (f_{pk}/f_{yk}) = 23 * 0.153 * 270/60 = 15.84$ in² > 9.64 in²
 Hence, no additional bonded reinforcement is required.

❖ **Span**

At section C in span (Fig. 7.2-1):
 $b_t = 34$ ft (10.36 m)
 $d = 241$ mm (9.50in)²⁶
 $f_{ctm} = 0.3 * 34.47(2/3) = 3.18$ MPa (461 psi)
 (i) $A_{smin} = 0.26 * f_{ctm} * b_t * d / f_{yk} =$
 $0.26 * 3.18 * 10.36 * 10^3 * 241 / 413.69 = 4990$ mm² (7.73 in²)

(ii) $A_{smin} = 0.0013 * b_t * d = 0.0013 * 10.36 * 10^3 * 241 =$
 3246 mm² (5.03in²)

Therefore, $A_{smin} = 4990$ mm² (7.73 in²)
 Contribution of reinforcement from bonded Prestressing:
 $A_{ps} * (f_{pk}/f_{yk}) = 23 * 0.153 * 270/60 = 15.84$ in² > 7.70 in²
 Hence, no additional bonded reinforcement is required.

❖ **Minimum reinforcement for crack control**

In EC2, the necessity of reinforcement for crack control is triggered, where computed tensile stresses exceed a code-specified threshold.

At all the three locations selected for code compliance, the hypothetical tensile stress of concrete is below the threshold for crack control. Hence, no crack control reinforcement is required

EXAMPLE 2

For demonstration of EC2²⁷ procedure for crack control, let the maximum hypothetical tensile stress in concrete exceed the threshold set in the code (3.21 MPa). Determine the required crack control rein-

forcement for the section reinforced with unbonded tendons.²⁸

Given:

$f_b = 3.5$ MPa (508 psi) (tension at bottom)
 $f_t = 5.2$ MPa (754psi) (compression at top)
 Depth of section = 241 mm (9.5 in)
 Width of section = 10,360 mm (34 ft)
 Required: Reinforcement for crack control
 $\sigma_s = f_{yk} = 413.69$ MPa (60 ksi)
 $k = 1$
 Depth of tension zone at bottom, using Fig. 7.4-1 =
 $3.5 * 241 / (3.5 + 5.2) = 97$ mm (3.82 in)
 $A_{ct} = 97 * 10360 = 1.004e+6$ mm²
 $k_c = 0.4 * [1 - (\sigma_c / (k_1 (h/h^*) f_{ct,eff}))]$
 $\sigma_c = N_{ED} / bh = 1.10$ MPa (average precompression)
 $h^* = 241$ mm (9.5 in)
 $k_1 = 1.5$ (since section is in compression)
 $f_{ct,eff} = f_{ctm} = 0.3 * (34.47)^{(2/3)} = 3.18$ MPa (461 psi)
 $k_c = 0.4 * [1 - (1.10 / (1.5 (241/241) 3.18))] = 0.31$
 $A_{smin} = k_c k f_{ct,eff} A_{ct} / \sigma_s$
 $A_{smin} = 0.31 * 1 * 3.18 * 1.004e+6 / 413.69 = 2375$ mm²
 (3.68 in²)

The minimum rebar required from different codes is summarized in TABLE 7.4-1

7.5 Deflection Check

The accurate determination of probable deflection is complex (see Chapter 4; Section 4.10.6). Further, once a value is determined, the judgment on its adequacy at design time is subjective, and depends on a number of construction variables - such as age of concrete at time of installation of nonstructural members that are likely to be damaged from deflection. In common construction, deflection compliance is generally based on recommended deflection to

TABLE 7.4-1 Summary of Minimum Rebar (in²) (T161US)

Code	Unbonded		Bonded	
	Support	Span	Support	Span
ACI/IBC	2.96	2.42	0	0
EC2	0	0	0	0

Note: Support is point A; span is point C

²⁶ It is questionable, whether in this relationship the effective depth, rather than total depth, should be used. The argument for using the total depth relies on the origin of the relationship, being based on the cross-sectional geometry, as contrasted to reinforcement amount and distribution

²⁷ EN 1992-1-1:2004(E), Section 7.3.2(3)

²⁸ For members reinforced with grouted tendons, the cross-sectional area of grouted tendons can be used to contribute to the minimum required area for crack control.

span ratios. The ratios were discussed in Section 4.5 of this example and are summarized below:

(i) For visual and functional effects, total long-term deflection from the day supporting shutters are removed not to exceed (span/250 EC2) and (span/240 USA). Camber can be used to offset the impact of displacement.

(ii) Immediate deflection under design live load not to exceed (span/500 for EC2) or (span/480 for USA).

Both ACI 318/IBC and EC2, tie the acceptability of deflection to displacement subsequent to the installation of members that are likely to be damaged. This requires knowledge of construction schedule and release of structure for service.

