CHAPTER 6

TWO-WAY FLOOR DESIGN EXAMPLE

Post-Tensioned Concrete Frame under Construction
(California, P634)

FOREWORD

This example walks you through the 10 steps of design of a post-tensioned floor level of a multistory building. Each of the 10 steps is commented in detail to provide you with the background information necessary to follow the calculations.

Many aspects of the example selected, such as the arrangement of its floor supports are highly irregular. The objective in selecting an irregular structure is to expose you to the different design scenarios that you may encounter in real life structures, but you do not find covered in standard textbooks. Design conditions that are not directly encountered in this example, but are important to know, are introduced and discussed as comments.

The floor slab is provided with both column drops for punching shear, and drop panels for additional strength in resisting high negative moments over the supports. The design example also features different number of strands along the length of the structure and change in tendon profile from span to span.

Design operations that are considered common knowledge, such as the calculation of moments and shears, once the geometry of a structure, its material properties and loading are known, are not covered in detail. You are referred to your in-house frame programs for their evaluation, or other sections of the book, where the specific operations are addressed in greater detail.

The design example covers side by side both the unbonded and bonded (grouted) post-tensioning systems, thus providing a direct comparison between the design processes of the two options. In addition, in parallel, the design uses the current American building codes (ACI-318 and IBC) along with the European Code (EC2). Where applicable, reference is made to the UK's committee report TR43.

There are three methods commonly used for the design of a post-tensioned floor system—Simple Frame Methods (SFM),[1] Equivalent Frame Method (EFM); and Finite Element Method (FEM). Among the three, the EFM has been the primary method of design used by leading consulting firms over the years. However, due to its complexity, it does not lend itself to hand calculation of real structures in the environment of a consulting firm. Computer programs based on the EFM, such as ADAPT-PT are generally used. Recently, many consultants sacrifice the efficiency and the option of optimization that is feasible for designs based on EFM and opt for the benefits of FEM-based designs, such as the computer program Floor-Pro by ADAPT. These FEM-based designs can model the entire floor system and provide seamless integration of design process from architectural drawings to fabrication documents.

Hand calculations, such as the one presented herein, use the SFM.

Two text fonts are used in the following. The numerical work that forms part of the actual calculations uses the font shown below:

This font is used for the numerical work that is part of the design.

The following text font is used, wherever comments are made to supplement the calculations:

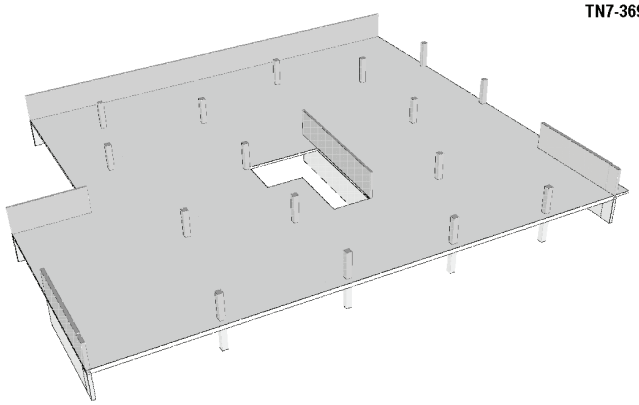
This font is used to add clarification to the calculations.

DESIGN STEPS

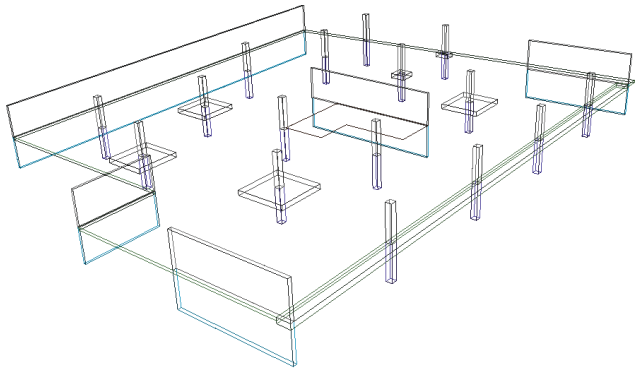
1. GEOMETRY AND STRUCTURAL SYSTEM
   - 1.1 Overview
   - 1.2 Geometry and Support Conditions
   - 1.3 Support Lines and Tributaries
   - 1.4 Idealized Design Strip
2. MATERIAL PROPERTIES
   - 2.1 Concrete
   - 2.2 Nonprestressed Reinforcement
   - 2.3 Prestressing

[1] The Simple Frame Method (SFM) in UK and the literature based on UK practice is referred to as "Equivalent Frame Method." It is based strictly on the cross-sectional geometry of the slab frame being designed. The term Equivalent Frame Method in the US literature is based on an approximation that is intended to simulate the two-way action of a floor slab. It is described in various versions of ACI 318.

TN7-369



(a) 3D Solid View of the Floor System (P471)



(b) See through View of the Floor System (P472)

O-1 3D Views of the Floor System

3. LOADS
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   - 3.2 Superimposed Dead Load
   - 3.3 Live Load
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7. CODE CHECK FOR SERVICEABILITY
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1 - GEOMETRY AND STRUCTURAL SYSTEM

1.1 Overview

Nahid building is a multi-story structure supported on walls and columns. The lateral loads are resisted by shear walls in two directions. The floor of the building is a two-way post-tensioned slab resting on columns and walls. The calculations that follow represent the design of one region of the floor slab identified by gridline B, and referred to as "design strip B." The remainder of the floor slab can be designed in a similar manner. The design is performed using the current versions of IBC; ACI-318; EC2 and TR-43.

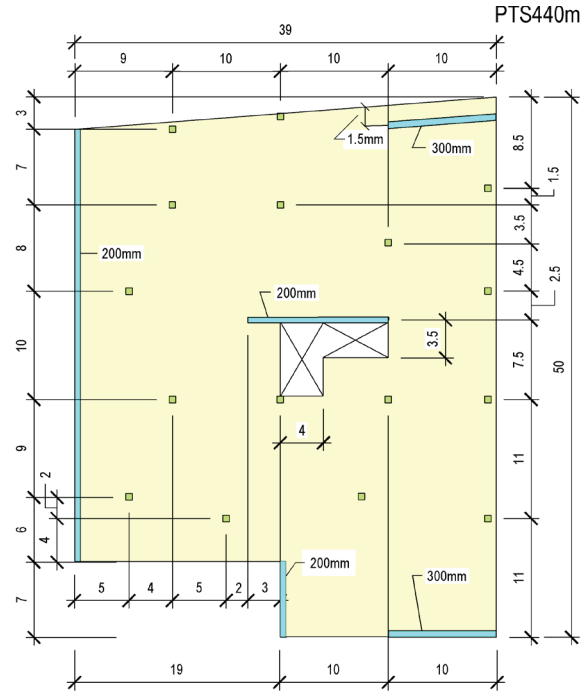
1.2 Geometry and Support Conditions

Dimensions and Support Conditions

Floor slab dimensions are shown in Fig. 1.1-1.

- ❖ Slab thickness and locations of Column drops/Panels are shown in Fig. 1.1-2;
- ❖ Dimensions of column drops/panels shown in Fig. 1.1-3;
- ❖ Columns are 600 mm x 600 mm and extend above and below the slab; and
- ❖ Columns are assumed fixed at connection to the slab and at their far ends.

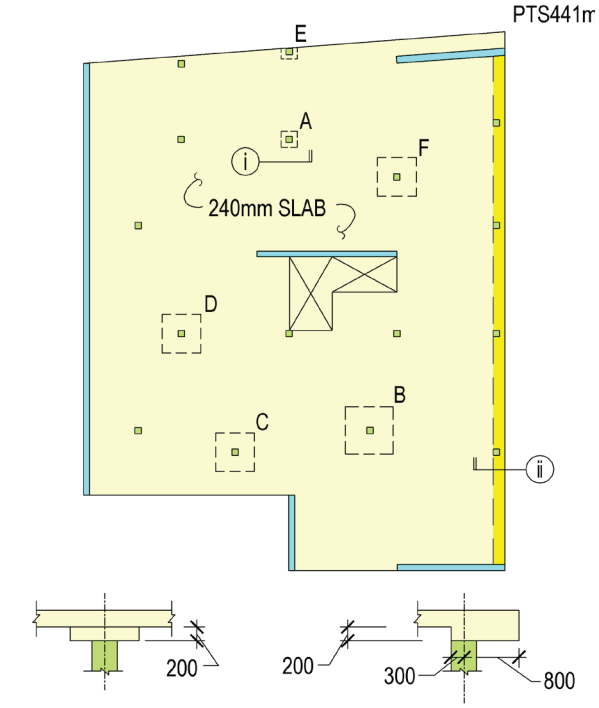
The maximum span to depth ratio for the 240 mm slab selected is less than 45, which is the upper value commonly used for similar structures. A preliminary analysis, not included in this work, showed that the slab thickness selected was not adequate for punching shear at selected column locations (marked as locations A through E in Fig. 1.2-2, and along the column supported right edge of the slab). As a result, the right edge is provided with a down turned edge beam (section ii in Fig. 1.2-2). The remainder of the locations are provided each with a column drop to resist punching shear. Further calculation of the preliminary design concluded that the required reinforcement over four of the interior columns was



dimensions in m, uno

Floor Slab Dimensions (m)

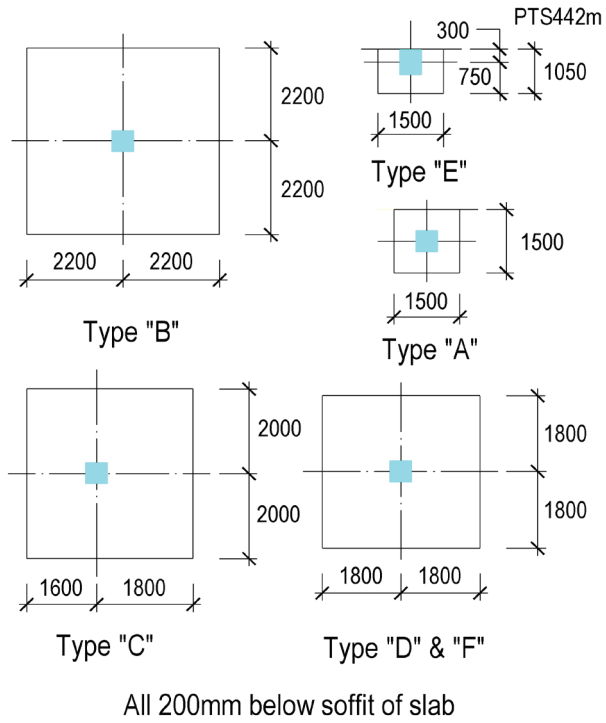
FIGURE 1.1-1



(i) Section - Typical (ii) Section

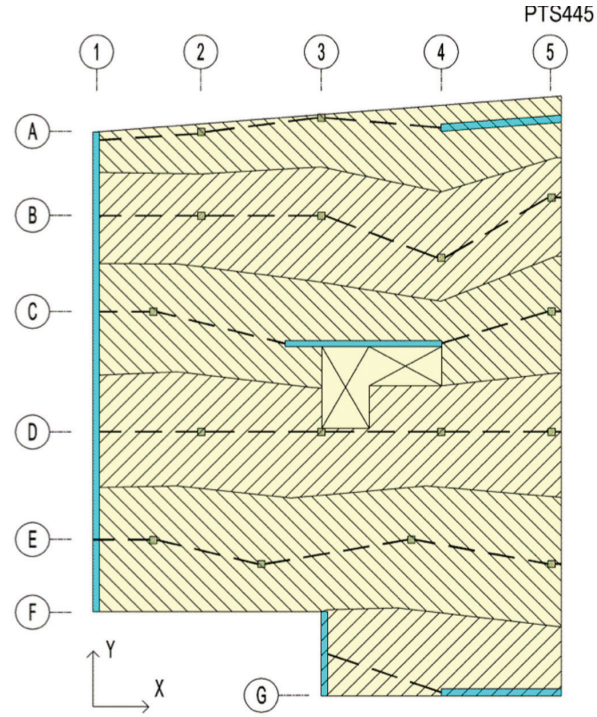
Thickness Geometry of the Floor (mm)

FIGURE 1.1-2



Plan Geometry of Drop Caps and Drop Panels (mm)

FIGURE 1.1-3



Tributaries for Design Strips in X - Direction

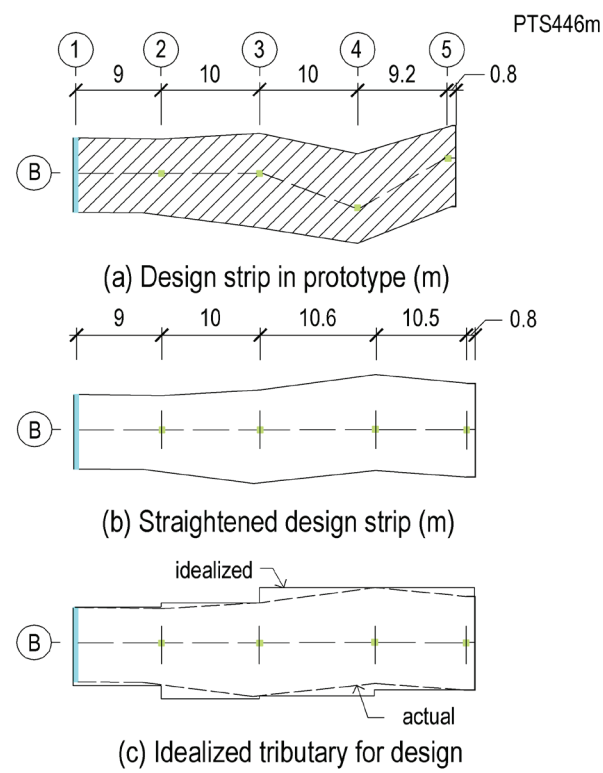
FIGURE 1.3-1

excessive (more than 4200 mm2) [Aalami, 1989]. To avoid congestion of top reinforcement, the column drops at these locations were enlarged to qualify them as drop panels. These locations are marked as B, C, D and F in Fig.1.2-2. While it is practical to eliminate column drops at locations A and E through provision of punching shear reinforcement, the drop panels cannot be eliminated without causing congestion in top rebar.

1.3 Support Lines and Tributaries

The breakdown of a floor into support lines, tributaries and design strips in two principal directions are explained in Chapter 3, as the first step in definition of load paths for design. The outcome is the subdivision of floor into design strips in each of the two orthogonal directions. In this example, we se-lect and complete the design of one of the design strips in X-direction. The remainder of the design strips will be treated in a similar manner.

The design strips in X-direction are shown in Fig. 1.3-1. Each design strip is extracted from the floor system and modeled in isolation as an idealized single design strip, such as the design strip for support line B shown in Fig. 1.3-2a.



