CHAPTER 7

POST-TENSIONED BEAM DESIGN STEP-BY-STEP CALCULATION

Beam Frame and One-Way Slab Construction (California, P466)

FORWORD

The example selected represents the frame of a one-way slab and beam construction—typical of parking structures, or floors, where span in one direction is twice or more than the span in the orthogonal direction. The beam frame selected has three spans, each with a different length. The third span is purposely selected to be short, compared to the other two. Also, the optimum post-tensioning for the design leads to different amount of post-tensioning in each span, and a different profile from span to span.

The objective in selecting a somewhat complex structure is to expose you to the different design scenarios that are encountered in real life structures, but are not generally featured in text books—in particular, where span lengths in a continuous member are widely different.

The example walks you through the 10 steps of design. Design aspects that are not covered in the example selected, but are important to know, are introduced and discussed as comments or inserted examples.

Design operations that are considered common knowledge, such as the calculation of moments and shears, once the geometry of a structure, its material and loading are known, are not detailed. You are referred to your in-house frame programs.

The example covers side by side both the unbounded and bonded (grouted) post-tensioning systems—thus providing a direct comparison between the design processes of the two options. In addition, in parallel, the design uses the current American building codes (ACI 318[1] and IBC[2]) along with the European Code (EC2[3]).

1 ACI 318-11
2 IBC 12; International Building Code 2012
3 EN 1992-1-1:2004(E)

The common method of analysis for beam frames and one-way slabs is the Simple Frame Method (SFM). While it is practical to use the method for hand calculations, for expediency in design, the iterative nature of optimization for post-tensioning lends itself well to the application of specialty computer programs, such as ADAPT-PT.[4]

Two text fonts are used in the example. The numerical work that forms part of the actual calculations uses the font shown below:

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The following font is used, where comments are made to add clarification to the calculations:

This font is used to add clarification to the calculations.

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REFERENCES

1. GEOMETRY AND STRUCTURAL SYSTEM:

The concrete frame of the structure consists of one-way slabs supported on parallel beams as shown in Fig. 1-1.

1.1 Dimensions and Support Conditions:

- Geometry is as shown in Fig. 1-1(a) and (b)
- Beam cross section as shown in Fig. 1-1(c)
- Total tributary width = 17 ' typical
- Columns extend below the deck only; first and last columns are assumed hinged at the bottom

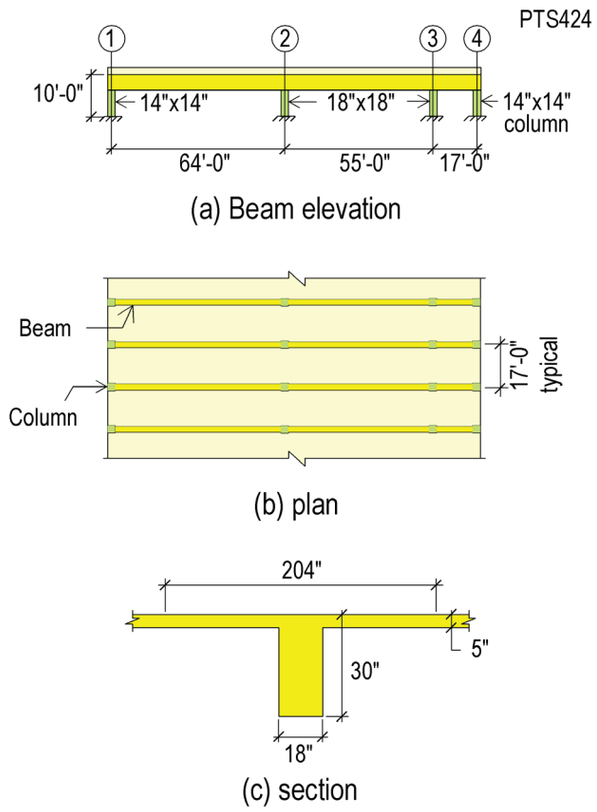


FIGURE 1-1

[4] www.adaptsoft.com

TABLE 1.3-1 Section Properties (T131US)

	Spans 1 and 2		Span 3	
	Axial effects	Bending effects	Axial effects	Bending effects
Area in^2	1470	940	1470	705
I in.^{4}	-----	77238	-----	60591
Y_t in.	7.09	9.68	7.09	12.07
Y_b in.	22.91	20.32	22.91	17.93
S_{top} in.^{3}	-----	7979	------	5020
S_{bot} in.^{3}	------	3801	-------	3379

I = moment of area (moment of inertia);
Y_t = distance of centroid to top fiber of section;
Y_b = distance of centroid to bottom fiber of section;
S_{top} = section modulus for top fiber; (I/Y_t); and
S_{bot} = section modulus for bottom fiber; (I/Y_{bot}).

End columns are assumed hinged and detailed as hinged at the connection to the footing, in order to reduce the adverse effects from the overall shortening of the floor.

1.2 Effective Width of Flanges:
When using hand calculation in design of flanged beams, an effective flange width is selected to account for the bending effects of the structure. ACI 318-11[5] explicitly states that the effective width used for analysis of conventionally reinforced flanged beams does not apply, when the same is post-tensioned, but does not offer an alternative. Chapter 4, Section 4.8.3 outlines the reason behind ACI 318's standing, and explains the applicable procedure. Briefly, for the effect of axial forces (post-tensioning) the entire cross-sectional area is to be used. But, for computation of flexural stresses in hand calculation a reduced flange width is applicable.