For engineering assessment of long-term displacement, ACI 318 recommends a multiplier factor of 2²⁹.

Deflections are calculated using a frame analysis program for each of the load cases: dead, live and post-tensioning. Gross cross-sectional area and linear elastic material relationship are used.. Point C at the middle of span 4 is selected for deflection check. The values for this point are as follows:

- Span 4 deflection (from frame analysis)
- Dead load = 0.20 in.
- Post-tensioning = -0.08 in.
- Dead load + PT = 0.12 in.
- Live load deflection = 0.04 in.
- There is no topping on the finished slab.

❖ Long-term deflection
Multiplier factor assumed for effects of creep and shrinkage on long-term deflection = 2³⁰
Load combination for long-term deflection, using a factor of 0.3 for sustained (quasi-permanent) live load :
(1.0*DL + 1.0*PT + 0.3*LL)*(1 + 2)
Long-term deflection: (1 + 2)*(0.12 + 0.3*0.04) = 0.40 in
Deflection ratio = 0.40/(34.50*12) = 1/1035 < 1/250 OK

❖ Instantaneous deflection due to design live load:
Live load deflection = 0.04 in.
Deflection ratio = 0.04/(34.50*12) = 1/10350 OK

Deflection does not generally govern the design for members dimensioned within the limits of the recommended values in ACI 318 and balanced within

the recommended range, and when subject to loading common in building construction. For such cases, deflections are practically always within the permissible code values.

8 - CODE CHECK FOR STRENGTH

8.1 Load Combinations

❖ ACI 318/IBC
1.2*DL + 1.6*LL + 1*Hyp
1.4*DL + 1*HYP
❖ EC2
1.35*DL + 1.5*LL + 1 * Hyp

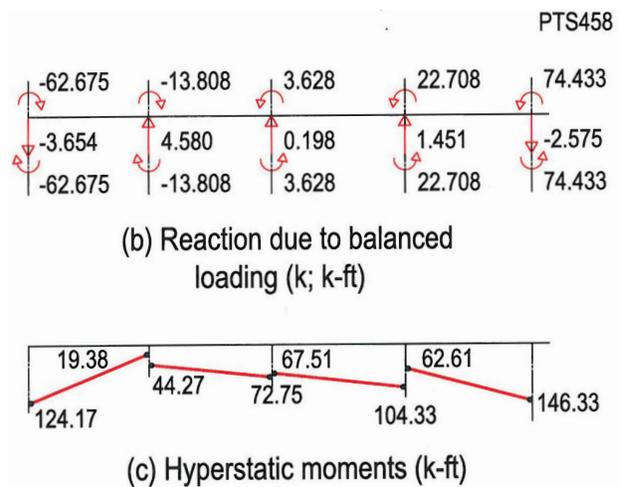
For strength combination, the hyperstatic (Hyp) actions due to prestressing are used. The background for this in Chapter 4, Section 4.11.2.

8.2 Determination of Hyperstatic Actions

The hyperstatic moments are calculated from the reactions of the frame analysis under balanced loads from prestressing (Loads shown in Fig. 6.4-1). The reactions obtained from a standard frame analysis are shown in Fig. 8.2-1(a). The reactions shown produce hyperstatic moments in the frame as shown in Fig. 8.2-1(b).

The hyperstatic (secondary) reactions must be in self-equilibrium, since the applied loading (balanced loads) are in self-equilibrium.

Check the validity of the solution for static equilibrium of the hyperstatic actions using the reactions shown in Fig. 8.2-1a:



Post - Tensioning Actions on Design Strip

FIGURE 8.2-1

²⁹ ACI 318-11 R9.5.2.5

³⁰ ACI 318 multiplier factor

$$\Sigma \text{ Vertical forces} = -3.65 + 4.58 + 0.20 + 1.45 - 2.58 = 0 \quad \text{OK}$$

$$\Sigma \text{ Moments about support 1} = -62.675 * 2 - 13.808 * 2 + 3.628 * 2 + 22.708 * 2 + 74.433 * 2 + 4.580 * 30 + 0.198 * 62.75 + 1.451 * 97.5 - 2.575 * 132 = -0.031 \text{ k-ft} \approx 0 \quad \text{OK}$$

8.3 Calculation of Design Moments

The design moment (M_u) is a factored combination of dead, live and hyperstatic moments.

Using ACI/IBC Design moments are :

$$M_{U1} = 1.2 * M_D + 1.6 * M_L + 1.0 * M_{HYP}$$

$$M_{U2} = 1.4 * M_D + 1.0 * M_{HYP}$$

By inspection, the second load combination does not govern, and will not be considered in the following.

The factored moments for the codes considered are listed in the following table.