Construction of Design Strip in Plan

FIGURE 1.3-2

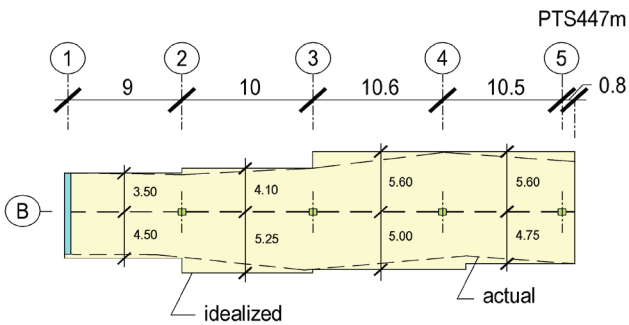
1.4 Idealized Design Strip

❖ Design Strip Dimensions

The extracted design is "straightened" to simplify analysis (Fig. 1.3-2b). The tributaries of each span of the extracted design strip are adjusted to the maximum width of the respective span on each side of the support line. The dimensions of the final design strip are shown in Figs. 1.4-1 and 1.4-2a.

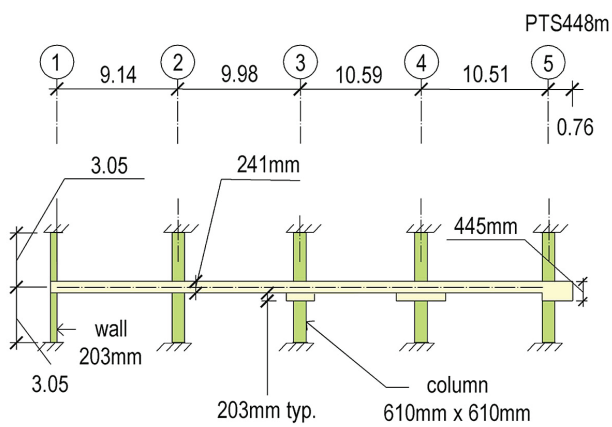
For gravity design of the structure, the practice in selection of boundary conditions of the extracted design strip is verbalized in ACI/IBC as follows. The strip is modeled with one level of supports immediately above and below the level under consideration. The far ends of the supports are assumed fixed against rotation.

The elevation of the idealized design strip and a three dimensional view of it are shown in Figs. 1.4-2, 1.4-3.



Plan of Design Strip B (m)

FIGURE 1.4-1



Design Strip in Elevation

FIGURE 1.4-2

❖ Section Properties

The section properties of each span are calculated using the gross cross-sectional area of the idealized design strip as shown in Figs. 1.4-1 and 1.4-2.

The stiffening of the slab due to the added thickness of the column drops and drop panels are accounted for in the calculation through their section properties. In SFM adopted in this example, the added stiffness in the slab immediately over the support is not included in the analysis. However, the EFM of analysis allows for the aforementioned increase in stiffness.

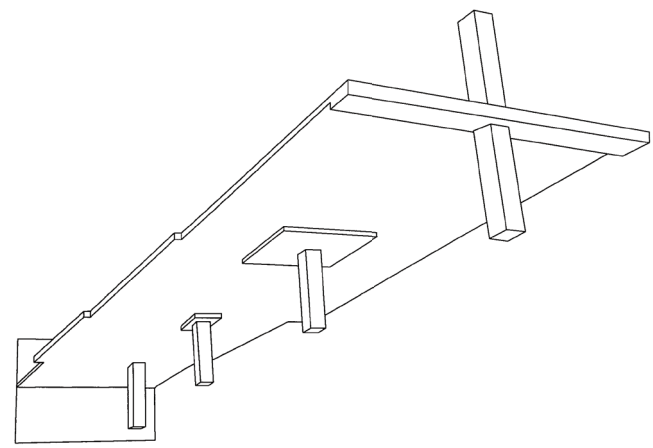


FIGURE 1.4-3 View of the Design Strip (P473)

2 - MATERIAL PROPERTIES

2.1 Concrete

f'_c, f_{ck} (28 day cylinder strength)[2] = 40 MPa
Weight = 24 kN/m [3]
Elastic Modulus $4700\sqrt{f'c}$ = 29725 MPa [ACI]
= $22^* 10^{3*} [(f_{ck} + 8)/ 10]^{0.3}$ [3] [EC2, TR-43]
= 35220 MPa
Creep coefficient = 2
Material factor, γ_c = 1 [ACI]; 1.50 [EC2, TR-43]

The creep coefficient is used to estimate the long-term deflection of the slab.

2.2 Nonprestressed (Passive) Reinforcement

f_y = 460 MPa
Elastic Modulus = 200000 MPa
Material factor, γ_c = 1 [ACI]; 1.15 EC2
Strength reduction factor (bending), φ = 0.9 [ACI];
= 1 [EC2, TR-43]

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2 Where cube strength is specified, the following conversion is used: cylinder strength = 0.8 times cube strength
3 EN 1992-1-1:2004(E) Table 3.1

TABLE 1.4-1 Section Property of the Design Strip (T157SI)

Span	Tributary Width (m)	Depth (mm)	I (mm^4)	Y_b (mm)	Y_t (mm)
1	8.0	240	9.216e+9	120	120
2 Mid	9.35	240	1.077e+10	120	120
2 Right	9.35	440	2.458e+10	294	146
3 Left	10.6	440	2.620e+10	297	143
3 Mid	10.6	240	1.221e+10	120	120
3 Right	10.6	440	4.177e+10	271	169
4 Left	10.35	440	4.134e+10	271	169
4 Mid	10.35	240	1.192e+10	120	120
4 Right	10.35	440	7.347e+10	220	220
Cantilever	10.35	440	7.347e+10	220	220

Where

I = second moment of area of the tributary section; and

Y_t, Y_b = top and bottom distances of the centroid of the section to the extreme fibers respectively

2.3 Prestressing (Figs 2.3-1 through 2.3-3)
Material—low relaxation, seven wire ASTM 416 strand
Nominal strand diameter = 13 mm
Strand area = 99 mm^2
Elastic Modulus = 200000 MPa
Ultimate strength of strand (f_{pu}) = 1860 MPa
Material factor, γ_c = 1 [ACI]; 1.15 [EC2, TR-43]

System
Unbonded System
Angular coefficient of friction (μ) = 0.07
Wobble coefficient of friction (K) = 0.003 rad/m
Anchor set (wedge draw-in) = 6 mm
Stressing force = 80% of specified ultimate strength
Effective stress after all losses[4] = 1200 MPa

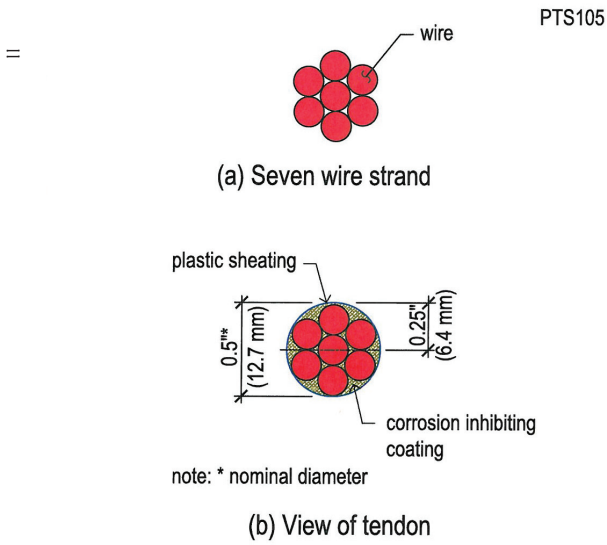
Bonded System
Use flat ducts 20x80mm; 0.35 mm thick metal sheet housing up to five strands
Angular Coefficient of Friction (μ) = 0.2
Wobble Coefficient of Friction (K) = 0.003 rad/m
Anchor Set (Wedge Draw-in) = 6 mm
Offset of strand to duct centroid (z) = 3 mm
Effective stress after all losses = 1100 MPa

3 - LOADS

3.1 Selfweight
Slab=(240/1000)* 2400* 9.81/1000
= 5.65 kN/m^2

3.2 Superimposed Dead Load
Superimposed dead load = 2.00 kN/m^2
Total Dead Load = SW + SDL = 7.65 kN/m^2
Span 1 DL = 7.65* 8.00 m = 61.20 kN/m
Span 2 DL = 7.65* 9.35 m = 71.53 kN/m
Span 3 DL = 7.65* 10.60 m = 81.09 kN/m
Span 4 DL = 7.65* 10.35 m = 79.18 kN/m
Added dead load due to column drop, drop panel and transverse beam:
Column drop DL (support 3)
= 0.2*1.5*2400*9.81/1000 = 7.06 kN/m
Load extends 0.75 m on each side of support 3)
Drop panel DL (support 4) =.2*3.6*2400*9.81/1000
= 16.95 kN/m (Load extends 1.8 m on each side of support 4)
Added beam depth (cantilever)

[4] For hand calculation, an effective stress of tendon is used. The effective stress is the average stress along the length of a tendon after all immediate and long-term losses. The value selected for effective stresses is a conservative estimate. When "effective stress" is used in design, the stressed lengths of tendons are kept short, as it is described later in the calculations.



Unbonded Tendon

FIGURE 2.3-1 Section View of an Unbonded Tendon

0.2*10.35*2400*9.81/1000 = 48.74 kN/m
(Load extends from 0.2 m left of support 5 to slab edge)

3.3 Live Load[5] **:** 3 kN/m^2
Span 1 LL = 3* 8 = 24 kN/m
Span 2 LL = 3* 9.35 = 28.05 kN/m
Span 3 LL = 3* 10.60 = 31.80 kN/m
Span 4 LL = 3* 10.35 = 31.05 kN/m
Cantilever LL = 3* 10.35 = 31.05 kN/m
LL/DL ratio = 3/7.65 = 0.39
< 0.75 ∴ Do not skip live load

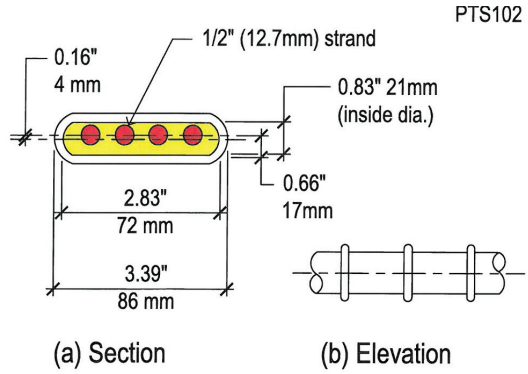
Live load is generally skipped (patterned), in order to maximize the design values. However, for two-way floor systems, ACI 318-11 does not require live load skipping,[6] provided the ratio of live to dead load does not exceed 0.75. In this example, as in most concrete floor systems for residential and office buildings, the ratio of live to dead load is less than 0.75. Hence, the live load will not be skipped.

The loading diagrams are shown in Fig. 3-1.

4 - DESIGN PARAMETERS

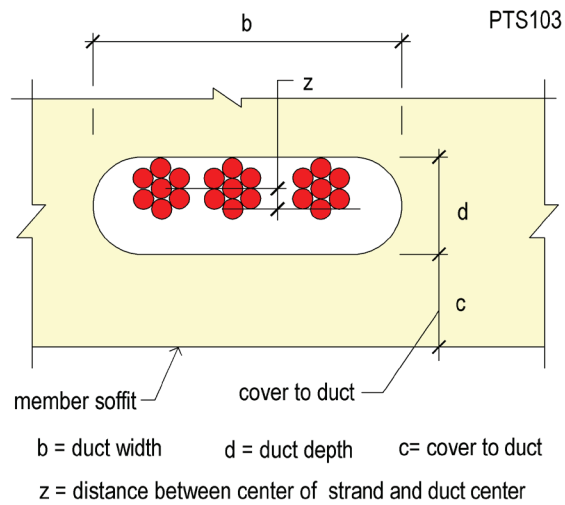
[5] Live load for residential floors is generally 2 kN/m2. For commercial buildings it is somewhat more. Herein, conservatively 3kN/m2 is assumed. Live load is generally reduced based on the floor area it covers. Reduction of live load is described in IBC 2012 (Chapter 16). In this design example, live load is not reduced.

[6] ACI 318-11 (13.7.6)



Example of a Grouted Flat Duct Used in Building Construction

FIGURE 2.3-2 Plastic Flat Corrugated Duct Grouted Tendon



Section through a Flat Duct at Low Point

FIGURE 2.3-3

4.1 Applicable Codes

The design is carried out according to each of the following codes. Further, reference is made to the Committee Report TR-43, where appropriate.

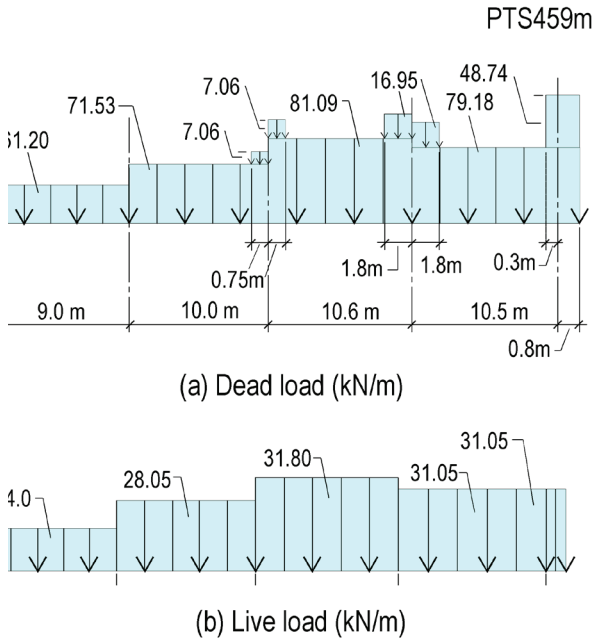
- IBC-2009 (ACI 318-2011)
- EC2(EN 1992-1-1:2004)

4.2 Cover to Rebar and Prestressing Strands

Minimum rebar cover = 20 mm top and bottom

Unbonded System

The slab is assumed to be in a non-corrosive environment. Cover to its reinforcement is based on a 2-hour fire rating with the exterior spans considered



Loading on Design Strip

FIGURE 3.1

restrained. This requires a minimum cover of 20 mm, using IBC-12. Hence, the CGS (Center of Gravity of Strand) of 13 mm strand is 27 mm from top and bottom fibers of concrete outline. The existing concrete wall at one end of the design strip, and the down turned beam at the other end of it are considered adequate to provide restraint against in plane expansion of the slab for fire resistivity. Hence, the end spans are considered "restrained."[7]

Minimum strand cover = 20 mm
CGS. all spans = 27 mm

Bonded System
Minimum top and bottom rebar cover = 20 mm
For post-tensioning tendons: (Fig. 4.2-1)
Cover to duct = 20 mm
Distance to centroid of strand
= 20 + 10 + 3 = 33 mm
Where, 10mm is half duct diameter and z=3 mm
CGS. all spans = 33 mm

4.3 Allowable Stresses

A. Based on ACI 318-11/IBC 2009[8]

Allowable stresses in concrete are the same for bonded and unbounded PT systems

❖ For Sustained Load Condition
Compression = $0.45^* f'_c = 0.45^*40 = 18$ MPa
Tension = $0.5^* \sqrt{f'_c} = 3.16$ MPa

❖ For Total Load Condition
Compression = $0.60^* f'_c = 24$ MPa
Tension = $0.5^* \sqrt{f'_c} = 3.16$ MPa

❖ For Initial Condition (at Tranfer of Prestressing)
Compression = $0.60^* f'_{ci} = 0.6^* 30 = 18$ MPa
Tension = $0.25^* \sqrt{f'_c} = 1.58$ MPa

In ACI 318/IBC 2012 the allowable stresses for two-way systems and one-way systems are different. The values stated are for two-way systems. These values may not be exceeded. Using ACI-318, two-way systems are deemed to be essentially crack-free when in service. Cracking, if any is not of design significance.