❖ For axial effects (precompression) use the entire tributary of the frame.
❖ For bending effects use the "effective width" value associated with the bending of flanged beams.

Also, note that the effective width concept is associated with the distribution of elastic stresses in the flange of a beam. It is to be used for the "serviceability limit" design (SLS) of a post-tensioned member. For safety checks (ULS) the effective width does not apply.

For conventionally reinforced concrete, ACI 318-11[6] recommends the least of the following:

(i) Eight times the flange thickness on each side of the stem;

(ii) one quarter of the span; or

(iii) the beam's tributary.

Tributary width = 17 x 12 = 204"

(i) Sixteen times flange thickness plus stem width
= 16 x 5" + 18" = 98"

(ii) One quarter of span
For span 1 = (64 x 12) / 4 = 192"
For span 2 = (55 x 12) / 4 = 165"
For span 3 = (17 x12) / 4 = 51"

(iii) Tributary width
For all spans = 204"
Assume the following:
Spans 1 and 2 = 98"
Span 3 = 51"

1.3 Section Properties:
The section properties for the axial effects are the same for all spans. For bending effects, however, due to different effective widths, the section properties differ. The section properties calculated are listed in Table 1.3-1

[5] ACI 318-11, Section 18.1.3

[6] ACI 318-11, Section 8.12.2

2. MATERIAL PROPERTIES:

2.1 Concrete:

Cylinder strength f'_c ,f_{ck} = 4000 psi
Weight = 150 pcf
Material factor, γ_c = 1 – ACI, 1.50 [EC2]
Modulus of Elasticity = $57000\sqrt{f'_c}$ = 3605 ksi [ACI]
= $22 \times 10^3 [(f_{ck} + 8) / 10]^{0.3}$ [7] [EC2];
= 32194.71 MPa (4,670 ksi)
Creep Coefficient = 2

2.2 Nonprestressed (Passive) Reinforcement:

f_y = 60 ksi
Elastic Modulus = 29000 ksi
Material factor, γ_c = 1 - ACI, 1.15 - EC2
Strength reduction factor, φ = 0.9 – ACI, 1 – EC2

2.3 Prestressing:

Material - Low relaxation, seven wire ASTM 416 strand
Strand diameter = ½ in.
Strand area = 0.153 in.^{2}
Elastic Modulus = 28000 ksi
Ultimate strength of strand (f_{pu}) = 270 ksi
Material factor, γ_c = 1 - ACI, 1.15 – [EC2]

System

Unbonded System

For configuration of an unbonded tendon refer to Fig. 2.3-1 in Chapter 6.
Angular coefficient of friction (μ) = 0.07
Wobble coefficient of friction (K) = 0.001 rad/ft
Anchor set (Wedge Draw-in) = 0.25 inch
Stressing force = 80% of specified ultimate strength
Effective stress after all losses = 175[8] ksi

Bonded System

For configuration of a grouted tendon and its geometry see Fig. 2.3-2 and 3 in Chapter 6.
Use flat ducts 20x800mm; 0.35 mm thickness metal sheet housing up to five strands
Angular coefficient of friction (μ) = 0.02
Wobble coefficient of friction (K) = 0.025 rad/ft
Anchor set (Wedge Draw-in) = 0.25 inch
Offset of strand to duct centroid (z) = 1/8 inch
Effective stress after all losses = 160 ksi

[7] EN 1992-1-1:2004(E) Table 3.1

[8] For hand calculation, an effective stress for tendon is used. The effective stress is the average stress along the length of a tendon subsequent to immediate and long-term losses. The value selected for effective stresses is a conservative estimate. When "effective stress" is used in design, the stressed lengths of tendons are kept short, as it is described later in Section 6 of this Chapter.

3. LOADING:

3.1 Dead Load:

Selfweight:
Slab = (5/12') x 0.150kcf x 17' = 1.063 klf
Stem = (25 / 12') (18 / 12') 0.150 pcf = 0.469 klf
Total dead load of self weight = 1.532 klf
Superimposed dead load from mechanical, sealant and overlay:
0.010 ksf x 17' = 0.170 klf
Total Dead Load = 1.702 klf

3.2 Live Load: 50 psf, reducible per IBC

1st Span:
Reduction = 0.08 ((64' x 17') – 150) = 75.04% > 40% max
Live load = (1.0-0.40) 0.05 ksf x 17' = 0.510 klf

2nd Span:
Reduction = 0.08 ((55' x 17') – 150) = 62.8% > 40% max
Live load = (1.0 – 0.40) 0.05ksf x 17' = 0.510 klf

3rd Span:
Reduction = 0.08 ((17' x 17') – 150) = 11.12%
Live load = (1.0-0.1112) 0.05ksf x 17' = 0.755 klf
MaxLL/DL ratio = 0.755 / 1.702 = 0.44 < 0.75
∴Do not skip live load.

The following relationships were used for the reduction of live loading, using IBC:
R = 0.08(A – 150)
Where, R = reduction factor not to exceed 40%; and
A = member tributary in square feet.

Strictly speaking, live loading must be skipped (patterned) to maximize the design values. But, when the ratio of live to dead loading is small (less than 0.75), it is adequate to determine the design actions based on live loading on all spans(ACI 318-11 [9]). This is specified for slab construction, but it is also used for beams.