8.4 Strength Design for Bending and Ductility

The strength design for bending consists of two provisions, namely

- ❖ The design capacity shall exceed the demand. A combination of prestressing and non-stressed steel provides the design capacity
- ❖ The ductility of the section in bending shall not be less than the limit set in the associated building code. The required ductility is deemed satisfied, if failure of a section in bending is initiated in post-elastic response of its reinforcement, in contrast to crushing of concrete. For the codes covered in this example this is achieved through the limitation imposed on the depth of the compression zone (see Fig. C8.4-1). The depth of compression zone is generally limited

to 50% or less than the distance from the compression fiber to the farthest reinforcement (d_r). Since the concrete strain (ϵ_c) at crushing is assumed between 0.003 and 0.0035, the increase in steel strain (ϵ_s) will at minimum be equal to that of concrete at the compression fiber. This will ensure extension of steel beyond its yield point (proof stress) and hence a ductile response.

For expeditious hand calculation, the flexural capacity of a post-tensioned member in common building structures can be approximated by assuming a conservative maximum stress for prestressing tendons. For detailed application of the code-proposed formulas refer to Chapter 12; Section 12.2. Strain compatibility for the calculation of section capacity is the preferred option, but its application for hand calculation is not warranted in routine designs.

There are two reasons, why a simplified method for ULS design of post-tensioned sections in daily design work are recommended. These are:

- ❖ Unlike conventionally reinforced concrete, where at each section along a member non-stressed reinforcement must be provided to resist the design moment; in prestressed members this may not be necessary, since prestressed members possess a positive and negative base capacity along the entire length of the prestressing tendons (Fig. C8.4-2b). Non-stressed reinforcement is needed at sections, where the moment demand exceeds the base capacity of the section.
- ❖ In conventionally reinforced concrete, the stress used for rebar at ULS is well-defined as f_y . For prestressed sections, however, the stress in tendon at ULS is expressed in terms of a somewhat involved

TABLE 8.3-1 Factored (Design) Moments (T162)

Span 4	Point A	Point B	Point C
M_D k-ft	-615.08	-215.92	197.90
M_L k-ft	-124.00	-44.52	40.60
M_{HYP} k-ft	62.61	75.50	104.47
ACI 318-11/IBC 2012 : 1.2DL+1.6LL+1Hyp			
M_{U1} k-ft	-873.89	-254.84	406.91
EC2 : 1.35DL+1.5LL+1Hyp			
M_{U1} (k-ft) (kN-m)	-953.74 (-1293.10)	-282.77 (-383.38)	432.53 (586.43)

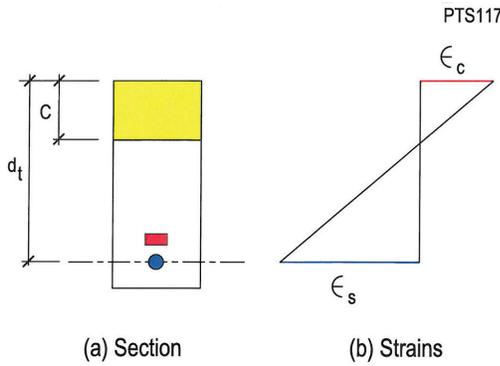


FIGURE C8.4-1 Distribution of Strain over Section

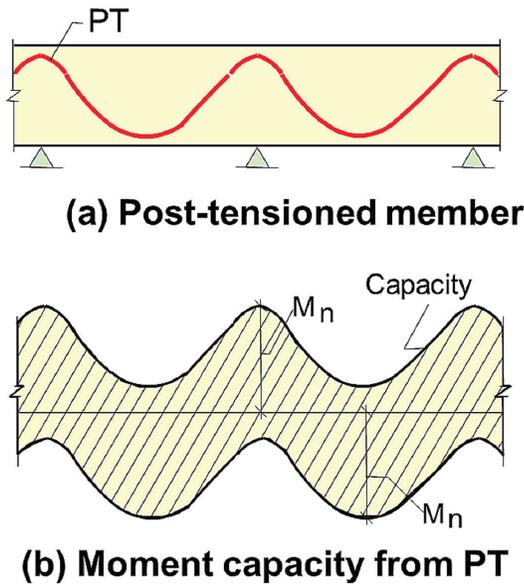


FIGURE C8.4-2 Demand and Capacity Moments (P500)

relationship, compared to RC design - hence the tendency to use a simplified, but conservative alternative for routine hand calculations. For repetitive work, computer programs are recommended.

The simplified procedure for section design is explained in Chapter 12, Section 12.3.2.

Briefly, for an expedient hand calculation, the capacity of a post-tensioned member can be approximated for common building structures by assuming an ultimate stress for the prestressing tendons. The approximate solution applies at locations where tendons are closest to member surface ($d_p \sim h$), and the following conditions are satisfied.