B. Based on EC2[9]

EC2 does not specify "limiting" allowable stresses in the strict sense of the word. There are stress thresholds that trigger crack control. These are the same for both bonded and unbounded systems

❖ For "Frequent" Load Condition
Concrete
Compression = $0.60^* f_{ck} = 0.6^* 40 = 24$ MPa
Tension (concrete) $F_t = f_{ct,eff} = f_{ctm}$[10]
$F_t = 0.30^* f_{ck}^{(2/3)} = 0.30^* 40^{(2/3)}$
= 3.51 MPa (Table 3.1, EC2)
Tension (non-prestressed steel) = $0.80^* f_{yk} = 0.8^*460$
= 368 MPa
Tension (prestressing steel) = $0.75^* f_{pk}$
= $0.75^* 1860 = 1395$ MPa

❖ For "Quasi-permanent" Load Condition
Compression = $0.45^* f_{ck} = 0.45^* 40 = 18$ MPa
Tension (concrete) = 3.51 MPa
same as frequent load combination

Unlike ACI 318/IBC, provisions in EC2 permit[11] overriding the allowable hypothetical tension stress in concrete, provided cracking is controlled not to exceed the selected "design crack width."

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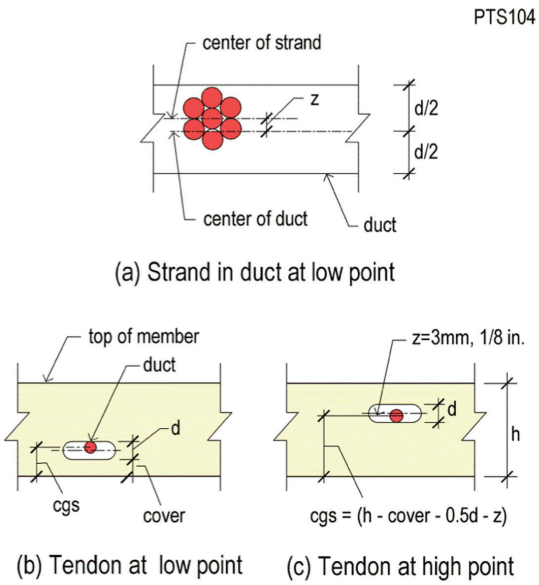
7 In IBC-12, where a span is free to expand in its own plane, it is considered "unrestrained," and is required to have a larger cover for fire resistivity than a span that is not free to expand (restrained). IBC Table 720.1

8 ACI 318-11, Sections 18.3

9 EN 1992-1-1:2004(E),Section 7.2

10 EN 1992-1-1:2004(E), Section 7.3.2(4)

11 EN 1992-1-1:2004(E), Section 7.3.2(4)



Position of Center of Gravity (cgs) of Strand at Extreme Positions in Member

FIGURE 4.2-1

❖ For "Initial" Load Condition (Table 3.1; EC2)
Tension (Unbonded) = $f_{ct,eff} = f_{ctm}$
$0.30^* f_{ci}^{(2/3)} = 0.30^* 30^{(2/3)} = 2.90$ MPa
Compression[12] = $0.60^* f_{ci} = 0.6^*30 = 18$ MPa

C. Based on TR-43[13] Using Panel Width

TR-43 Report provides two sets of allowable stresses. One is based on the traditional selection of design strips based on the full tributary and referred to as "full panel width," such as the one used in this design example and commercial software. The other set of allowable stresses is based on narrower design strips selected to more closely capture the local behavior of a slab. The latter, referred to as "design strip approach" is an option for processing solutions obtained from Finite Element analyses. For practical reasons, most engineers and automated commercial software tools use the "full panel width" option of allowable stresses, in particular, since both options are deemed to result in the same design.

For flat slabs, allowable stresses are the same for both bonded and unbonded systems, as well as for "frequent" and "quasi-permanent" load combinations.

There are two thresholds for hypothetical tension stresses. If the hypothetical tension stresses are below the first threshold, no bonded reinforcement need be added. If the hypothetical tension stresses exceed the first threshold, but are less than the second, a specified amount of bonded reinforcement must be provided. The hypothetical stresses are not permitted to exceed the upper threshold.[14] Grouted tendons can be considered as bonded reinforcement, as it is explained in greater detail in Section 7.4.

Tension (without bonded reinforcement)
For full panel[15] = $0.3\ f_{ctm,fl}$
$f_{ctm,fl}$ = larger of $(1.6 - h/1000)\ f_{ctm}$ or f_{ctm}[16]
= larger of $(1.6 - 0.24)\ f_{ctm}$ or f_{ctm}
= larger of $1.36^* f_{ctm}$ or f_{ctm}
$f_{ctm} = 0.30^* f_{ck}^{(2/3)}$ (Table 3.1, EC2)
$= 0.30^* 40^{(2/3)} = 3.51$ MPa
Allowable lower threshold = $0.3^*1.36^* 3.51$
= 1.43 MPa

Tension (with bonded reinforcement)
For full panel = $0.9\ f_{ctm,fl}$
$= 0.9^*1.36^* 3.51 = 4.30$ MPa
Compression (support) = $0.3^* f_{ck} = 0.3^*40 = 12$ MPa
Compression (span) = $0.4^* f_{ck} = 0.4^*40 = 16$ MPa

❖ For "initial" load condition[17]
Tension = $0.4\ f_{ctm}$
where f_{ctm} refers to strength at stressing
$f_{ctm} = 0.30^* f_{ci}^{(2/3)} = 0.30^* 30^{(2/3)} = 2.90$ MPa
Allowable tension stress = $0.4^*2.90 = 1.16$ Mpa
Allowable compression stress = $0.40^* f_{ci} = 12$ Mpa

4.4 Crack Width Limitation

A. Based on ACI 318-11/IBC 2012

No explicit limit is imposed by the code for crack width calculation and or its control for two-way floor systems, since the designs are deemed to be essentially within the pre-cracking range of concrete.

B. Based on EC2

In EC2, the allowable crack width depends on whether the post-tensioning system used is "bonded," or "unbonded," and the load combination being considered.[18]

❖ Prestressed members with bonded tendons: 0.2 mm; to be checked for frequent load case.

12 EN 1992-1-1:2004(E), Section 5.10.2.2(5)
13 TR-43 Second Edition, Table 4. For tensile stress, stress limit without bonded reinforcement is considered.
14 The amount of bonded reinforcement to be added is explained in Section 7.4 "Minimum Reinforcement."
15 TR-43, 5.8.1 Table 3
16 EN 1992-1-1:2004(E),Eqn.3-23
17 TR-43 Second Edition, Table 5.
18 EN1992-1-1-2004 (E) Table 7.1N

❖ Prestressed members with unbonded tendons: 0.3 mm; to be checked at quasi-permanent load case.

C. Based on TR-43
For both prestressed systems[19] = 0.2 mm

4.5 Allowable Deflection

A. Based on ACI 318-11/IBC 2012[20]

In all major codes, the allowable deflection is tied to (i) the impact of the vertical displacement on occupants; (ii) the installed non-structural objects such as partitions, glass, or floor covering; and (iii) functional impairment, such as proper drainage. Details of the allowable values, their measurement and evaluation are given in reference [ADAPT TN292]. For perception of displacement by sensitive persons, consensus is limit of *L/250*, where L is the deflection span. It is important to note that this is the displacement that can be observed by a viewer.

The allowable values are:
Since in this design example carpet is assumed to be placed directly on the finished floor, the applicable vertical displacement is the total deflection subsequent to the removal of forms.

Total allowable deflection: L/240

The second deflection check is for potential damage to non-structural brittle construction, such as partitions, from displacement subsequent to installation of such members. The value recommended by ACI-318 is *L/480*. This is vertical displacement resulting from the full application of design live load together with the long-term deflection subsequent to the installation of construction likely to be damaged by deflection. Such installations are application of plaster on concrete masonry unit partitions or installation of dry wall (gypsum boards). Raw framing or masonry units that are not finished are not considered to be subject to the deflection limitations.

Total deflection subsequent to finish on partitions together with application of live load: L/480
Where, L is the length of deflection span. For this design example, the partitions are assumed to have been installed/finished 60 days after the floor is cast.

B. Based on EC2[21]

The interpretation and the magnitude of allowable deflections in EC2 are essentially the same as that of ACI 318. The impact of vertical displacement on the function of the installed members and the visual impact on occupants determine the allowable values. The following are suggested values:

Deflection subsequent to finishing of floors from Quasi-permanent combination: L/250
Deflection subsequent to installation of construction that can be damaged from load combination Quasi-permanent: L/500.

C. Based on TR-43[22]
TR-43 refers to EC2 for allowable deflections.

In summary, the allowable deflection from the two codes and the committee report are essentially the same. Conservatively, it can be summarized as follows:

Quasi Permanent Load Combination
Total deflection: L/250
Deflection subsequent to installation of construction that can be damaged: L/500

Brittle partitions are assumed to have been installed 60 days subsequent to date of casting the slab.

5 - ACTIONS DUE TO DEAD AND LIVE LOADS

Actions due to dead and live loads are calculated by a generic frame program, using the idealized frame dimensions shown in Fig. 5-1. The stiffness of each of the spans is based on the second moment of area given in Table 1.3-1. At locations of the column drop, drop panel, and transverse beam, the stiffness used includes the local thickening of the slab.

In Fig. 5-1 the column drop and drop panel are shown centered about the mid-depth of the slab, since it is assumed that most frame programs used by consultants. The shortcoming becomes critical when designing post-tensioned members, where the eccentricity of tendons with respect to that of the section is of central importance. Later in this design example, it will be illustrated how to account for this shortcoming, and obtain correct values with due allowance for eccentricities. The computer programs ADAPT-PT or ADAPT-Builder, automatically account for the shift in the centroid of a column drop/panel below that of the slab., These computer programs do not require an adjustment.

[19] TR-43 Second Edition, Section 5.8.3.
[20] ACI 318-11, Section 18.3.5
[21] EN 1992-1-1:2004(E), Section 7.4.1
[22] TR-43 Second Edition, Section 5.8.4

For hand calculations, a simple frame analysis is used (Simple Frame Method—SFM). The simple frame method of analysis lacks the specific features of the Equivalent Frame Method (EFM) as listed below:

(i) Increased stiffness of slab over slab/support interface is not accounted for. The stiffness of a slab over its support is assumed to be the same as that at the face of support;

(ii) Increased stiffness of the column within the slab, or within the column drop/panel is not accounted for. In other words, the stiffness of a column is assumed constant over its entire analysis length. Note that the analysis length of a column extends to the centroid of slab; and

(iii) The analysis does not account for the two-way action of the slab, as is implemented in the Equivalent Frame Method. The stiffness of the structure is strictly based on the cross-sectional geometry of the design strip.

The SFM is adequate when hand calculation is used for design. The EFM is more accurate, but it is too complex for hand calculation in the environment of a production oriented consulting office. It is important to note that the SFM provides a safe design, but not necessarily the most economical alternative. The EFM generally leads to smaller column moments, when compared to the SFM.

Examples of the EFM in the literature are generally limited to flat plates mostly without column drop or drop panel, and with uniform tributaries. The use of computer programs with EFM formulation is the practical way for design of complex floor systems with column drop, and/or drop panel, irregular tributaries and non-uniform loads.

The moments calculated from the frame analysis refer to the center line values. These are reduced to the face-of-support using the static equilibrium of each span. The computed moments from the analysis using Simple Frame Method (SFM) are shown in Fig. 5-2 and Fig.5-3. The values at each face-of-support and at midspan are summarized in Table 5-1.

The critical design moments are not generally at midspan. But, for hand calculation, the midspan is selected. The approximation is acceptable when spans and loads are relatively uniform.

6 - POST-TENSIONING

6.1 Selection of Design Parameters

Unlike conventionally reinforced slabs, where given geometry, boundary conditions, material properties and loads result in a unique design, for post-tensioned members in addition to the above a minimum of two other input assumptions are required, before a design can be concluded. A common practice is (i) to assume a level of precompression and (ii) target to balance a percentage of the structure's dead load. In this example, based on experience the level of precompression suggested is larger than the minimum required by ACI-318 code (0.86 MPa). Other major building codes do not specify a minimum precompression. Rather, they specify a minimum reinforcement. Use the following assumption to initiate the calculations.

1. Minimum average precompression = 1.00 MPa
2. Maximum average precompression = 2.00 MPa
3. Target Balanced Loading = 60% of total dead load, up to 80% where beneficial

The minimum precompression is used as the entry value (first trial) for design. The stipulation for a maximum precompression does not enter the hand calculation directly. It is stated as a guide for a not-to-exceed upper value. In many instances, floor slabs that require more than the maximum value stated can be re-designed more economically.

For deflection control the selfweight of the critical span is recommended to be balanced to a minimum

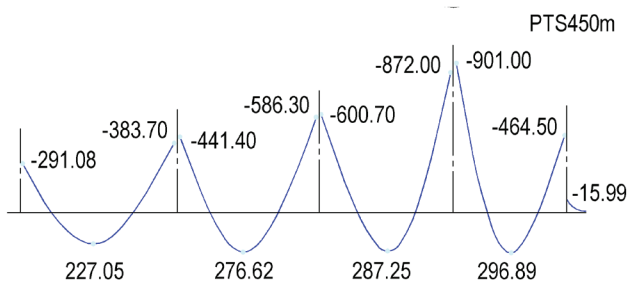
Table 5-1 Moments at Face-of-Support and Midspan (T158SI)

Span	Location	M_D (kN-m)	M_L (kN-m)
Span #1	Left FOS*	-291.08	-114.10
	Midspan	227.05	89.03
	Right FOS	-383.70	-150.50
Span #2	Left FOS	-441.40	-173.10
	Midspan	276.62	108.50
	Right FOS	-586.30	-229.60
Span #3	Left FOS	-600.70	-234.80
	Midspan	287.25	112.30
	Right FOS	-872.00	-335.70
Span #4	Left FOS	-901.00	-347.40
	Midspan	296.89	116.20
	Right FOS	-464.50	-180.90
Cantilever	Left FOS	-15.99	-3.88

* FOS = face-of-support

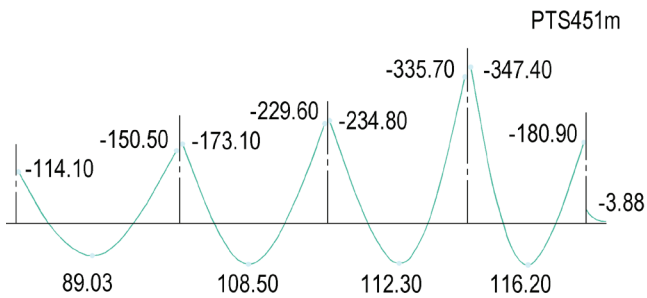
of 60%. Non-critical spans need not be balanced to the same extent.