4. DESIGN PARAMETERS:

4.1 Applicable Codes

The design is carried out according to each of the following codes.

- ACI 318-2011; IBC-2012
- EC2 (EN 1992-1-1:2004)

4.2 Cover to rebar and prestressing strands

Unbonded and bonded system

[9] ACI 318-11, Section 13.7.6

Minimum rebar cover = 2.00 in. top and bottom

The cover selected is higher than the minimum code requirement to allow for installation of slab bars over the beam cage. Refer to Fig. 4.2-1 in Chapter 6 for determination of CGS.

Minimum distance to center of prestressing CGS = 2.75 in., all spans

The cover and hence distance to the CGS (Center of Gravity of Strand) is determined by the requirements for fire resistivity. The distance 2.75" selected is slightly higher than the minimum required. Its selection is based on ease of placement.

4.3 Allowable Stresses:

A. Based on ACI 318-11/IBC 2012[10]

Allowable stresses in concrete are the same for bonded and unbonded systems

❖ For sustained load condition

Compression = 0.45 f'_c = 1800 psi

❖ For total load condition

Compression = 0.60 f'_c = 2400 psi

Tension (Transition condition of design is selected)

For top fibers select $9\sqrt{f'_c}$, Tension, top = 569 psi

For bottom fibers $12\sqrt{f'_c}$, Tension, bottom = 759 psi

❖ For initial condition:

Compression = 0.60 f'_{ci} = 0.6 x 3000 = 1800 psi

Tension = $3\sqrt{f'_c}$ = 164 psi

For one-way systems, ACI 318 [ACI 318, 2011] defines three conditions of design, namely uncracked (U), transition (T) and cracked (C). The three conditions are distinguished by the magnitude of the maximum hypothetical tension stress in concrete at the farthest tension fiber. For the current design example the transition (T) condition is selected. For this condition, hypothetical tension stresses can exceed 7.5 $\sqrt{f'_c}$ but not larger than 12 $\sqrt{f'_c}$. However, since the surface of the parking structure being designed is exposed, the design example uses a stress limit of 9 $\sqrt{f'_c}$ for the top surface and the maximum value allowed by the code for the bottom surface. This is not a code requirement. Based on the code, 12 $\sqrt{f'_c}$ or larger would have been acceptable. The selection of a lower value for the top surface is based on good engineering practice.

[10] ACI 318-11, Sections 18.3 and 18.4

B. Based on EC2[11]

EC2 does not specify "limiting" allowable stresses in the strict sense of the word. There are stress thresholds that trigger crack control. These are the same for both bonded and unbonded systems.

❖ For "frequent" load condition

Concrete:

Compression = 0.60 f_{ck} = 16.55 MPa (2400 psi)

Tension (concrete) = $f_{ct,eff} = f_{ctm}$[12]

= 0.30 $f_{ck}^{(2/3)}$ (Table 3.1, EC2)

= 0.30 x $27.58^{(2/3)}$ = 2.74 MPa (397 psi)

Tension (mild steel) = 0.80 f_{yk} = 330.95 MPa (48,001 psi)

Tension (prestressing steel) = 0.75 f_{pk} = 1395 MPa (202,331 psi)

❖ For "quasi-permanent" load condition

Compression = 0.45 f_{ck} = 12.41 MPa (1800 psi)

Tension (concrete) = 2.74 MPa (397 psi) same as frequent load combination

Unlike ACI 318/IBC, provisions in EC2 permit[13] overriding the allowable hypothetical tension stress in concrete, provided cracking is controlled not to exceed the selected "design crack width."

❖ For "initial" load condition

Tension (unbonded and bonded) = $f_{ct,eff} = f_{ctm}$

= 0.30 $f_{ci}^{(2/3)}$ (Table 3.1, EC2) = 0.30 x $20.69^{(2/3)}$ = 2.26 MPa (328 psi)

Compression[14] = 0.60 f_{ci} = 0.6 x 20.69 = 12.41 MPa (1800 psi)

For both unbonded and grouted post-tensioning systems, EC2[15] recommends fct,eff as the limit for hypothetical tensile stresses, before it becomes necessary to provide reinforcement for crack control. For stresses below this threshold, the minimum reinforcement provisions of EC2 will suffice.

4.4 Crack Width Limitation:

A. Based on ACI 318-11/IBC 2012

Crack width control and limitation applies when member is designed for the "cracked" regime. No requirements are stipulated, if as in this example, the stresses are kept within the transition or uncracked regimes.

[11] EN 1992-1-1:2004(E), Section 7.2

[12] EN 1992-1-1:2004(E) , Section 7.3.2(4)

[13] EN 1992-1-1:2004(E) , Section 7.3.2(4)

[14] EN 1992-1-1:2004(E) , Section 5.10.2.2(5)

[15] EN 1992-1-1:2004(E), Section 7.3.2(4)

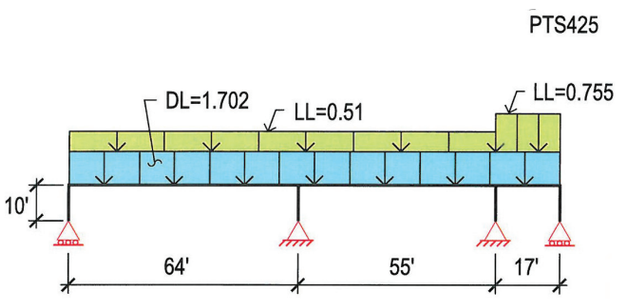
B. Based on EC2[16]
In EC2, the allowable crack width depends on whether the post-tensioning system used is "bonded," or "unbonded," and the load combination being considered.[17]