- ❖ $f'_c \geq 4000$ psi (27.58 MPa);
- ❖ $P/A \leq 250$ psi (1.72 MPa);

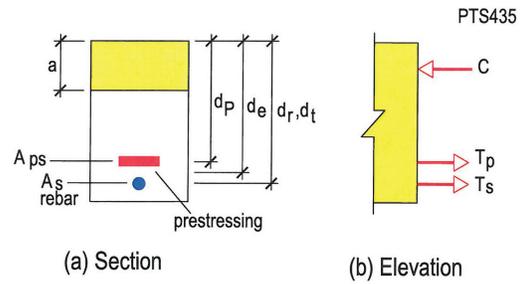


FIGURE C8.4-3 Distribution of Basic Forces on a Rectangular Section at ULS

- ❖ $c/d_t \leq 0.375$, where d_t is the distance from the compression fiber to the farthest location designated for reinforcement; and
- ❖ Tendon Length $\leq 125'$ (38.10m) for single end stressing
 $\leq 250'$ (76.20 m) for double end stressing.

Unbonded tendons

Tendon stress (f_{ps}) for strength can be approximated to be 215 ksi (1482 MPa) if span to depth ratio does not exceed 35 ; and 195 ksi (1344 MPa) if span to depth ratio exceeds 35

Grouted tendons

Tendon stress (f_{ps}) for strength can be approximated to be 255 ksi (1758 MPa)
 Figure C8.4-3 illustrates the forces and dimensional parameters used in the calculations.

A. Based on ACI 318-11; IBC 2012:

Unbonded Tendon:

- ❖ At Point A (face of support):

$$A_{ps} = 23 * 0.153 = 3.52 \text{ in}^2$$

$$d_p \approx 15.80''$$

At centerline d_p is 16.50'' and at face of support it is calculated as 15.80''.

$$A_s = 2.96 \text{ in}^2 \text{ (from minimum computation)}$$

$$d_r = 17.5'' - 0.75'' - 0.625/2 = 16.44''$$

$$\text{Span}/d > 35.0', f_{ps} = 195 \text{ ksi}$$

$$\text{Total Tension Force} = T_p + T_{sr}$$

$$= 3.52 * 195 + 2.96 * 60 \text{ ksi}$$

$$= 686.4 + 177.60 = 864 \text{ kips}$$

$$a = 864 \text{ kips} / (144 * 0.85 * 5) = 1.41 \text{ in.}$$

$$c = 1.41 / 0.80 = 1.76''$$

$$d_t = 16.44''$$

$$c/d_t = 1.76 / 16.44 < 0.375 \text{ OK to use approximation}$$

$$\Phi = 0.90$$

$$\Phi M_n = 0.9 * [686.4 * (15.80 - 1.41/2) + 177.6 * (16.44 - 1.41/2)] / 12$$

$$= 986.68 \text{ k-ft} > M_u = 873.89 \text{ k-ft OK}$$

❖ At Point C (Midspan):

Similar computation to point A concludes that the strength provided by the existing reinforcement at point C exceeds the strength demand; hence $A_s = 2.42$ in² OK

In form of an example, at the end of the bonded tendon below, the computation for the case where the provided reinforcement is not adequate is outlined.

Bonded Tendon:

❖ At Point A (face of support):

$$A_{ps} = 23 * 0.153 = 3.52 \text{ in}^2$$

$$d_p \approx 15.59''$$

At centerline d_p is 16.25" and at face of support it is calculated as 15.59".

$$A_s = 0 \text{ in}^2 \text{ (From minimum computation)}$$

$$d_r = 17.5'' - 0.75'' - 0.625/2 = 16.44''$$

$$f_{ps} = 255 \text{ ksi}$$

$$\text{Total tension force} = T_p + T_{Sr}$$

$$= 3.52 * 255 \text{ ksi} + 0 * 60 \text{ ksi}$$

$$= 897.6 + 0 = 897.60 \text{ kips}$$

$$a = 897.60 \text{ kips} / (144'' * 0.85 * 5) = 1.47 \text{ in.}$$

$$c = 1.47 / 0.80 = 1.84''$$

$$d_t = 16.44''$$

$$c/d_t = 1.84/16.44 < 0.375 \text{ OK to use approximation}$$

$$\Phi = 0.90$$

$$\Phi M_n = 0.9 * [897.60 * (15.59 - 1.47/2)] / 12$$

$$= 1000 \text{ k-ft} > M_U = 873.89 \text{ k-ft OK}$$

❖ At Point C (Midspan):

$$A_{ps} = 3.52 \text{ in}^2$$

$$d_p \approx 8.25''$$

$$A_s = 0 \text{ in}^2$$

$$d_r = 9.5'' - 0.75'' - 0.75/2 = 8.38''$$

$$M_U = 406.91 \text{ k-ft}$$

Similar computation to point A concludes that the strength provided by the existing reinforcement exceeds the strength demand; hence no added rebar required

Example:

This example illustrates the simplified computation, where the provided rebar is not adequate.