Effective stress in prestressing strand
For unbonded tendons: f_{se} = 1200 MPa
For bonded tendons: f_{se} = 1100 MPa



Moments Due to Dead Loading (kNm)

FIGURE 5-1



Moments Due to Live Loading (kNm)

FIGURE 5-2

The design of a post-tensioned member can be based either on the "effective force", or the "tendon selection" procedure. In the effective force procedure, the average stress in a tendon after all losses is used in design. In this case, the design concludes with the total effective post-tensioning force required at each location. The total force arrived at the conclusion of design is then used to determine the number of strands required, with due allowance for friction and long-term losses. This provides an expeditious and simple design procedure for hand calculations. In the "tendon selection" procedure, the design is based on the number of strands with due allowance for the immediate and long-term losses. In the following, the "effective force" method is used to initiate the design. Once the design force is determined, it is converted to the number of strands required. A graphical presentation of the preceeding assumptions is given in Chapter 4, Fig 4.8.7.1-2.

The effective stress assumed in a strand is based on the statistical analysis of common floor slab dimensions for the following conditions (Fig. C6.1-1):

(i) Members have dimensions common in building construction;

(ii) Tendons equal or less than 38 m long stressed at one end. Tendons longer than 38m, but not exceeding 76m are stressed at both ends. Tendons longer than 76m are stressed at intermediate points to limit the unstressed lengths to 38m for one-end stressing or 76m for two-end stressing, whichever is applicable;

(iii) Strands used are the commonly available 13 or 15 mm nominal diameter with industry common friction coefficients as stated in material properties section of this design example; and

(iv) Tendons are stressed to $0.8f_{pu}$.

For other conditions, a lower effective stress is assumed, or tendons are stressed at intermediate points. In the current design, the total length of the tendon is 41 m. It is stressed at both ends. Detailed stress loss calculations, not included herein, indicate that the effective tendon stress is 1250 MPa for the unbonded system and also larger than assumed for the grouted system.

6.2 Selection of Post-Tensioning Tendon Force and Profile

The prestressing force in each span will be chosen to match a whole number of prestressing strands. The following values are used:

1. The effective force along the length of each tendon is assumed to be constant. It is the average of force distribution along a tendon.

Unbonded tendons:
Force per tendon = 1200* 99 mm²/1000
= 118.8 ≈ 119.0 kN/ tendon
Use multiples of 119 kN when selecting the post-tensioning forces for design.

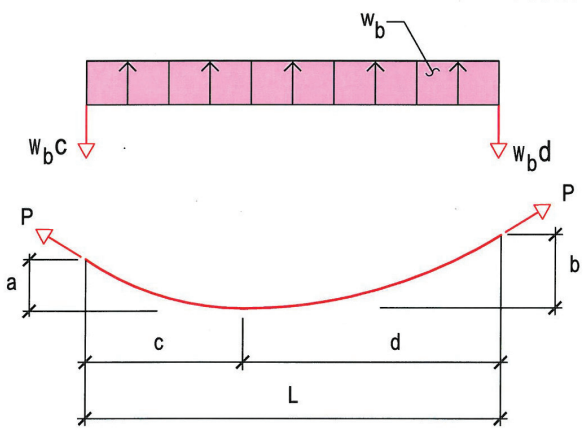
Bonded tendons:
Force per tendons = 1100* 99 mm²/1000
= 108.9 ≈ 109.0 kN/ tendon
Use multiples of 109 kN when selecting the post-tensioning forces for design.

2. Tendon profiles are chosen to be simple parabola.

These produce a uniform upward force in each span.

For ease of calculation the tendon profile in each span is chosen to be simple parabola from support centerline to support centerline (Fig. C6.2-1). The position of the low point is selected such as to generate a uniform upward force in each span. The relationship given in Fig. C6.2-1 defines the profile. For exterior spans, where the tendon high points are not generally at the same level, the resulting low point will not be at midspan. For interior spans, where tendon high points are the same, the low point will coincide with midspan. Obviously, the chosen profile is an approximation of the actual tendon layout used in construction. Sharp changes in curvature associated with the simple parabola profile assumed are impractical to achieve on site. The tendon profile at construction is likely to be closer to reversed parabola, for which the distribution of lateral tendon forces will be somewhat different as discussed henceforth. Tendon profiles in construction and the associated tendon forces are closer to the diagrams shown in Fig. C6.2-2 for two common cases.

PTS107



$c/L = [\sqrt{a/b} \;/\; (1 + \sqrt{a/b}\;)]$

$w_b = 2aP/c^2$

Geometry and Actions of a Parabolic Tendon

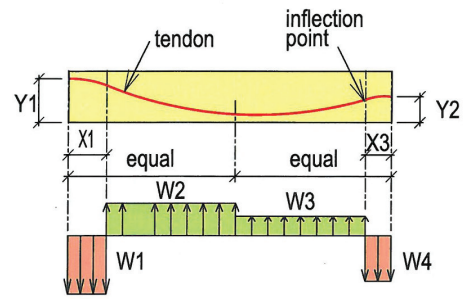
FIGURE C6.2-1

For the beam/cantilever at the right end, the profile selected is a straight line, due to short length of the overhang (Fig.C6.2-3).

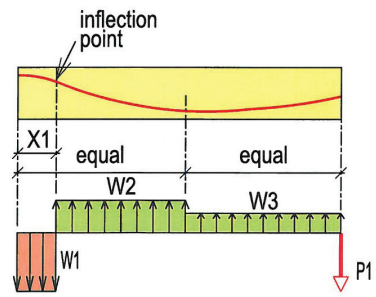
6.3 Selection of Number of Strands

Determine the initial selection of number of strands for each span based on the assumed average precompression and the associated cross-sectional area of each span's tributary. Then, adjust the number of strands selected, based on the uplift they provide.

PTS109



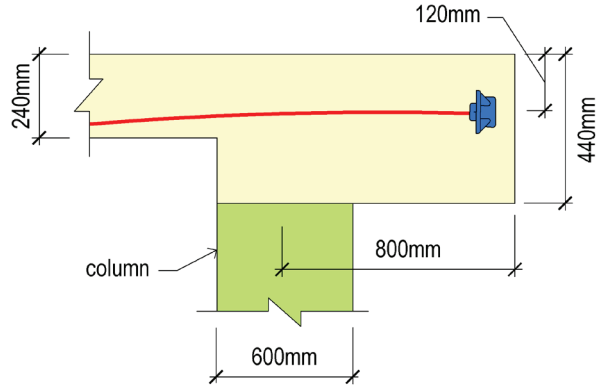
(a) Reversed parabola with two inflection points



(b) Reversed parabola with one inflection point

FIGURE 6.2-2 Two Examples of Common Tendon Profiles

PTS108m



Tendon Profile at Overhanhg

FIGURE C6.2-3 View of Overhang at Right End of Design Strip

Unbonded Tendon

Span 1 Area = 8.0 m* 1000* 240 mm = 1.92e+6 mm^2

TABLE 6.3-1 Tendon Selection Based on Minimum Precompression (T159SI)

Span	Tributary (m)	Thickness (mm)	Area (mm^2)	Force (kN)	Tendons	Selected tendons
1	8.00	240	1.920e+6	1920	17	20
2	9.35	240	2.244e+6	2244	20	20
3	10.6	240	2.544e+6	2544	23	23
4	10.35	240	2.484e+6	2484	22	23
Cant.	10.35	440	4.554e+6	4554	41	23

Span 1 Force = 1.0 MPa* 1.92e+6/1000 = 1920 kN
No. of Tendons = 1920/119.0 = 16.13; say 17
Calculated values for other spans are shown in table below

Bonded Tendon
Span 1 Area = 8.0 m* 1000* 240 mm = 1.92e+6 mm^2
Span 1 Force = 1.0 MPa* 1.92e+6/1000 = 1920 kN
No. of Tendons = 1920/109.0 = 17.61; say 18

It is noted that the number of strands required to satisfy the same criterion differs between the unbonded and bonded systems. Due to higher friction losses, when using bonded systems, more strands are generally needed to satisfy the in-service condition of design. For brevity, without compromising the process of calculation, in the following the same number of strands is selected for both systems.

The number of strands in Table 6.3-1 is based on a minimum precompression of 1.0 MPa at the midsection of each span. The added cross-sectional area of column drops, drop panels and transverse beams are disregarded in the calculation of the force for minimum precompression. The selected number of tendons is chosen to avoid an overly complicated tendon layout. Again, the precompression limit is disregarded for the cantilever, since the large value obtained is due to the depth of the beam having been used in the calculations, as opposed to slab depth.

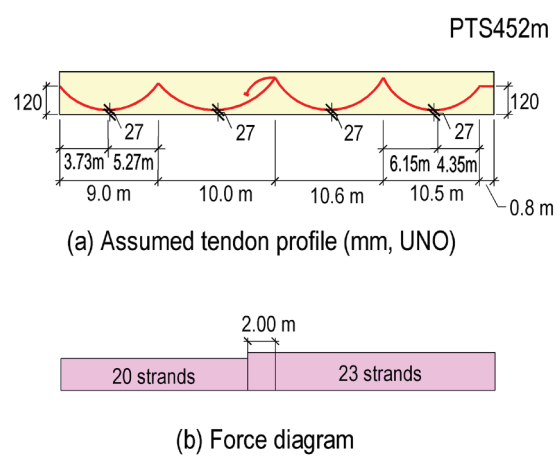
The tendon profile and force selected for unbonded tendons is shown in Fig. 6.3-1

6.4 Calculation of Balanced Loads

Balanced loads are the forces that a tendon exerts to its concrete container. It is generally broken down to forces normal to the centerline of the member (causing bending) and axial to it (causing uniform precompression) and added moments at locations of change in location of centroidal axis. Figure C6.2-2 shows two examples of balanced loading for members of uniform thickness.

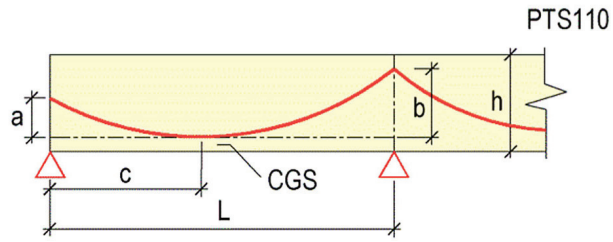
Span 1
Refer to Fig C6.2-1 and Fig. 6.4-1
a = 120—27 = 93 mm
b = 213—27 = 186 mm
L = 9.00 m
$c = \{[93/186]^{0.5}/[1 + (93/186)^{0.5}]\}* 9.00 = 3.73$ m
W_b/tendon = 2 P*a/c^2 = 119.0 kN* (2*93/1000)/3.73^2
= 119.0 kN/tendon* 0.013/m =1.59 kN/m/tendon
For 20 tendons W_b = 1.59/ tendon* 20 tendons
= 31.8 kN/m
% of DL Balanced = 31.8/ 61.20 = 52%
(less than 60% target, but considered acceptable)
Balanced load reaction, left = 31.8 kN/m* 3.73
= 118.61 kN ↓
Balanced load reaction, right= 31.8 kN/m* 5.27
= 167.59 kN ↓

The profiles of the first and last spans are chosen such that the upward force on the structure due to the tendon is uniform. This is done by choosing the loca-



Post Tensioning Profile and Force

FIGURE 6.3-1 Tendon Profile and Selected Force



Tendon Elevation in First Span

FIGURE 6.4-1

tion of the tendon low point such that in each span the profile is a continuous parabola (Fig. C6.2-1). Both spans appear to be critical and will be designed for maximum drape, in order to utilize the maximum amount of balanced loading. If the low point of the tendon is not selected at the location determined by "*c*", two distinct parabolas result. Figure C6.2-2 illustrates the condition, where the low point is not at center of a tendon span.

Span 2

Span 2 has 20 continuous strands and three short strands (added tendons) that extend from span 3 to span 2 and terminate at its right end. The balanced load from each is calculated separately.

Continuous Tendons

a = 186 mm

L = 10.0 m

For a symmetrical parabola of span "*L*," drape "*a*," and uniform force "*P*," the force normal to L is given by $8P*a/L^2$.

W_b/ tendon = $8*P*a/L^2 = (8*119*186/1000)/10^2$
= 1.77 kN/m

For 20 tendons W_b = 1.77* 20 tendons = 35.41 kN/m

% DL Balanced = 35.41/71.53 = 50% OK

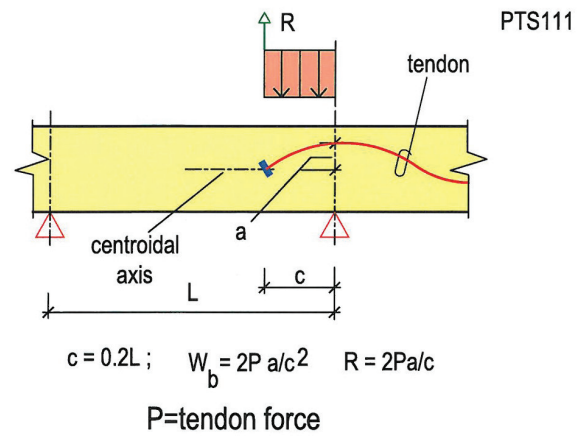
Balanced load reaction:

Left = 35.41 kN/m* 5m = 177.05 kN ↓

Right = 35.41 kN/m* 5m = 177.05 kN ↓

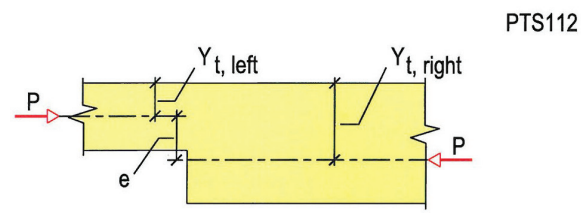
Added Tendons

Increase in the number of strands from 20 to 23, from the third span on, results in 3 strands from the third span to terminate in the second span. The terminated three strands are dead-ended in the second span. The dead end is located at a distance 0.20*L from the right support, at the centroid of the design strip (Fig. C6.4-1). The tendons are assumed horizontal over the support and concave downward toward the dead end. Hence the vertical balanced loads of these tendons will be downward, with a concentrated upward force at the dead end.