Frequent load condition:
For prestressed members with bonded tendons = 0.2 mm (0.008 in); no cracking check for unbonded systems
Quasi-permanent load condition:
For prestressed members with unbonded tendons = 0.3 mm (0.01 in); no cracking check for grouted systems

4.5 Allowable Deflection:
The allowable deflection is tied to (i) the visual impact of the vertical displacement of a member on the occupants; (ii) the possible damage to installed non-structural members. such as partitions, glass, or floor covering; and (iii) functional impairment, such as proper drainage. Details of the allowable values, their measurement and evaluation are given in Chapter 4, Section 4.10.6. For perception of displacement by sensitive persons, consensus is limit of L/250, where L is the deflection span. For a parking structure, depending on the code used L/250 or L/240 applies.

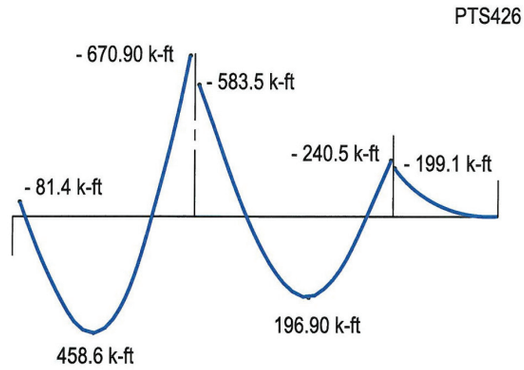
Based on ACI 318-11/IBC 2009[18]
Total allowable downward displacement below level - L/240

A. Based on EC2[19]
Total allowable downward displacement below level - L/250



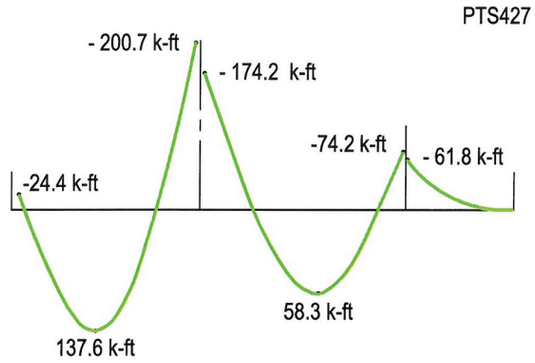
Structural Frame and Its Dead and Live Loading (k/ft)

FIGURE 5-1



DL Moment Distribution (k-ft)

FIGURE 5-2 DL Moments at Face-of-Support and Midspan



Live Load Moment Distribution

FIGURE 5-3 LL Moments at Face-of-Support and Midspan

5. ACTIONS DUE TO DEAD AND LIVE LOADING

The structural system of the frame and its dead and live loads are shown in Figs. 5-1 through 5-3.

Actions due to dead and live loads are calculated using a generic frame analysis program. The members are assumed prismatic and of uniform cross section throughout the length of each span. Spans 1 and 2 have the same geometry. Centerline to centerline distances are used for span lengths. No allowance is made in the hand calculation for stiffening of members over support. A number of commercially available software account for this stiffening, and increase the moment of inertia of the beam over the support region [ADAPT-PT, 2013]

The computed moments from the frame analysis are reduced to the face of the support using statics of each span. The face-of-support moments and the moments at midspan are summarized in Table 5-1.

16 EN 1992-1-1:2004(E), Table 7.1N
17 EN1992-1-1-2004 (E) Table 7.1N
18 ACI-318-11, Sections 18.3.5
19 EN 1992-1-1:2004(E) , section 7.4.1

TABLE 5-1 Moments at Face-of-Support and Midspan (T132US)

	Span 1			Span 2			Span 3		
	Left	Mid	Right	Left	Mid	Right	Left	Mid	Right
M_D kip-ft	-81.4	458.6	-670.9	-583.5	196.9	-240.5	-199.1	-46.2	4.5
M_L kip-ft	-24.4	137.6	-200.7	-174.2	58.3	-74.2	-61.8	-7.2	1.8
$M_D + M_L$ Total load kip-ft	-105.8	596.2	-871.6	-757.7	255.2	-314.7	-260.9	-53.4	6.3
$M_D + 0.3M_L$ Sustained load kip-ft	-88.7	499.9	-731.1	-635.8	214.4	-262.8	-217.6	-48.4	5.0

The critical design moments are not generally at midspan. But, for hand calculation, the midspan location is selected. The approximation is acceptable when spans and loads are essentially uniform.

6. POST-TENSIONING

6.1 Selection of Design Parameters

Unlike conventionally reinforced members, where a given geometry, boundary conditions, material properties and loads result in a unique design, for post-tensioned members, in addition to the above a minimum of two other input assumptions are required, before a design can be concluded. Common practice is (i) to assume a value for the average precompression and (ii) target to balance a percentage of the structure's dead load. In this example, based on experience the level of precompression suggested will be more than the minimum required by ACI 318 (125 psi; 0.84 MPa). Other major building codes do not specify a minimum precompression. Rather, they specify a minimum reinforcement. Use the following assumption to initiate the calculations.

1. Minimum average precompression = 150 psi
2. Maximum average precompression = 300 psi
3. Target balanced loading = 60 % of total dead load for critical span; other spans can be balanced less.