Consider point C, and assume, using unbonded tendons and $M_u = 550$ k-ft

$$A_{ps} = 3.52 \text{ in}^2$$

$$d_p \approx 8.5'' ; d_r \approx 8.38''$$

$$A_s = 2.42 \text{ in}^2$$

$$\text{Total tension force} = 3.52 * 195 + 2.42 * 60 = 831.6 \text{ kips}$$

$$a = 831.6 / (0.85 * 5 * 408) = 0.48 \text{ in.}$$

$$c = 0.48 / 0.80 = 0.60 \text{ in.}$$

$$c/d_t = 0.60/8.5 < 0.375; \text{ use } \Phi = 0.90$$

$$\Phi M_n = 0.9 * [686.4 * (8.5 - 0.48/2) + 145.2 * (8.38 - 0.48/2)] / 12$$

$$= 513.87 \text{ k-ft} < M_u = 550 \text{ k-ft NG ; add supplemental rebar}$$

Since supplemental rebar must be added the depth of compression zone must be prorated to approximate the added compressive stress in the section.

$$\text{Prorated } a = 0.48 * 550 / 513.87 = 0.51''$$

$$M_{\text{supplemental}} = 550 - 513.87 = 36.13 \text{ k-ft}$$

$$A_{\text{supplemental}} = 36.13 / [0.90 * 60 * (8.38 - 0.51/2) / 12] = 0.99 \text{ in}^2$$

Since this is an approximate method for expeditious hand calculation, add 10% more to the supplemental rebar in lieu of iterating the solution.

$$A_s = 2.42 + 0.99 * 1.1 = 3.51 \text{ in}^2$$

B. Based on EC2:**Unbonded Tendons:**

❖ At Point A (face of support):

$$A_{ps} = 23 * 99 = 2277 \text{ mm}^2 \text{ (3.53 in}^2\text{)}$$

$$d_p \approx 401 \text{ mm (15.80'')}$$

At centerline d_p is 420 mm and at face of support it is calculated as 401 mm.

$$A_s = 0 \text{ in}^2 \text{ (From minimum computation)}$$

$$d_r = 445 - 19 - 16/2 = 418 \text{ mm (16.46'')}$$

$$\text{Span}/d > 10.67 \text{ m (35.0')} \quad f_{ps} = 1344 \text{ Mpa (195 ksi)} < 0.9f_{pk} = 1675 \text{ Mpa}$$

$$\text{Total Tension Force} = T_p + T_{Sr}$$

$$= (2277 * 1344 / 1.15 + 0 * 413.69 / 1.15) / 1000$$

$$= 2661.12 + 0 = 2661.12 \text{ kN (598.25 kips)}$$

$$a = 2661.12 * 1000 / (3658 * 1 * 34.47 / 1.5) = 32 \text{ mm (1.26'')}$$

$$x = a / \lambda = 32 / 0.80 = 40 \text{ mm (1.57'')}$$

$$d_r = d_t = 418 \text{ mm (16.46'')}$$

$$x/d_t = 40/418 < 0.45 \text{ OK}$$

$$M_n = [2661.12 * (401 - 32/2)] / 1000$$

$$= 1024.53 \text{ kN-m (755.65 k-ft)} < M_U = 1293.10 \text{ kN-m (953.74 k-ft)} \quad \text{No Good}$$

Add Supplemental Rebar:

Since supplemental rebar must be added the depth of compression zone must be prorated to approximate the added compressive stress in the section.

$$\text{Prorated } a = 32 * 1293.10 / 1024.53 = 40 \text{ mm (1.57'')}$$

$$M_{\text{supplemental}} = 1293.10 - 1024.53 = 268.57 \text{ kN-m} \\ (198.09 \text{ k-ft})$$

$$A_{\text{supplemental}} = 268.57 \times 10^6 / [413.69 / 1.15 \times (418 - 40/2)] \\ = 1876 \text{ mm}^2 + 188 \text{ (10\% more for conservatism)} = \\ 2064 \text{ mm}^2 (3.2 \text{ in}^2)$$

Since this is an approximate method for expeditious hand calculation, add 10% more rebar in lieu of iterating the solution.

$$A_s = 0 + 2064 = 2064 \text{ mm}^2 (3.2 \text{ in}^2)$$

No. of Bars = $2064 / 201 = 10.3$; Say 11-16 mm (11-# 5) bars

$$A_s = 11 \times 201 = 2211 \text{ mm}^2 (3.43 \text{ in}^2)$$

$$T_s = (2211 \times 413.69 / 1.15) / 1000 = 795.36 \text{ kN} (178.80 \text{ kips})$$

$$M_n = [2661.12 \times (401 - 40/2) + 795.36 \times (418 - 40/2)] / 1000 = 1330.44 \text{ kN-m} (981.28 \text{ k-ft}) > M_U = 1293.10 \text{ kN-m} (953.74 \text{ k-ft}) \quad \text{OK}$$