Geometry and Actions of Tendons Terminated in Span

FIGURE C6.4-1



Section at Change in Centroidal Axis

FIGURE C6.4-2

a = 93 mm

c = 0.20* 10 = 2.00 m

W_b = (3* 119.0* 2* 93/1000)/2.0^2 16.60 kN/m ↓

Concentrated force at dead end = 16.60* 2.0
= 33.20 kN ↑

PT-induced Moments Due to Shift in Centroid

Because the centroid of the design strip section is shifted at the face of the column drop, drop panel and the edge beam, there will be a moment due to axial force from prestressing at each of these locations. These moments must be included in the balanced loading to obtain a complete and correct solution. The moments are simply the post-tensioning force in the section multiplied by the shift in the section's centroid (see Fig. 6.4-2).

Moment at Face of Column Drop

M = P* shift in centroid (e) = P* ($Y_{t\text{-Left}}$-$Y_{t\text{-Right}}$)

= 23* 119.0* (120-146)/1000 = -71.16 kN-m
M = 23* 119.0* (146-143)/1000 = 8.21 kN-m

Span 3
a = 186 mm
L = 10.60 m
W_b/ tendon = 8*P*a/ L^2 = (8*119.0*186/1000)/10.60^2
= 1.58 kN/m
For 23 tendons W_b = 1.58* 23 tendons = 36.34 kN/m
% DL Balanced = 36.34/81.09 = 45% ≈ 50% OK
Balanced load reaction:
Left = 36.34 kN/m* 5.3m = 192.60 kN ↓
Right = 36.34 kN/m* 5.3m = 192.60 kN ↓

PT-induced Moments Due to Shift in Centroid
Moment at face of left column drop:
M = P* shift in centroid
= 23* 119.0 kN* (143-120)/1000
= 62.95 kN-m
Moment at face of right drop panel:
M = P* ($Y_{t\text{-}Left}$-$Y_{t\text{-}Rigth}$)
= 23* 119.0* (120-169)/1000 = -134.11 kN-m
Moment at centerline of right support:
M = 23* 119.0* (169-169) ≈ 0 kN-m

Span 4
Refer to Figure C6.2-1
a = 120-27 = 93 mm
b = 213-27 = 186 mm
L = 10.50 m
C = {$[93/186]^{0.5}$/[1 + $(93/186)^{0.5}$]}* 10.5 = 4.35 m
Wb/ tendon = 119.0 kN* (2* 93/1000)/4.35^2
= 1.17 kN/m/tendon
For 23 tendons W_b = 1.17 kN/m/tendon* 23 tendons
= 26.90 kN/m
% DL Balanced = (26.90/79.18)*100 = 34 %

The dead load in the fourth span tends to produce an upward "lift" on adjacent spans. Since the fourth span is next to a somewhat larger, more heavily loaded third span, it is advantageous to design the fourth span with a lower level of balanced loading and allow its non-prestressing load to counteract the actions in the adjoining longer span. For this reason, the level of dead load balanced in the fourth span (34%) is acceptable, even though it is well below the target amount of 60% for the critical span. The above values will be assumed for a first try. If the stress check to follow will not be satisfactory the prestressing force will be adjusted.

Balanced Load Reaction
Left = 26.90* 6.15 = 165.44 kN ↓
Right = 26.9* 4.35 = 117.02 kN ↓
Moment at drop panel face
Left of span
M = 23* 119.0* (169-120)/1000 = 134.11 kN-m
Right of span
M = 23* 119.0* (120-220)/1000 =-273.70 kN-m

Cantilever
Tendon is horizontal and straight. Hence no upward force from tendon.
Moment due to dead end anchored away from centroid:
M = 23* 119.0* (220-120)/1000 = 273.70 kN-m

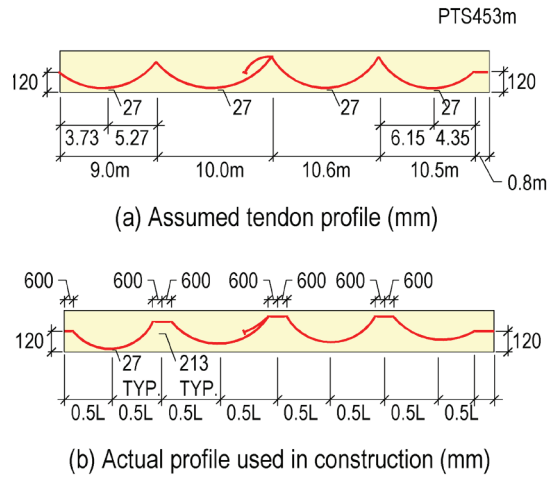
There is no vertical force over the length of the cantilever from the tendon profile of that span. However, the eccentricity of the tendon at edge of the slab results in a constant moment over the entire length of the cantilever.

The complete balanced loading consisting of up and down forces (part "a" of the figure) and the associated moments (part "b" of the figure) are shown in Fig. 6.4-4. In addition to the forces shown in the figure, there is an axial compressive force that is shown in Fig. 6.3-1b.

The actions shown in Fig. 6.4-3 represent the forces from the simplified tendon profile assumed for hand calculation and shown in Fig. C6.4-2a. In construction where unbonded system is uses, tendons in the design strip under consideration will be banded over the support line. In the perpendicular direction, the tendons will be distributed uniformly. The profile used for construction together with the one selected for hand calculation is shown in Fig. C6.4-2.

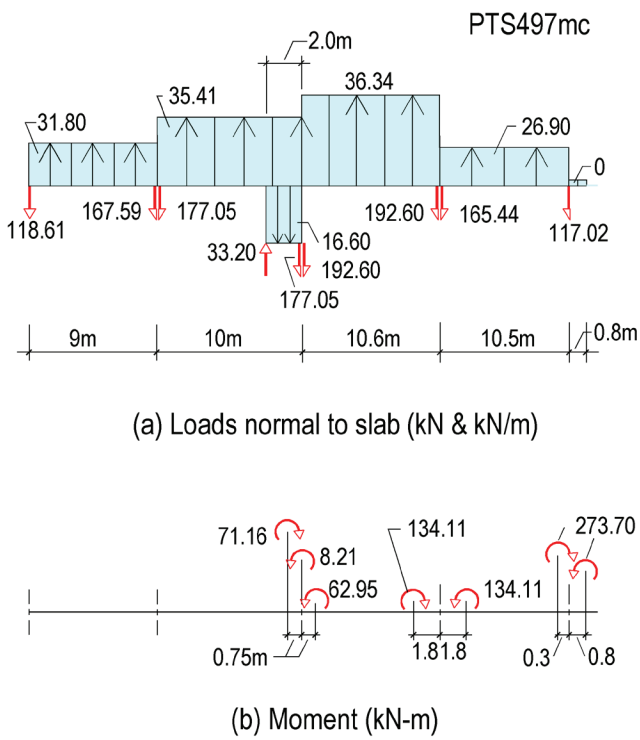
The forces exerted by a tendon to its container (concrete slab in this case) are always in static equilibrium, regardless of the geometry of tendon and the configuration of the member that contains the tendon. To guarantee a correct solution, it is critical to perform an equilibrium check for the balanced loads calculated (Fig. 6.4-3) before proceeding to the next step.

Equilibrium Check
Sum of forces in the vertical direction:
∑Forces ↑= -118.61 +(9*31.8)-167.59-177.05+(10*35.41)-(2*16.60)+33.20-177.05-192.60+(10.6*36.34)-192.60-165.44+ (10.5*26.90)-117.02 = 0.006 kN ≈ 0 OK
Sum of moments about the third support:
$\sum M_{3rd\ Support}$ = 118.61*19.0 + (167.59+177.05)*10.0-33.20*2.0- 31.8*9.0*14.5-35.41*10^2/2+16.60*2^2/2+36.34*10.6^2/2 +



Presentation of Simplified and Actual Tendon Profiles

FIGURE C6.4-2



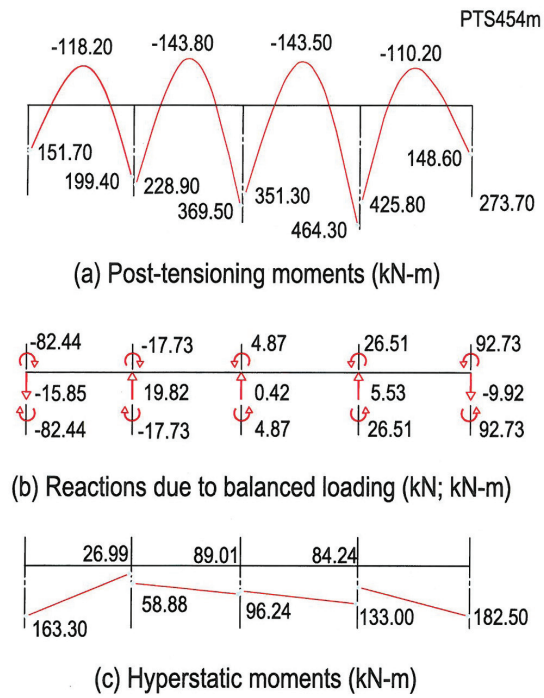
Balanced Loading

FIGURE 6.4-3

26.90*10.5*15.85-(192.60+ 165.44)*10.6-117.02*21.1-71.16+8.21+134.11+62.95-134.11-273.70+273.70 = 0.46 ≈ 0 kN-m OK

6.5 Determination of Actions due to Balanced (Post-Tensioning) Loads

The distributions of post-tensioning moments due to balanced loading, and the corresponding reactions at



Post-Tensioning Actions on Design Strip

FIGURE 6.5-1

the slab/support connections, are shown in Fig. 6.5-1. These actions are obtained by applying the balanced loads shown in Fig. 6.4-3 to the frame shown in Fig. 5 1.

Actions due to post-tensioning are calculated using a standard frame program. The input geometry and boundary conditions to the standard frame program are the same as used for the dead and live loads.

7 CODE CHECK FOR SERVICEABILITY

7.1 Load Combinations

The following lists the recommended load combinations of the building codes covered for serviceability limit state (SLS).

❖ ACI, IBC
Total load condition 1*DL + 1*LL + 1*PT
Sustained load condition 1*DL + 0.3*LL + 1*PT[23]

[23] ACI-318 specifies a "sustained" load case, but does not stipulate the fraction of live load to be considered "sustained." It is left to the judgment of the design engineer to determine the applicable fraction. The fraction selected varies between 0.2 and 0.5. The most commonly used fraction is 0.3, as it is adopted in this design example.

❖ EC2, TR43
Frequent load condition 1*DL + 0.5*LL + 1*PT
Quasi-permanent load condition 1*DL + 0.3*LL + 1*PT

For serviceability, the actions from the balanced loads due post-tensioning (PT) are used. The background for this is explained in detail in reference [Aalami, 1990].

7.2 Stress Check

For hand calculation, the critical locations for stress check are selected based on engineering judgment. The selected locations may or may not coincide with the locations of maximum stress levels. This will introduce a certain degree of approximation in design, which reflects the common practice for hand calculations. Computer solutions generally calculate stresses at multiple locations along a span, thus providing greater accuracy. For brevity, only three locations will be selected for this design example. Point *A* is at the face-of-support; Point *B* is at the face of the drop panel; and point *C* is at the midspan (Fig. 7.2-1).

Using the Moment diagrams of Fig. 5-2 and 5-3 as guide, several critical locations are identified for the stress check. These are shown as sections A, B and C in Fig. 7.2-1.

Stresses
$\sigma = (M_D + M_L + M_{PT})/S + P/A$
$S = I/Y_c$

Where, M_D, M_L, and M_{PT} are the moments across the entire tributary of the design strip. S is the section modulus of the entire tributary; *A* is the cross-sectional area of the entire tributary; *I* is the second moment of area of the entire tributary; and Y_c is the distance of the centroid of the entire tributary to the farthest tension fiber of the entire tributary.

The parameters for stress check at point A are:
$S_{top} = 4.134e+10/169 = 2.446e+8\ mm^3$
$S_{bot} = 4.134e+10/271 = 1.525e+8\ mm^3$
$A = 10.35*1000*240 + 3600*200$
$= 3.204e+6\ mm^2$
$P/A = -2737*1000/3.204e+6 = -0.85$ MPa

A. Based on ACI 318-11/IBC 2012
Stress checks are performed for the two load conditions of total load and sustained loads.

At Point A

❖ Total Load Combination
Stress limit in compression: 0.60* 40 = 24 MPa
Stress limit in tension: $0.5*\sqrt{40} = 3.16$ MPa
$M_D + M_L + M_{PT} = (-901-347.40 + 425.80)$
$= -822.60$ kN-m

Bottom fiber
$\sigma = -822.60*1000^2/1.525e+8-0.85 = -6.24$ MPa Compression < -24 MPa OK

Top Fiber
$\sigma = 822.60*1000^2/2.446e+8-0.85$ Mpa = 2.51 MPa Tension < 3.16 MPa OK

❖ Sustained Load Combination
Stress limit in compression: 0.45* 40 = 18 MPa
Stress limit in tension: $0.5*\sqrt{40} = 3.16$ MPa
$M_D + 0.3\ M_L + M_{PT} = (-901-0.3*347.40 + 425.80)$
$= -579.42$ kN-m

Bottom Fiber
$\sigma = -579.42*1000^2/1.525e+8-0.85 = -4.65$ MPa Compression< -18 MPa OK

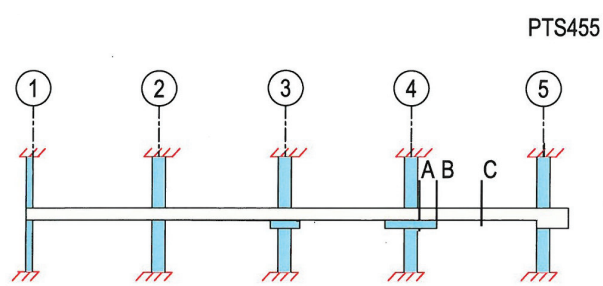


FIGURE 7.2-1

Top Fiber
$\sigma = 579.42*1000^2/2.446e+8-0.85 = 1.52$ MPa Tension < 3.16 MPa OK

B. Based on EC2
Stress checks are performed for the two load conditions of frequent load and quasi-permanent loads.

❖ Frequent Load Condition
$\sigma = (M_D + 0.5\ M_L + M_{PT})/S + P/A$

At Point A
Stress Thresholds
Compression = 0.60* 40 = -24 Mpa
Tension = f_{ctm} = 3.51 MPa
$M_D + 0.5M_L + M_{PT}$
$= (-901-0.5*347.40 + 425.80) = -648.90$ kN-m

Top Fiber
σ = 648.90*1000^2/2.446e+8-0.85 = 1.80 MPa Tension < 3.51 MPa OK
Bottom Fiber
σ = -648.90* 1000^2/1.525e+8-0.85 =-5.10 MPa Compression <-24 MPa OK

❖ Quasi-permanent load condition:
$\sigma = (M_D + 0.3\ M_L + M_{PT})/S + P/A$

At Point A
Stress Thresholds
Compression = 0.45* 40 = -18 MPa
Tension = f_{ctm} =3.51 MPa
$M_D + 0.3M_L + M_{PT}$
= (-901-0.3*347.40 + 425.80) = -579.42 kN-m

Top Fiber
σ = 579.42*1000^2/2.446e+8-0.85 = 1.52 MPa Tension < 3.51 MPa OK

Bottom Fiber
σ = -579.42* 1000^2/1.525e+8-0.85 = -4.65 MPa Compression <-18 MPa OK

C. Based on TR-43
❖ Frequent load condition:
$\sigma = (M_D + 0.5\ M_L + M_{PT})/S + P/A$

At Point A
Stress Limits
Compression (support) = 0.3* f_{ck} = 0.3*40 = 12 MPa
Tension (without bonded reinforcement)
= 0.3 $f_{ctm,fl}$ = 0.3*1.36*3.51 = 1.43 MPa
Tension (with bonded reinforcement) = 0.9 $f_{ctm,fl}$
= 0.9*1.36*3.51 = 4.30 MPa
$M_D + 0.5M_L + M_{PT}$ = (-901-0.5*347.40 + 425.80)
= -648.90 kN-m

Top fiber
σ = 648.90*10002/2.446e+8-0.85 MPa = 1.80 MPa
Tension > 1.43 MPa
but less than 4.30 MPa; hence bonded reinforcement required.[24]

Bottom fiber
σ = -648.90* 1000^2/1.525e+8-0.85 = -5.10 MPa Compression <-12 MPa OK

Other points are evaluated in a similar manner. The outcome is listed in the following table (Table 7.2-1):

The following illustrates the calculation of moments at interior of a span, such as point B for span under consideration.