Spans, other than the critical, need not be balanced to the same extent. As it will become apparent further in the calculations, for the current beam frame it is beneficial if the tendon exerts a downward force on the third span, as opposed to an upward force in the critical (first) span.

4. Effective stress in prestressing strand:

For unbonded tendons: f_{se} = 175 ksi
For bonded tendons: f_{se} =160 ksi

The design of a post-tensioned member can be based either on the "effective force", or the "tendon selection" procedure. In the effective force procedure, the average stress in a tendon after all losses is used in design. In this case, the design concludes with the total effective post-tensioning force required for each design member. The total force arrived at the conclusion of design is then used to determine the number of strands required, with due allowance for friction and long-term stress losses. This method provides an expeditious and simple design procedure for hand calculations. In the "tendon selection" procedure, the design is based on the number of strands with allowance for the immediate and long-term stress losses. In the following, the "effective force" method is used to initiate the design. Once the design force is determined, the force is converted to the number of strands required.

The background to the effective stress assumed is described in Chapter 4, Section 4.8.7.1. It is based on the following conditions:

(i) Members have dimensions common in building construction;

(ii) Tendons equal or less than 125 ft long stressed at one end. Tendons longer than 125 ft, but not exceeding 250 ft are stressed at both ends. Tendons longer than 250 ft are stressed at intermediate points to limit the maximum of the unstressed lengths to 125 ft for one-end stressing, or 250 ft for two-end stressing, whichever be applicable;

(iii) Strands are the common commercially available generic 0.5 or 0.6 inch extruded tendons with industry common friction coefficients as stated in material properties section of this example; and

(iv) Tendons are stressed to 0.8fpu.

For the preceding conditions, the effective stress (f_{se}) is assumed as 175 ksi for unbonded tendons, and 160 ksi for bonded tendons, since friction loss in bonded tendons is larger. For other conditions, a lower effective stress is assumed, or tendons are stressed at intermediate points. In the current design, the total length of the longest tendon is 136 ft. It is stressed at both ends. Detail calculations indicate that the effective tendon stress is actually 182 ksi for the unbonded systems, and larger than the value assumed for bonded systems.

6.2 Selection of Post-tensioning Tendon Forces and Profile

The prestressing force in each span will be chosen to match a whole number of prestressing strands. The following values are used:

For unbonded tendons

Force per tendon = 175 ksi x 0.153 in^2 = 26.77 kips/tendon

Use multiples of 26.77 kips when selecting the post-tensioning forces for design.

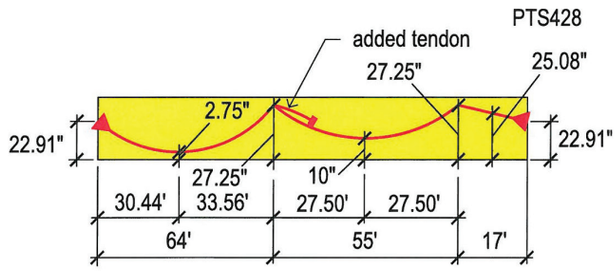
For bonded tendons

Force per tendon =160 ksi x 0.153 in^2 = 24.48 kips/tendon

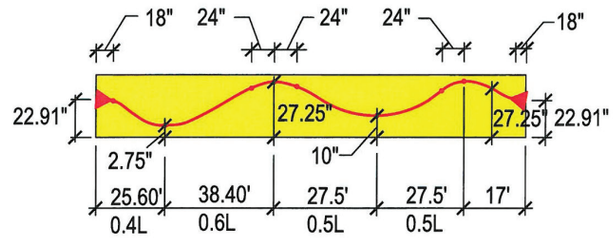
Use multiples of 24.48 kips when selecting the post-tensioning forces for design.

Tendon profiles are chosen to be simple parabola. These produce a uniform upward force in each span.

For ease of calculation the tendon profile in each span is chosen to be concave upward, simple parabola from centerline to centerline of supports (Refer to Fig. C6.2-1 in Chapter 6). The position of the low point is selected such as to generate a uniform upward force in each span. The relationship given in Fig. C6.2-1 defines the profile. For exterior spans, where the tendon high points are not generally the same, the resulting low point will not be at midspan. For interior spans, where tendon high points are the same, the low point will coincide with midspan. Obviously, the chosen profile is an approximation of the actual tendon profile used in construction. Sharp changes in curvature associated with the simple parabola profile assumed are impractical to achieve on site. The tendon profile at construction is likely to be closer to reversed parabola, for which the distribution of lateral tendon forces will be somewhat different. Tendon profiles in construction and the associated tendon forces are closer to the diagrams shown in Fig. 6.2-1 in Chapter 6.



(a) Simplified tendon profile (in.; ft)



(b) Actual tendon profile used in construction (in.; ft)

Comparison of Simplified and Actual Tendon Profiles

FIGURE 6.2-1

6.3 Selection of Number of Strands

Select the initial number of strands of each span based on the assumed average precompression, using the cross-sectional area of each span's tributary. If needed, adjust the number of strands selected, based on the uplift they provide.