❖ At Point C (Midspan):

$$M_U = 586.43 \text{ kN-m}$$

$$A_{ps} = 2277 \text{ mm}^2 (3.53 \text{ mm}^2)$$

$$d_p \approx 216 \text{ mm} (8.5")$$

$$A_s = 0 \text{ in}^2$$

$$d_r = d_t = 241 - 19 - 19/2 = 213 \text{ mm} (8.37")$$

A similar computation to point A concludes that 353 mm² rebar is required. Add 388 mm² (0.60 in²)

Bonded Tendons:

❖ At Point A (face of support):

$$A_{ps} = 23 \times 99 = 2277 \text{ mm}^2 (3.53 \text{ mm}^2)$$

$$d_p \approx 396 \text{ mm} (15.59")$$

At centerline d_p is 413 mm and at face of support it is calculated as 396 mm

$$A_s = 0 \text{ in}^2 \text{ (From minimum computation)}$$

$$d_r = 445 - 19 - 16/2 = 418 \text{ mm} (16.46")$$

$$f_{ps} = 1758 \text{ MPa} (255 \text{ ksi}) > 0.9f_{pk} = 1675 \text{ MPa}$$

Hence, $f_{ps} = 1675 \text{ MPa}$

$$\text{Total tension force} = T_p + T_{sr}$$

$$= (2277 \times 1675 / 1.15 + 0 \times 413.69 / 1.15) / 1000$$

$$= 3316.50 + 0 = 3316.50 \text{ kN} (745.58 \text{ kips})$$

$$a = 3316.50 \times 1000 / (3658 \times 1 \times 34.47 / 1.5) = 39 \text{ mm} (1.54")$$

$$x = a / \lambda = 39 / 0.80 = 49 \text{ mm} (1.93")$$

$$d_r = d_t = 418 \text{ mm} (16.46")$$

$$x / d_t = 49 / 418 < 0.45 \text{ OK}$$

$$M_n = [3316.50 \times (396 - 39/2)] / 1000$$

$$= 1248.66 \text{ kN-m} (920.96 \text{ k-ft}) < M_U = 1293.10 \text{ kN-m} (953.74 \text{ k-ft}) \text{ No Good}$$

Add Supplemental Rebar:

Since supplemental rebar must be added the depth of compression zone must be prorated to approximate the added compressive stress in the section.

$$\text{Prorated } a = 39 \times 1293.10 / 1248.66 = 40 \text{ mm} (1.57")$$

$$M_{\text{supplemental}} = 1293.10 - 1248.66 = 44.44 \text{ kN-m} (32.78 \text{ k-ft})$$

$$A_{\text{supplemental}} = (44.44 \times 10^6) / [(413.69 / 1.15) \times (418 - 40/2)]$$

$$= 310 \text{ mm}^2 + 31 \text{ (10\% more for conservatism)} = 341 \text{ mm}^2 (0.53 \text{ in}^2)$$

Since this is an approximate method for expeditious hand calculation, add 10% more rebar in lieu of iterating the solution.

$$A_s = 0 + 341 = 341 \text{ mm}^2 (0.53 \text{ in}^2)$$

No. of bars = $341 / 201 = 1.70$; Say 2-16 mm (2-# 5) bars

$$A_s = 2 \times 201 = 402 \text{ mm}^2 (0.62 \text{ in}^2)$$

$$T_s = (402 \times 413.69 / 1.15) / 1000 = 144.61 \text{ kN} (32.51 \text{ kips})$$

$$M_n = [3316.50 \times (396 - 40/2) + 144.61 \times (418 - 40/2)] / 1000$$

$$= 1304.56 \text{ kN-m} (962.19 \text{ k-ft}) > M_U = 1293.10 \text{ kN-m} (953.74 \text{ k-ft}) \text{ OK}$$

❖ At Point C (Midspan):

$$A_{ps} = 2277 \text{ mm}^2 (3.53 \text{ mm}^2)$$

$$d_p \approx 210 \text{ mm} (8.25")$$

$$A_s = 0 \text{ in}^2$$

$$d_r = d_t = 241 - 19 - 19/2 = 213 \text{ mm} (8.37")$$

$$f_{ps} = 1675 \text{ MPa}$$

$$\text{Total tension force} = (2277 \times 1675 / 1.15 + 0 \times 413.69 / 1.15) / 1000$$

$$= 3316.50 + 0 = 3316.50 \text{ kN} (745.58 \text{ kips})$$

$$a = 3316.50 \times 1000 / (10363 \times 1 \times 34.47 / 1.5) = 14 \text{ mm} (0.55")$$

$$x = a / \lambda = 14 / 0.80 = 18 \text{ mm} (0.71")$$

$$x / d_t = 18 / 213 < 0.45 \text{ OK}$$

$$M_n = [3316.50 \times (210 - 14/2)] / 1000$$

$$= 673.25 \text{ kN-m} (496.56$$

$$\text{k-ft}) > M_U = 586.43 \text{ kN-m} (432.53 \text{ k-ft}) \quad \text{OK}$$

❖ Cracking moment larger than moment capacity:

Where cracking moment of a section is likely to exceed its design capacity in flexure, reinforcement is added to raise the moment capacity. In such cases, the contribution of each reinforcement is based on the strength it provides. If the value of reinforcement required is expressed in terms of cross-sectional area of nonprestressed reinforcement, the applicable value for compliance will be $(A_s + A_{ps} \times f_{py} / f_y)$.