Centerline moments and shears for *DL, LL* and *PT* obtained from frame analysis, along with the externally applied loads are shown below for the fourth span. The calculation of the values at the face-of-support follows simple statics of the free-body diagram shown below. In the following the calculation of moment at the face of drop panel in the fourth span is detailed. Other locations follow a similar procedure (Fig. 7.2-2).

Moment due to DL at the face of drop panel distance 1.80m from the fourth support

M_{DL} = 388.16* 8.7-575.16-48.72*0.3*8.55-79.18* 8.7^2/2 = -319.70 kN-m
Moment due to LL
M_{LL} = 146.19* 8.7-223.37-31.05* 8.7^2/2
= -126.60 kN-m
Moment due to PT
M_{PT} = -126.80* 8.7 +459.17+ 26.90* 8.7^2/2-
273.70+134.11 = 234.45 kN-m

7.3 Crack Width Control

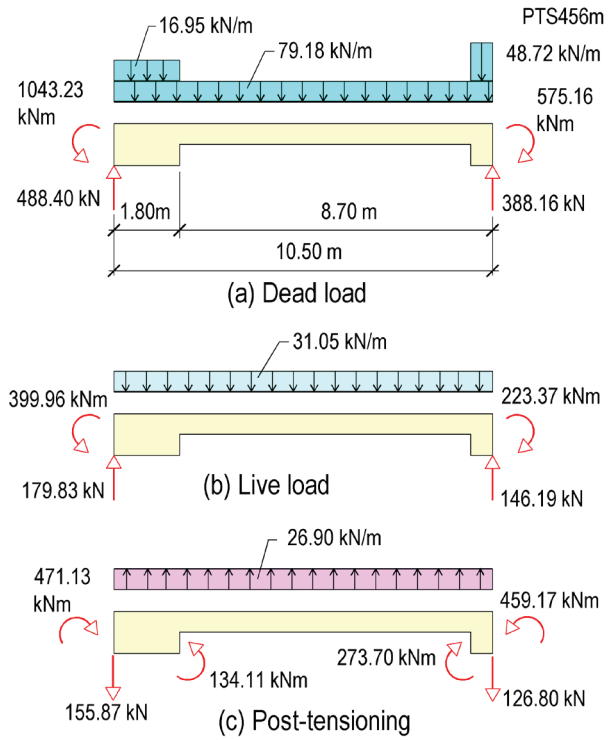
A. Based on ACI 318-11/IBC 2012
ACI 318-11/IBC 2012 do not stipulate specific measures to follow for crack control of slabs designed as two-way systems. The limit imposed on tensile stresses keeps the slabs essentially crack free, when in service.

B. Based on EC2 and TR-43[25]
The allowable crack width for members reinforced with unbonded tendons (Quasi-permanent load combination) is 0.3 mm, and for bonded tendon (Frequent load combination) is 0.2 mm. Since in this example the maximum computed tensile stress is within the threshold limit, crack width calculation is not required. If the computed tensile stress exceeds the threshold, EC2 recommends to limit the bar diameter and bar spacing to the values given in Table 7.2N or 7.3N of EC2 to control the width of probable cracks. The following example illustrates the point.

EXAMPLE
To illustrate the procedure for crack control recommended in EC2, as an example let the maximum tensile stress exceed the threshold value by a large margin.

[24] The required bonded reinforcement is calculated in Section 7.4-Minimum Reinforcement

[25] EN 1992-1-1:2004(E) , Section 7.3.3, and TR-43 2nd Edition, Section 5.8.3



Free Body Diagram of Fourth Span

FIGURE 7.2-2

Given: computed hypothetical farthest fiber tensile stress in concrete f = 30MPa
Required : reinforcement design for crack control

Calculate stress in steel at location of maximum concrete stress: σ_s = (f/Ec)*Es
Where f is the hypothetical tensile stress in concrete under service condition
σ_s = (30/35220)*200000 = 170 MPa (this is a hypothetical value)
Crack spacing can be limited by either restricting the bar diameter and/or bar spacing. Use the maximum bar spacing from Table 7.3 N for the σ_s of 170 MPa.
From Table: for 160 MPa-300 mm
200 MPa-250 mm
By interpolation, maximum spacing for 170 MPa is 287 mm.
Limit the spacing of reinforcement to 285 mm or less (280 mm) in order to control cracking. Note that based on the magnitude of the computed tensile stress in concrete the area of the required reinforcement is calculated separately,

7.4 Minimum Reinforcement

There are several reasons why the building codes specify a minimum reinforcement for prestressed members. These are:

❖ **Crack control, where potential of cracking exists:** Bonded reinforcement contributes in mitigating local cracks. The contribution of bonded reinforcement to crack control is gauged by the stress it develops under service load. Change of stress in bonded reinforcement from applied strain is a function of its modulus of elasticity and its cross-sectional area. Hence, the area of reinforcement considered available for crack control is *(As + Aps)*, where *Aps* is the area of bonded tendons. It is recognized that both bonded and unbonded prestressing provide precompression. While the physical presence of an unbonded tendon may not contribute to crack control, the contribution through the precompression it provides does. However, for code compliance and conformance with practice, the contribution of unbonded tendons is not included in the aforementioned sum.

❖ **Ductility:** One of reason ACI-318 specifies a minimum bonded reinforcement over supports of members reinforced with unbonded tendons is to provide ductility at the location. Where unbonded tendons are used, the required minimum area is provided through As only. Current ACI 318/IBC do not specify a minimum of non-stressed bonded reinforcement in post-tensioned members reinforced with bonded tendons.

Use 16 mm bars (Area = 201 mm^2; Diameter = 16 mm) for top and bottom, where required
d = 240- 20-16/2 =212 mm

A. Based on ACI 318-11/IBC 2012[26]

❖ Unbonded Tendons

Supports

ACI 318[27]/IBC require a minimum area of passive (non-stressed reinforcement to be placed over the supports, where unbonded tendons are used. The minimum area is expressed in terms of the cross-sectional geometry of the design strip, and the strip orthogonal to it. A_{cf} is the larger gross cross-sectional area of the design strips in the two orthogonal directions for the support under consideration. Figure C7.4-1 illustrates the applicable locations to

[26] ACI 318-11, Section 18.9
[27] ACI 318-11, Section 18.9.3

determine the cross-sectional areas. Line *PP* refers to the section in the design strip direction and *FF* to the section orthogonal to it.

A_S = 0.00075* Acf
At section A (Fig. 7.2-1):
In direction of design strip:
A_S = 0.00075* 0.5*(10600*240 +10500*240)
= 1899 mm^2
In the orthogonal direction to the design strip the spans adjacent to the support under consideration are 10.60 and 10.35 m. Hence,
A_S = 0.00075* 0.5*(10600*240 +10350*240)
= 1886 mm^2
A_s = 1899 mm^2 applies
Number of bars = 1899/201 = 9.4
Use 10-16mm bars = 10* 201 mm2 = 2010 > 1899 mm^2 provided top
Spans

The minimum passive reinforcement at midspan for unbonded tendons depends on the value of computed (hypothetical) tension at the bottom fiber. If the hypothetical tension stress is less than $0.166\ f'_c{}^{0.5}$, based on ACI 318,[28] no midspan minimum bottom rebar is required. It is re-iterated that the computed tensile stress is not permitted to exceed $0.5\ f'_c{}^{0.5}$.

At Point C in Span
At midspan A_s = N_c/(0.5*f_y) if hypothetical tensile stress > 0.166*$\sqrt{f'_c}$
where N_c is the total of tension force in the tensile zone of the section
Computed hypothetical tensile stress: f_{ct} = 1.95MPa
Stress Limit = 0.166* √40 = 1.05 MPa
1.95 MPa > 1.05 MPa ∴ Minimum steel is required
Compressive stress at top: fc = — 4.15 MPa
The relationships given in Fig. 7.4-1 will be used to determine the force of tensile zone (N_c)
Depth of tension zone from bottom
= 1.95* 240/(1.95+4.15) = 77 mm
N_c = 77mm* 1.95 MPa* 10350/(2* 1000)
= 777.03 kN
A_s = 777.03*1000 /(0.5*460) = 3378 mm^2
Number of bars = 3378/ 201 =16.8
Use 17-16 mm bars = 17* 201 = 3417 > 3378 mm^2 OK

❖ Bonded (Grouted) Tendons
There is no requirements for minimum reinforcement based on either geometry of the design strip, nor its

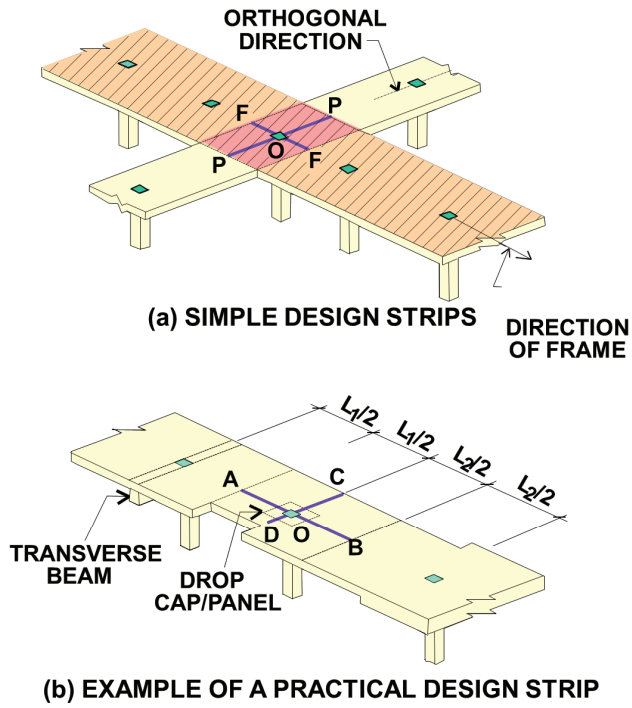


FIGURE C7.4-1 Illustration of Sections for Minimum Rebar (P563)

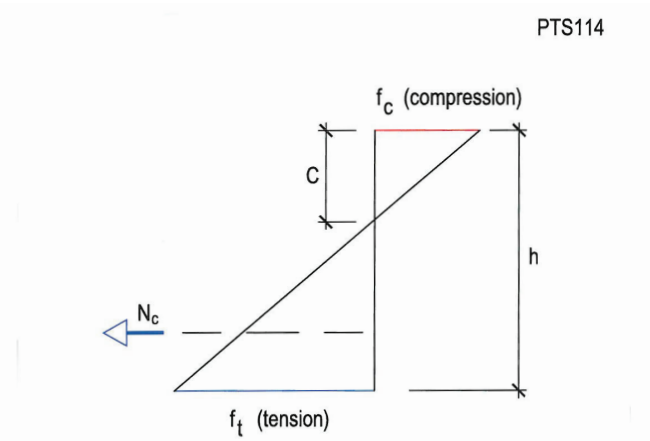


FIGURE 7.4-1 Distribution of Strain Over Section of Member

hypothetical tensile stresses. The minimum requirement is handled through the relationship between the cracking moment of a section and its nominal strength in bending. This is handled in the "strength" check of the member.

B. Based on EC2[29]
EC2 specifies the same requirement for minimum reinforcement at supports and spans, and also for both unbonded and bonded tendons. Two checks apply. One is based on the cross-sectional geometry of

[28] ACI 318-11 Section 18.9.3.2

[29] EN 1992-1-1:2004(E), Section 9.3.1 & 7.3.2

TABLE 7.2-1 Summary of Service Stress Checks (T160SI)

Span 4		Section A	Section B	Section C
Based on ACI 318-11/IBC 2009				
Sustained load	f_t (MPa)	**1.52**	**0.14**	**-3.33**
	f_b (MPa)	**-4.65**	**-2.34**	**1.13**
	F_t (MPa)	3.16	3.16	-18
	F_b (MPa)	-18	-18	NA
		OK	**OK**	**OK**
Total load	f_t (MPa)	**2.51**	**1.03**	**-4.15**
	f_b (MPa)	**-6.24**	**-3.23**	**1.95**
	F_t (MPa)	3.16	3.16	-24
	F_b (MPa)	-24	-24	3.16
		OK	**OK**	**OK**
Based on EC2				
Frequent Load	f_t (MPa)	**1.8**	**0.4**	**-3.56**
	f_b (MPa)	**-5.1**	**-2.6**	**1.36**
	F_t (MPa)	3.51	3.51	-24
	F_b (MPa)	-24	-24	3.51
		OK	**OK**	**OK**
Quasi-Permanent Load	f_t (MPa)	**1.52**	**0.14**	**-3.33**
	f_b (MPa)	**-4.65**	**-2.34**	**1.13**
	F_t (MPa)	3.51	3.51	-18
	F_b (MPa)	-18	-18	3.51
		OK	**OK**	**OK**
Based on TR-43				
Frequent Load	f_t (MPa)	**1.8**	**0.4**	**-3.56**
	f_b (MPa)	**-5.1**	**-2.6**	**1.36**
	F_t (MPa)	4.3	4.3	-16
	F_b (MPa)	-12	-12	4.3
		OK	**OK**	**OK**

Note: F_t and F_b are the respective top and bottom fiber allowable (threshold) stresses; F_c is allowable compressive stress.

the design strip and its material properties and the other on computed stresses. In the former, the minimum reinforcement applies to the combined contributions of stressed and non-stressed reinforcement. Hence, the participation of each is based according to the strength it provides, the prestressing steel is accounted for with higher values. The reinforcement requirement for crack control is handled separately.