Unbonded tendon:

150 psi x 1470 in^2 / 1000 = 220.5 kips

Number of strands = 220.5 k/ 26.77 k = 8.24, ∴ say 9 strands

Force in 9 strands = 240.93 kips

Bonded tendon:

150 psi x 1470 in^2 / 1000 = 220.5 kips

Number of strands = 220.5 k/ 24.48 k = 9.01, ∴ say 10 strands

Force in 10 strands = 244.48 kips

It is noted that the number of strands required to satisfy the same criterion differs between the unbonded and bonded systems. Due to higher friction losses, bonded systems generally more strands to satisfy the

stress criteria of design. For brevity, without compromising the process of calculation, in the following the same number of strands is selected for both systems.

6.4 Calculation of Balanced Loads and Design Forces

Balanced loads are the forces that a tendon exerts to its concrete container. It is generally broken down to forces normal to the centerline of the member (causing bending), forces along the member's centerline (causing uniform precompression), and added moments at locations of change in centroidal axis of the member. The breakdown of balanced load to the described components is described in Chapter 4, Section 4.8.1.

Span 1:

The profile of the first span is chosen to provide a uniform upward force over the entire span. This is done by selecting the location of the tendon low point such as to create a continuous simple parabola (Fig. C6.2-1). Span 1 is the longest span, and is considered the critical span. Being the critical design span, its tendon is profiled with the maximum drape, in order to utilize the largest amount of balanced loading it can provide. If the low point of the tendon is not selected at the location determined by c, two distinct parabolas result. The upward force from a single parabolic profile selected is shown in Fig. C6.2-1.

Try maximum drape and force based on the assumed P/A.
a = 22.91" – 2.75" = 20.16"
b = 27.25" – 2.75" = 24.50"
L = 64'
$c = \{[20.16 / 24.5]^{0.5} / [1 + (20.16 / 24.5)^{0.5}]\} \times 64' = 30.44'$
$W_b = 240.93 \text{ kips} \times (2 \times 20.16 / 12) / 30.44^2 = 0.874$ klf
% DL balanced = (0.874 / 1.702) 100 = 51% < 60% No Good
Prorated number of strands = 60% / 51% x 9 = 10.6 strands
Try 12 strands x 26.77 kips = 321.24 kips[20]
W_b = (321.24 kips / 240.93 kips) x 0.874 ft. = 1.165 klf ↑

% DL balanced = (1.165/1.702)100 = 68.4% OK
Balanced load reaction, left = 1.165 klf x 30.44' = 35.46 k ↓
Balanced load reaction, right = 1.165 klf (64 – 30.44') = 39.10 k ↓

[20] Normally, in this situation 11 strands will be selected. The 12-strand selection made is in anticipation that for the bonded option more strands will be required.

Fig. 6.4-1 shows the distribution of balanced loading for span 1.

Span 2:

Continuous Tendons:

This span is shorter than the critical span. Therefore, the 9 tendons necessary for the assumed minimum precompression of 150 psi is used. In addition, recognizing that balancing a lower percentage of self-weight will be beneficial to the critical span (span 1); the minimum of 60% uplift used as guideline for tendon selection is waived for this span. A smaller percentage for balanced loading is acceptable. The span is balanced for 50% of the dead load, for which the necessary drape is selected. Note that the tendon low point is located at midspan.

Force of 9 strands is 9x0.153x175 = 240.93 k
W_b = 50% x 1.702 klf = 0.851 klf ↑
$a = W_b L^2/(8 P) = [(0.851 \times 55^2) / (8 \times 240.93)]$ 12" = 16.03"; assume 17"
CGS = 27.25 – 17.0 = 10.25"; assume 10"
Hence,
a = 17.25"
$W_b = (17.25/12)\ 8 \times 240.93/55^2 = 0.916$ klf
Balanced load reactions = 0.916 klf x 27.5' = 25.19 k ↓ (left and right)

Added Tendons:

Reduction of tendons from 12 in span 1 to 9 in span 2 results in 3 tendons from span 1 to terminate in span 2. The terminated three tendons are dead-ended in span 2. The dead-end is located at a distance 0.20*L* from the left support, at the centroid of the member section. The tail of the terminated tendons is assumed to be in the shape of a half parabola with its apex horizontal over the support and concave downward to the dead end. Hence, the distributed balanced loading of these tendons will be downward, with a concentrated upward force at the dead end (Fig. 6.4-2). The governing relationships for the forces of added tendons are given in Chapter 6, Fig. C6.4-1.

$W_b = (2aP)/L^2$
a = 27.25" – 22.91" = 4.34"
L = 0.20 x 55' = 11.0'
$W_b = (2 \times 4.34 / 12) \times 3 \times 26.77 / 11.0^2 = 0.480$ klf ↓
Concentrated force at dead end = 0.480 klf x 11.0' = 5.28 k ↑

Span 3:

The tendon profile in this span is chosen to be straight from the high point at the interior support, to the cen-

troid of the section at the exterior support. The objective is to avoid uplift in the short span. As a matter of fact, for this beam a downward force in the third span would be beneficial to the design of the interior span.

CGS left = 27.25"
CGS right = 22.91"
CGS center = (27.25 + 22.91) / 2 = 25.08
Vertical components of balanced loading are concentrated forces acting at the supports only. They are equal and opposite.

W_b = 240.93 k x (27.25 – 22.91) / (17' x 12") = 5.13 kips ↑(right); ↓ (left)

The complete tendon profile, effective force and balanced loading diagram are shown in Fig. 6.4-2.