❖ Bonded (grouted) tendons

TABLE 8.4-1 Summary of Required Reinforcement for Strength Limit State (in²) (T163US)

Code	Unbonded		Bonded	
	Support	Span	Support	Span
ACI/IBC	2.96	2.42	0	0
EC2	3.2	0.60	0.53	0

Note: Support is point A; Span is point C

TABLE 8.4-2 Cracking Moment Values and Parameters (T164US)

Basic parameters and analysis	Span 4	Section A	Section C
	S_{top} (in ³)	15302	----
	S_{bot} (in ³)	---	6137
	P (kips)	615.71	615.71
	P / A (psi)	-122	-158
	$f_r + (P/A)$	652	688
	M_{cr} (k-ft)	831.41	351.85
	1.2 M_{cr} (k-ft)	998	422
	ΦM_n (k-ft)	1026	513
	Status	OK	OK

ACI318³¹/IBC requires that for members reinforced with bonded tendons the total amount of prestressed and nonprestressed reinforcement shall be adequate to develop a factored load moment at least 1.2 times the cracking load computed on the basis of the modulus of rupture of the section. In practice, this is taken as cracking moment of the section M_{cr} .

The necessity and amount of rebar is defined as a function of cracking moment of a section (M_{cr}). For prestressed members

$$M_{cr} = (f_r + P/A) * S$$

Where, f_r is the modulus of rupture defined³².

$$f_r = 7.5 \sqrt{f'_c} = 7.5 \sqrt{5000} = 530 \text{ psi}$$

P/A is the average precompression, and S the section modulus. Table 8.4-2 summarizes the leading values and the outcome.

Since at both the face-of-support (section A) and mid-span (section C) the design capacity of the section with prestressing alone exceeds 1.2* M_{cr} , no additional rebar is required from this provision

In design situations like above, where the objective is to establish whether a value is less or more than a target, it is expeditious to start the check using simplified, but conservative procedures. If the computed conservative value using the approximate procedure is too close to the threshold, the check can be followed with a more rigorous computation. Using a simple approximation:

Cover to strand CGS = 1.5 in; hence $d = h$ (thickness) - 1.5

Moment arm = 0.9d

Design force in strand = $A_{ps} * 270$ ksi ; $\Phi = 0.9$

At face-of-support, with 23 strands, 270 ksi strength $\Phi * M_n = 0.9 * 23 * 0.153 * 270 * (17.5 - 1.5) * 0.9/12 = 1026$ k-ft

Design moment at midspan is calculated in a similar manner.

The envelope of total reinforcement is given in Table 8.4-3.

8.5 Punching Shear Check and Design

For moment capacity, the reinforcement calculated

³¹ ACI 318-11 Section 18.8.2

³² ACI-318, Section 9.5.2.3

TABLE 8.4-3 Envelope of Reinforcement for Serviceability(SLS) and Strength Condition (ULS) (in2) (T165US)

Code	Unbonded		Bonded	
	Support	Span	Support	Span
ACI/IBC	2.96	2.42	0	0
EC2	3.2	0.60	0.53	0

TABLE 9-1 Stresses at Transfer of Post-Tensioning (T166US)

Span #4	Point A	Point B	Point C
M_D k-ft	-615.08	-215.92	197.90
M_{PT} k-ft	321.67	120	-79.12
P kips	615.71	615.71	615.71
P/A ksi	-0.122	-0.158	-0.158
f_t psi	52	-29	-391
f_b psi	-447	-334	27
ACI 318-11/IBC 2012			
F_t psi	184	-2250	-2250
F_b psi	-2250	-2250	184
	OK	OK	OK
EC2			
F_t psi (MPa)	380 (2.62)	-2250 (-15.51)	-2250 (-15.51)
F_b MPa (psi)	-2250 (-15.51)	-2250 (-15.51)	380 (2.62)
	OK	OK	OK

Note: Section properties I , A , S_{top} , S_{bot} are the same as used for service condition stress check

F_t and F_b are allowable stresses at top and bottom respectively

for a given section does not differ substantially, when using different building codes. The design values are in reasonable agreement. For punching shear, on the other hand, the treatment and outcome between the building codes covered herein differ significantly. Due to the larger variation, the subject matter is treated in greater detail separately (Chapter 4, Section 4.11.6).