❖ Unbonded and Bonded Tendons

Supports

At section A (Fig. 7.2-1):

$A_{smin} \geq (0.26^* f_{ctm}{}^* b_t{}^* d/f_{yk}) \geq 0.0013^* b_t{}^* d$

Since in EC2 the minimum reinforcement is a function of (bt^*d) cross-sectional area, at the face-of-support the cross-sectional area including the drop panel is used.

Cross-sectional Area

b_t = 10350 mm

Drop panel width = 3,600 mm

Drop panel depth below slab = 200 mm

Tributary cross-sectional area = 10,350*240 + 3,600*200 = 3.204^*10^6 mm^2

f_{ctm} = 0.3*40(2/3)= 3.51 MPa

(i) $A_{smin} = 0.26^* f_{ctm}{}^* b_t{}^* d/f_{yk}$

$= 0.26^* 3.51^* 3.204^*10^6/460 = 6{,}356\ mm^2$

(ii) $A_{smin} = 0.0013^* b_t^* d$

$= 0.0013^* 3.204^* 10^6 = 4{,}165.2\ mm^2$

Therefore, $A_{smin} = 6{,}356\ mm^2$

Contribution of reinforcement from bonded Prestressing:

$Aps^*(fpk/fyk) = 23^* 99^*1860/460$

$= 9207\ mm^2 > 6356\ mm^2$

Hence, no additional bonded reinforcement is required.

Span

At section C in span (Fig. 7.2-1)

$b_t = 10350$ mm

(i) $A_{smin} = 0.26^* f_{ctm}^*b_t^*d/f_{yk}$

$= 0.26^*3.51^* 10350^*212/460 = 4353$ mm2

(ii)$A_{smin} = 0.0013^*b_t^*d$

$= 0.0013^*10350^* 212 = 2852\ mm^2$

Hence, $A_{smin} = 4353\ mm^2$

Contribution of reinforcement from bonded Prestressing:

$Aps^*(fpk/fyk) =$

$23^* 99^*1860/460 = 9207\ mm^2 > 4353\ mm^2$

Hence, no additional bonded reinforcement is required.

❖ Minimum Reinforcement for Crack Control

In EC2 necessity of reinforcement for crack control is triggered, where computed tensile stresses exceed a code-specified threshold.

At all the three locations selected for code compliance, the hypothetical tensile stress of concrete is below the threshold for crack control. Hence, no crack control reinforcement is required.

EXAMPLE

For demonstration of EC2[30] procedure for crack control, let the maximum hypothetical tensile stress in concrete exceed the threshold set in the code (3.51MPa). Determine the required crack control reinforcement for the section reinforced with unbonded tendons.[31]

Given

$f_b = 3.7$ MPa (tension) at bottom

$f_t = -5.2$ MPa (compression) at top

Depth of section = 240 mm

Width of section = 10,350 mm

Required: Reinforcement for Crack Control

$\sigma_s = f_{yk} = 460$ MPa

k = 1

Depth of tension zone at bottom, using Fig. 7.4-1

$= 3.7^*240/(3.7 + 5.2) = 100$ mm

$A_{ct} = 100^*10350=10.35e+5\ mm^2$

$k_c = 0.4^* [1-(\sigma_c /(k1 (h/h^*) f_{ct,eff})]$

$\sigma_c = N_{ED} /bh = 1.10$ MPa (average precompression)

$h^* = h = 240$ mm

$k_1 = 1.5$ (since section is in compression)

Criteria

$f_{ct,eff} = f_{ctm} = 0.3^*(40)^{(2/3)} = 3.51$ MPa

Design

$k_c = 0.4^* [1-(1.10 /(1.5 (240/240) 3.51)] = 0.32$

$A_{smin} = k_c\ k\ f_{ct,eff}\ A_{ct} /\sigma_s$

$A_{smin} = 0.32^* 1^* 3.51^* 10.35e+5 /460 = 2499\ mm^2$

C. Based on TR-43

If the hypothetical tensile stress calculated for a panel (design strip as used in this example) exceeds the specified threshold given below, add non-prestressed rebar in addition to the prestressing to resist *Nc* [32]

(i) where unbonded tendons are used, and the hypothetical full tributary tensile stress exceeds 0.3 $f_{ctm,fl}$; and

(ii) where bonded tendons are used, and the hypothetical full tributary tensile stress exceeds 0.9 $f_{ctm,fl.}$

The amount of non-tensioned reinforcement depends on the tensile force (*Nc*) developed in the tensile zone of the location being considered. The area of (*As + Aps*) shall be adequate to resist Nc, where Aps is the area of available "bonded" reinforcement.

❖ Unbonded Tendons

TR43 specifies a minimum amount of non-prestressed reinforcement over the supports. The required minimum is based on both the cross-sectional geometry of the design strip and the computed tensile stresses.

At Support

Based on Geometry[33]

$A_{smin} = 0.00075$ Acf

Acf = cross-sectional area of the design strip in direction of analysis

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30 EN 1992-1-1:2004(E), Section 7.3.2(3)

31 For members reinforced with grouted tendons, the cross-sectional area of grouted tendons can be used to contribute to the minimum required area for crack control.

32 TR-43 2nd Edition, Section 5.8.1; Table 4

33 TR-43 2nd Edition, Section 5.8.8

A_S = 0.00075* 0.5*(10600*240 +10500*240)
= 1899 mm^2

Based on Computed Stresses[34]
Refer to Fig. 7.4-2 where f_{cc} is the concrete fiber stress in compression; f_{ct} is the extreme concrete fiber stress in tension.
Depth of tension zone: h-x = -f_{ct}*h/(f_{cc}-f_{ct})
f_{ct} = tensile stress = 1.80 MPa
f_{cc} = compressive stress =-5.10 MPa
h-x = 1.80*440/(5.10+1.80) = 115 mm
As = F_t /(5*f_{yk} /8)
Where F_t is the total tensile force over the tensile zone of the entire section
F_t = -f_{ct}* b* (h-x)/2
= 1.80*10350*115/(2*1000) = 1071.23 kN
As = 1071.23*1000/(5*460/8) =3726 mm^2
The applicable rebar for this condition is the calculated value less area of unbonded tendons. Hence
As = 3726-23*99 = 1,449 mm^2
Comparing (i) and (ii), As = 1,899 mm^2
Use 10-16mm bars = 10* 201 mm^2
= 2010 > 1,899 mm^2 OK

At Span
At span bonded reinforcement is required if:
computed stress is greater than 0.3 $f_{ctm,fl}$ = 1.43 MPa.
f_{ct} =calculated tensile stress = 1.36 MPa
Since the calculated tensile stress is less than 0.3fctm,fl, additional bonded reinforcement is not required.

❖ Grouted Tendons
At support (Point A)
Based on Geometry-the same as in unbonded tendons. Hence,
A_{smin} = 1899 mm^2
However, the area of grouted tendon counts toward the requirement
Available reinforcement = 23* 99 = 2277 mm^2
> 1899 mm^2 OK

Based on Computed Stresses
Refer to Fig. 7.4-2 where fcc is the concrete fiber stress in compression; fct is the extreme concrete fiber stress in tension.
f_{ct} = tensile stress = 1.80 MPa
f_{cc} = compressive stress =-5.10 MPa
h-x = 1.80*440/(5.10+1.80) = 115 mm
As = F_t /(5*f_{yk} /8)
Where F_t is the total tensile force over the tensile zone of the entire section.

[34] TR-43 2nd Edition, Section 5.8.7

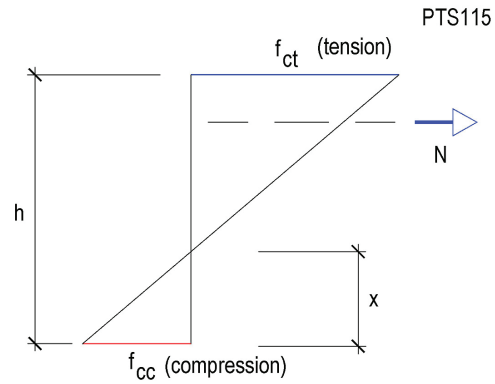


FIGURE 7.4-2 Stress Diagram

F_t = -f_{ct}* b* (h-x)/2 = 1.80*10350*115/(2*1000) = 1071.23 kN
As = 1071.23*1000/(5*460/8) =3726 mm^2
The applicable rebar for this case includes the contribution of bonded tendons. Hence
Aps 23*99 = 2,277 mm^2
As required = 3,726-2,277 = 1,449 mm^2
Comparing (i) and (ii), As = 1,449 mm^2 , since (i) is deemed satisfied
Use 10-16mm bars = 8* 201 mm^2 = 1,608 > 1,449 mm^2 OK

At Span (Point C)
Since the calculated stress is within the threshold value, no rebar needed
The minimum rebar required from different codes is summarized in TABLE 7.4-1

TABLE 7.4-1 Summary of Minimum Rebar (mm^2) (T161SI)

Code	Unbonded		Bonded	
	Support	Span	Support	Span
ACI/IBC	1899	3378	0	0
EC2	0	0	0	0
TR43	1899	0	1449	0

7.5 Deflection Check

Recognizing that (i) the accurate determination of probable deflection is complex [TN292]; and (ii) once a value is determined, the judgment on its adequacy at design time is subjective, and depends on unknown, yet important, parameters such as age of concrete at time of installation of nonstructural members that are likely to be damaged from large displacement, in common construction, deflection checks are generally performed following a simplified procedure. A rigor-

ous analysis is initiated, only where the parameters of design and applied loads are more reliably known. In most cases, post-tensioned members are sized according to recommended span/depth ratio proven to perform well in deflection.[35]

The simplified procedure includes:

(i) For visual and functional effects, total long-term deflection from the day supporting shutters are removed not to exceed a value that depending on the code used varies between (span/250 EC2) and (span/240 USA). Camber can be used to offset the impact of displacement.

(ii) Immediate deflection under design live load not to exceed (span/500 for EC2/TR43 designs) or (span/480 for USA).[36]

Both ACI 318/IBC and EC2 (EN 1992-1-1:2004(E)), tie the deflection adequacy to displacement subsequent to the installation of members that are likely to be damaged. This requires knowledge of construction schedule and release of structure for service. In the following the common design practice is followed.

For assessment of long-term displacement in the context of foregoing, ACI 318 recommends a multiplier factor of 2.[37]

Deflections are calculated using a frame analysis program for each of the load cases: dead, live and post-tensioning. Gross cross-sectional area and linear elastic material relationship are used. Point C at the middle of span 4 is selected for deflection check. The values for this point are:

Span 4 Deflection

Dead Load	5.5 mm
Post-Tensioning	-2.1 mm
Dead Load + PT	3.4 mm
Live Load Deflection	2.1 mm

❖ Long-term Deflection

Multiplier factor assumed for effects of creep and shrinkage on long-term deflection = 2[38]

Load combination for long-term deflection, using a factor of 0.3 for sustained "quasi-permanent" live load:
(1.0*DL + 1.0*PT + 0.3*LL)*(1 + 2)
Long-term deflection: (1 + 2)*(3.4 + 0.3*2.1) = 12.1 mm
Deflection ratio = 12.1/(10.5*1000) = 1/867 < 1/250 OK

❖ Instantaneous Deflection Due to Design Live Load

Live load deflection = 2.1 mm
Deflection ratio = 2.1/(10.5*1000) = 1/5000
<1/480 or 1/500 OK

Deflection does not generally govern the design for members dimensioned within the limits of the recommended values in ACI 318 and balanced within the recommended range, and when subject to loading common in building construction. For such cases, deflections are practically always within the permissible code values.

8 CODE CHECK FOR STRENGTH

8.1 Load Combinations

❖ ACI-318/IBC
1.2*DL + 1.6*LL + 1* Hyp
1.4*DL + 1*HYP

❖ EC2
1.35*DL + 1.5*LL + 1* Hyp

❖ TR43
1.35*DL + 1.5*LL + 0.9* Hyp

For strength combination, the hyperstatic (Hyp) actions (secondary) from prestressing are used. The background for this is explained in detail in reference [Aalami, 1990].

8.2 Determination of Hyperstatic Actions

The hyperstatic moments are calculated from the reactions of the frame analysis under balanced loads from prestressing (Loads shown in Fig. 6.4-4). The reactions obtained from a standard frame analysis are shown in Fig. 8.2-1a. The reactions shown cause the hyperstatic moments in the frame shown in Fig. 8.2-1b.

The hyperstatic (secondary) reactions must be in self-equilibrium, since the applied loading (balanced loads) are in self-equilibrium.

Check the validity of the solution for static equilibrium of the hyperstatic actions using the reactions shown in Fig. 8.2-1a:
ΣVertical Forces = -15.85 + 19.82 +0.42 + 5.529-9.924

---

35 TR-43 5.8.4; ADAPT-TN292
36 Both ACI 318 and EN 1992-1-1:2004(E) tie the deflection check for long-term values subsequent to installation of members that are likely to be damaged from added deflection.
37 ACI 318-11 R9.5.2.5
38 ACI 318 multiplier factor

= 0.005 ≈ 0 OK
ΣMoments about Support 1 = -82.44*2-17.73*2 +4.87*2 + 26.51*2 + 92.73*2 + 19.82*9 +0.42*19 + 5.529*29.6-9.924*40.1 = -0.05 kNm ≈ 0 OK

8.3 Calculation of Design Moments

The design moment (M_u) is the factored combination of dead, live and hyperstatic moments.

Using ACI/IBC
Design moments are:
$M_{U1} = 1.2^* M_D + 1.6^* M_L + 1.0^* M_{HYP}$
$M_{U2} = 1.4^* M_D + 1.0^* M_{HYP}$

The second combination governs, when the values from dead load are eight times or larger than live loads. This is a rare condition.

By inspection, the second load combination does not govern, and will not be considered in the following.

The factored moments for the codes considered are listed in the following table.