Verify the computed balanced loading:
(i) Sum of vertical forces must add up to zero:
-5.13 + 5.13 - 35.46 - 39.10 + 1.165 x 64 - 25.19 + 5.28 - 0.480 x 11 + 0.916 x 55 - 25.19 = 0 OK

(ii) Sum of moments of the forces must be zero. Taking moments about the first support gives:
-39.10 x 64 + 1.165 x 64^2 / 2 – 25.19 x 64 -0.48 x 11 x 69.5 + 5.28 x75 + 0.916 x 55 x 91.5 – 25.19 x 119 – 5.13 x 119 + 5.13 x136 = -0.23 k-ft OK

It is essential to verify that the balanced loading constructed satisfies the static equilibrium. And, that the concentrated forces over the supports are correctly

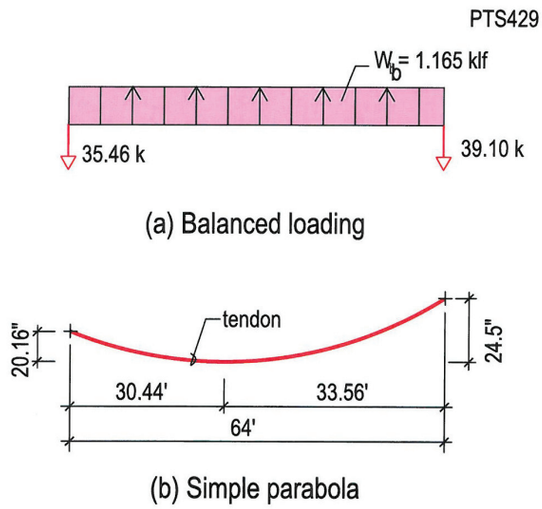


FIGURE 6.4-1

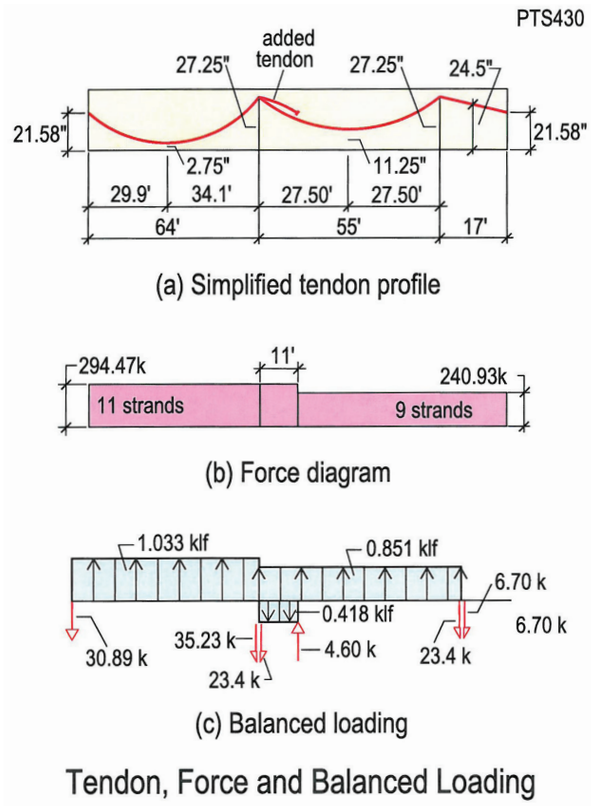


FIGURE 6.4-2

computed and accounted for. In particular, the force due to the short length of the terminated strands in the second span must be included to satisfy equilibrium. If equilibrium is not satisfied, it becomes imperative to ensure that the results err on the conservative side.

6.5 Determination of Actions due to Balanced (Post-Tensioning) Loads

The distribution of post-tensioning moments due to balanced loading is shown in Fig. 6.5-1. The actions are obtained by applying the balanced loads shown in Fig. 6.4-2(c) to the frame shown in Fig. 5-1. The moments

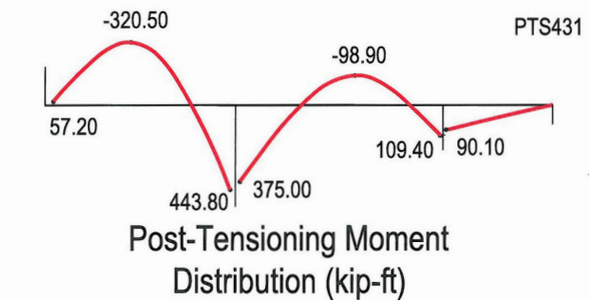


FIGURE 6.5-1

shown in the figure are values reduced to the face-of-support. Midspan moments are also shown in the figure.

7. CODE CHECK FOR SERVICEABILITY:

7.1 Load Combinations

The following lists the recommended load combinations of the building codes covered for serviceability limit state (SLS).

❖ [ACI, IBC]
Total load condition 1 DL + 1 LL + 1 PT
Sustained load condition 1 DL + 0.3 LL + 1 PT [21]

❖ [EC2]
Frequent load condition 1 DL + 0.5 LL + 1 PT
Quasi-permanent load condition 1 DL + 0.3 LL + 1 PT

For serviceability check, the actions from the balanced loads of post-tensioning (Fig. 6.5.1) are used. These are due to "balanced loading." The background for this is explained in detail in reference [Aalami, 1990].

7.2 Stress Check

Critical Locations for Stress Check:

For hand calculation, the critical locations for stress check are based on engineering judgment. Relying on personal judgment, the selected locations may or may not coincide with the locations of maximum stress levels. This will introduce a certain degree of approximation in design, which reflects the common practice for hand calculations. Computer solutions generally calculate stresses at multiple locations along a span, thus providing greater accuracy.