9 - CODE CHECK FOR INITIAL CONDITION

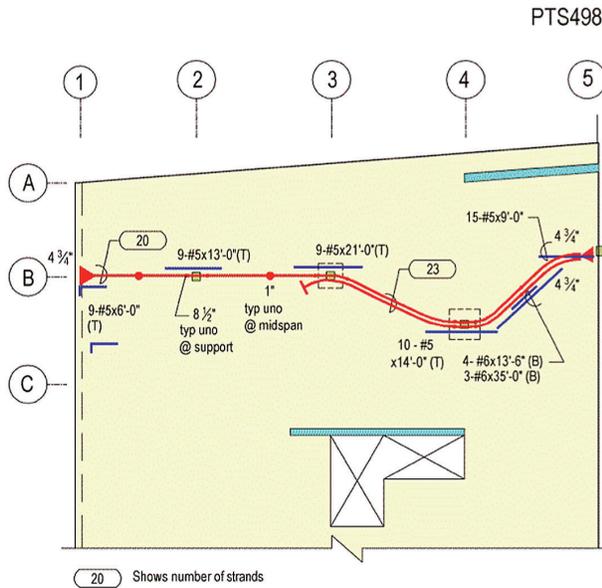
At stressing (i) concrete is at low strength ; (ii) prestressing force is at its highest value; and (iii) live load generally envisaged to be counteracted by prestressing is absent. As a result, the stresses experienced by a member can exceed the limits envisaged

acceptable for the condition. Hence, post-tensioned members are checked for both tension and compression stresses at transfer of prestressing. Where computed compression stresses exceed the allowable values, stressing is delayed until either concrete gains adequate strength, or the member is loaded. Where computed tension stresses are excessive, ACI/IBC³³ suggest adding non-stressed reinforcement to control cracking.

9.1 Load Combinations

The codes covered are not specific on the applicable load combination at transfer of prestressing. The fol-

³³ ACI-318, Section 9.5.2.3



Tendon and Reinforcement Layout for Band at Line B

FIGURE 10-1

lowing is the combination generally assumed among practicing engineers;

Load Case: $1.0 \cdot DL + 0 \cdot LL + 1.15 \cdot PT$

Specification of this design example calls for tendons to be stressed when concrete cylinder reaches 3750 psi³⁴.

$$f_{ci} = \frac{3}{4} \cdot 5000 = 3750 \text{ psi (25.85 MPa)}$$

Stress Check:

$$\sigma = \pm(M_D + 1.15 \cdot M_{PT})/S + 1.15 \cdot P/A$$

$$S = I/Y_c$$

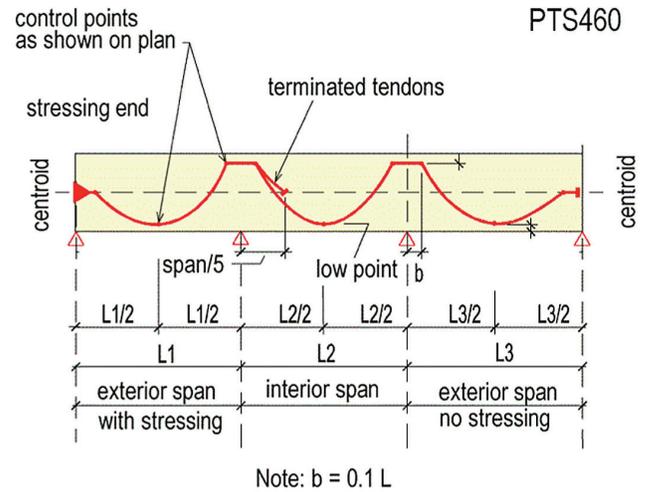
Allowable stresses

❖ Based on ACI-318-11; IBC 2012
Tension = $3 \cdot \sqrt{3750} = 184 \text{ psi}$
Compression = $0.60 \cdot 3750 = -2250 \text{ psi}$

❖ Based on EC2
Tension = $f_{ct,eff} = 2.62 \text{ MPa (380 psi)}$
Compression = $0.60 \cdot 25.85 = -15.51 \text{ MPa (-2250 psi)}$

Farthest fiber stresses are calculated in a similar manner to service condition as covered earlier. The outcome is summarized in Table 9-1.

³⁴ The value specified is on the high side. Most hardware is designed to be stressed at 2000 psi concrete cylinder strength or less.



Profile for Banded Slab Tendons

FIGURE 10-2

10 DETAILING

The final tendon and reinforcement layout for the design strip at line B is shown in figures 10-1 and 10-2 for unbonded tendons. Unbonded tendons are flexible and lend themselves to swerving on plan as shown in the figure. Bonded tendons are not as flexible. They are generally arranged along straight lines.

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