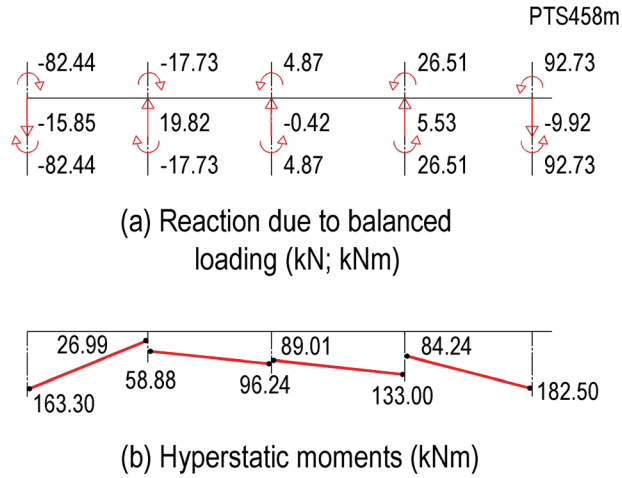
8.4 Strength Design (ULS) for Bending and Ductility

The strength design for bending consists of two provisions, namely

❖ The design capacity ($\Phi^* M_n$; R) shall exceed the demand. A combination of prestressing and non-stressed steel provides the design capacity.

❖ The ductility of the section in bending shall not be less than the limit set in the associated building code. The required ductility is deemed satisfied, if failure of a section in bending is initiated in post-elastic response of its reinforcement, as opposed to crushing of concrete. For the codes covered in this example this is achieved through the limitation imposed on the depth of the compression zone (see Fig. C-8.4-1). The depth of compression zone is generally limited to 50% or less than the distance from the compression fiber to the farthest reinforcement (dr). Since the concrete strain (e_c) at crushing is assumed between 0.003 and 0.0035, the increase in steel strain (e_s) will at minimum be equal to that of concrete at the compression fiber. This will ensure extension of steel beyond its yield point (proof stress) and hence a ductile response.

For expeditious hand calculation, the flexural capacity of a post-tensioned member in common building structures can be approximated by assuming a



Post-Tensioning Actions on Design Strip

FIGURE 8.2-1

conservative maximum stress for prestressing tendons. For detailed application of the code-proposed formulas refer to TN179. Application of strain compatibility for the calculation of section capacity is the accurate option (see TN178 for details), but its application for hand calculation is not warranted in daily design work of a consulting office, unless a software is used.

There are two justifications, why the simplified method for ULS design of post-tensioned sections in daily design work are recommended. These are:

(i) Unlike conventionally reinforced concrete, where at each section along a member non-prestressed reinforcement must be provided to resist the design moment, in prestressed members this may not be necessary, since prestressed members possess a

TABLE 8.3-1 Design Moments (T162SI)

Span 4	Point A	Point B	Point C
M_D (kN-m)	-901.00	-319.70	296.89
M_L (kN-m)	-347.40	-126.60	116.20
M_{HYP} (kN-m)	84.24	99.12	133.40
ACI 318-11/IBC 2012 : 1.2DL+1.6LL+1Hyp			
M_{U1} kN-m	-1552.80	-487.08	675.59
EC2 : 1.35DL+1.5LL+1Hyp			
M_{U1} kN-m	-1653.21	-522.38	708.50
TR 43 : 1.35DL+1.5LL+0.9Hyp			
M_{U1} kN-m	-1661.63	-532.29	695.16

base capacity along the entire length of prestressing tendons (Fig. C- 8.4-2a,b). Non-stressed reinforcement is needed at sections, where the moment demand exceeds the base capacity of the section.

(ii) In conventionally reinforced concrete, the stress used for rebar at ULS is a well-defined in the principal building codes. For prestressed sections, however, the stress in tendon at ULS is oftentimes expressed in terms of an involved relationship—hence the tendency to use a simplified, but conservative scheme for everyday hand calculation. For repetitive work, computer programs are recommended.

Using strain compatibility procedure[39] the required reinforcement for each of the three codes are calculated. The outcome is as follows.

❖ **Cracking Moment Larger than Moment Capacity:** Where cracking moment of a section is likely to exceed its design capacity in flexure, reinforcement is added to raise the moment capacity. In such cases, the contribution of each reinforcement is based on the strength it provides. If the minimum value is expressed in terms of cross-sectional area of reinforcement, the applicable value is ($A_s + A_{ps} * f_{py}/f_y$).

❖ Bonded (Grouted) Tendons

ACI-318[40]/IBC requires that for members reinforced with bonded tendons the total amount of prestressed and nonprestressed shall be adequate to develop a factored load at least 1.2 times the cracking load computed on the basis of the modulus of rupture of the section. In practice, this is taken as cracking moment of the section *Mcr*.

The necessity and amount of rebar is defined as a function of cracking moment of a section (Mcr). For Prestressed Members

Mcr = (fr + P/A)*S
Where, fr is the modulus of rupture defined[41]
Fr = 0.625 √f'c = 0.625 √40 = 3.95 MPa

P/A is the average precompression, and S the section modulus. The Table 7.4-1 summarizes the leading values and the outcome.

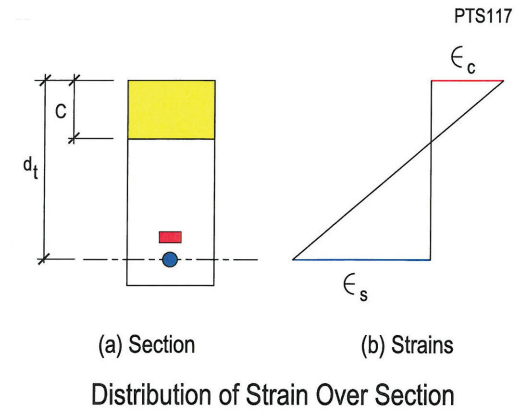


FIGURE C8.4-1 Distribution of Strain over Section

Since at both the face-of-support (section A) and mid-span (section C) the design capacity of the section with prestressing alone exceeds 1.2*Mcr, no additional rebar is required from this provision.

In design situations like above, where the design is initiated by determination of whether a value is less or more than a target, it is advisable to start the check using a simplified, but conservative procedure. If the computed value is close to the target, design check can be followed with a more rigorous computation.

Table 8.4-1 Summary of Required Reinforcement for Strength Limit State (mm2) (T163SI)

Code	Unbonded		Bonded	
	Support	Span	Support	Span
ACI/IBC	3120	1923	1029	0
EC2	4945	2590	3003	726
TR43	4945	2560	3003	726

Assume the following:
Cover to strand CGS = 40 mm;
hence d = h (thickness)-40
Moment arm = 0.9d
Design force in strand = *Aps** 1860 MPa; Φ = 0.9
At face-of-support, with 23 strands, 1860 MPa strength
Φ*Mn = 23* 99*0.9* 1860* (440-40)* 0.9/10^6 = 1372.21 kNm
The capacity is less than 1.2Mcr=1411 kNm for this section. Rebar has to be added.
Design moment at midspan is calculated in a similar manner.

39 ADAPT-TN178
40 ACI 318-11 Section 18.8.2
41 ACI 318-11 Section 9.5.2.3

TABLE 8.4-2 Cracking Moment Values and Parameters (T164SI)

Basic parameters and analysis	Span 4	Section A	Section C
	S_{top} (mm³)	2.45E+08	
	S_{bot} (mm³)		9.93E+07
	P (kN)	2737	2737
	P / A (MPa)	-0.85	-1.1
	fr + (P/A)	4.8	5.05
	Mcr (KNm)	1176	501.46
	1.2 Mcr (kNm)	1411	601
	Φ Mn (kNm)	1372	686
	Status	Added rebar	OK

TABLE 8.4-3 Envelope of Reinforcement for Serviceability (SLS) and Strength Conditions (ULS) (mm²) (T165SI)

Code	Unbonded		Bonded	
	Support	Span	Support	Span
ACI/IBC	3120	3378	1029	0
EC2	4945	2590	3003	726
TR43	4945	4235	3003	726

8.5 Punching Shear Check and Design

For moment capacity, the values obtained for a given section using different major building codes do not vary substantially. But, for punching shear, the treatment and outcome differ significantly. Due to the larger variation, the subject matter is treated in greater detail separately (Chapter 4, Section 4.11.6).

9 - CODE CHECK FOR INITIAL CONDITION

At stressing (i) concrete is at low strength; (ii) prestressing force is at its highest value; and (iii) live load generally envisaged to be counteracted by prestressing is absent. As result, the stresses experienced by a member can fall outside the envelope of the limits envisaged for the in-service condition. Hence, post-tensioned members are checked for both tension and compression stresses at transfer of prestressing. Where computed compression stresses exceed the allowable values, stressing is delayed until either concrete gains adequate strength, or the member is loaded. Where computed tension stresses are excessive, ACI/IBC[42] suggest adding non-stressed reinforcement to control cracking.

[42] ACI 318-11; Section 18.4

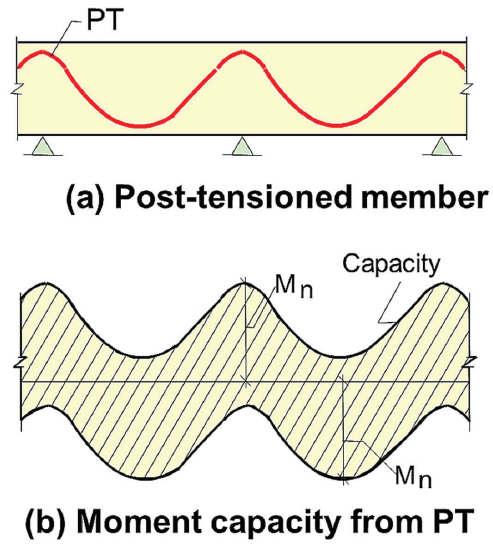
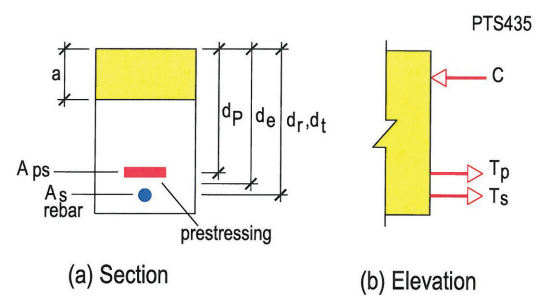


FIGURE C8.4-2 Demand and Capacity Moments (P500)



Distribution of Basic Forces on a Rectangular Section

FIGURE C8.4-3

9.1 Load Combinations

The codes covered are not specific on the applicable load combination at transfer of prestressing. The following is the combination generally assumed among practicing engineers;

Load Case: 1.0*DL + 0*LL + 1.15*PT

Specification of this design example calls for tendons to be stressed with concrete cylinder reaches 30 MPa. f'_{ci} = 30 MPa[43]

9.2 Stress Check

$\sigma = \pm(M_D + 1.15*M_{PT})/S + 1.15*P/A$

$S = I/Y_c$

Allowable Stresses

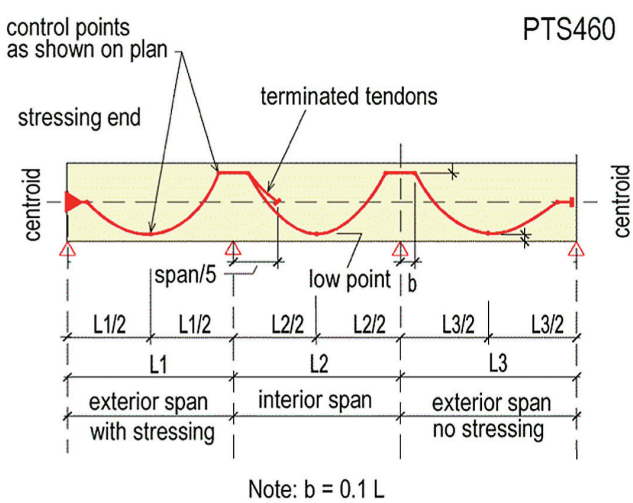
❖ Based on ACI 318-11; IBC 2009

[43] The value specified is on the high side. Most hardware are designed to be stressed at 20MPa concrete cylinder strength or less.

TABLE 9-1 Stresses at Transfer of Post-Tensioning (T166SI)

Span 4	Section A	Section B	Section C
M_D (kN-m)	-901.00	-319.70	296.89
M_{PT} (kN-m)	425.80	234.45	-110.20
P (kN)	2737	2737	2737
P / A (MPa)	-0.85	-1.10	-1.10
f_t (MPa)	0.70	-0.76	-2.98
f_b (MPa)	-3.67	-1.77	0.45
ACI-11/IBC 2009			
F_t (MPa)	1.37	-18	-18
F_b (MPa)	-18	-18	1.37
	OK	OK	OK
EC2			
F_t (MPa)	2.90	-18	-18
F_b (MPa)	-18	-18	2.90
	OK	OK	OK
TR-43			
F_t (MPa)	1.16	-12	-12
F_b (MPa)	-12	-12	1.16
	OK	OK	OK

Note: Section properties I, A, Stop, Sbot are the same as used for service condition stress check F_t and F_b are allowa-ble stresses at top and bottom respectively



Profile for Banded Slab Tendons

FIGURE 10-1

Tension = 0.25* √30 = 1.37 MPa
Compression = 0.60* 30 = -18 MPa

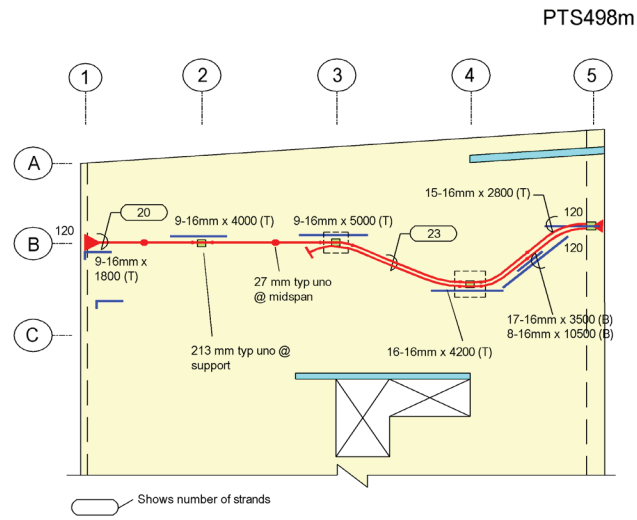


FIGURE 10-2 Layout Using ACI 318 Solution

❖ Based on EC2
Tension = f_{cteff} = 2.90 MPa
Compression = 0.60* 30 = -18 MPa

❖ Based on TR-43
Tension = $0.4f_{ctm}$ = 1.16 MPa
Compression = 0.40* 30 = -12 MPa

Farthest fiber stresses are calculated in a similar manner with to service condition as outlined earlier. The outcome is summarized in the following table.

10 - DETAILING

The final tendon and reinforcement layout for the design strip at line B is shown in figures 10-1 and 10-2 for unbonded tendons. Unbonded tendons are flexible and lend themselves to swerving on plan as shown in the figure. Bonded tendons are not as flexible. They are generally arranged along straight lines.

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