By inspection, locations marked in Fig. 7.2-1 as sections A through E are considered critical for design. These are the midspan locations and the face-of-support locations of the first interior column.

The moment values due to the combined action of dead and live loading (Table 5.1) and the moment distribution due to post-tensioning (Fig. 6.5-1) are used to determine the design values at the selected locations.

[21] ACI 318 specifies a "sustained" load case, but does not stipulate the fraction of live load to be considered "sustained." It is left to the judgment of the design engineer to determine the applicable fraction. The fraction selected varies be-tween 0.2 and 0.5. The commonly used fraction is 0.3, as it is adopted in this design example.

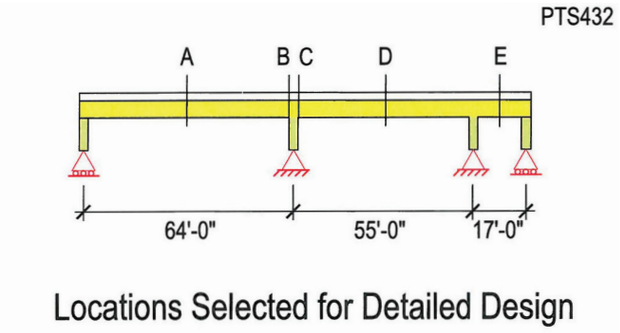


FIGURE 7.2-1

Stresses:
$\sigma = (M_D + M_L + M_{PT}) / S + P/A$
$S = I/Y_c$

Where, M_D, M_L, and M_{PT} are the moments across the entire tributary of the design strip. S is the section modulus of the cross section reduced through effective width defined for bending action; A is the area of the entire tributary; I is the second moment of area of the portion of the cross-section that is defined by the effective width for bending; and Y_c is the distance of the centroid of the reduced section (defined for bending) to the farthest tension fiber of the section.

The parameters for stress check at point A are:
Y_T = 9.68" (245.87 mm)
Y_B = 20.32" (516.13 mm)
S_{top} = 7979 in^3 (1.307e+8mm^3)
S_{bot} = 3801 in^3 (6.231e+7mm^3)
A = 1470 in^2. (948385 mm^2)
P/A = -321.24x1000/1470 = -219 psi (-1.51 MPa)
The area A is the tributary area. It is the same for all spans. Values used in the stress formula of the critical points selected are listed in Table 7.2-1.

A. Based on ACI 318-11/IBC 2012:
Stress checks are performed for the two load conditions of total load and sustained load.

Point A:

❖ Total load combination
$\sigma = (M_D + M_L + M_{PT}) / S + P/A$
Stress Limits: Top tension = 9 √4000 = 569 psi
Bottom tension = 12 √4000 = 759 psi
Compression = 0.60 x 4000 = -2400 psi
$M_D + M_L + M_{PT}$ = (458.6 + 137.6 – 320.5) = 275.70 k-ft
Bottom fiber:
σ = (275.70 x 12 x 1000 / 3801) – 219 = 651 psi Tension

< 759 psi OK
Top fiber:
σ = (-275.70 x 12 x 1000 / 7979) – 219 = -634 psi Compression < -2400 psi OK

❖ Sustained load combination:
$\sigma = (M_D + 0.3M_L + M_{PT}) / S + P/A$
Stress Limits: Top tension = 9 √4000 = 569 psi
Bottom tension = 12 √4000 = 759 psi
Compression = 0.45 x 4000 = -1800 psi
$M_D + 0.3\ M_L + M_{PT}$ = (458.6 + 0.3 x 137.6 – 320.5) = 179.38 k-ft
Bottom fiber:
σ = 179.38 x 12 x 1000 / 3801 – 219 = 347 psi Tension < 759 psi OK
Top fiber:
σ = -179.38 x 12 x 1000 / 7979 – 219 = -489 psi Compression < -1800 psi OK

B. Based on EC2:
Stress checks are performed for the two load conditions of frequent load and quasi-permanent. The outcome will determine whether crack width needs to be controlled or not. See section on "Allowable Stresses."

Point A:

❖ Frequent Load Condition:
$\sigma = (M_D + 0.5\ M_L + M_{PT})/S + P/A$

Stress thresholds:
Compression = 0.60 x 27.58 = -16.55 MPa (2400 psi)
Tension (concrete) = $f_{ct,eff} = f_{ctm}$ = 2.74 MPa (397psi)
$M_D + 0.5M_L + M_{PT}$ = (621.8 + 0. 5 x 186.60 – 434.53)
= 280.57 kN-m (206.94 k-ft)
Top:
σ = -280.57 x 10002 / 1.307e+8 – 1.51 = -3.66 MPa (-530psi) Compression
< -16.55 MPa (2400 psi) OK
Bottom:
σ = 280.57 x1000^2/6.231e+7 – 1.5 = 2.99 MPa (434 psi) Tension > 2.74 MPa (397psi) NG
Hence check and control crack width.[22]

❖ Quasi-Permanent Load Condition:
$\sigma = (M_D + 0.3\ M_L + M_{PT}) / S + P/A$
Stress Limits:
Compression = 0.45 x 27.58 = -12.41 MPa (1800 psi)
Tension (concrete) = $f_{ct,eff} = f_{ctm}$ = 2.74 MPa (397 psi)

[22] EN 1992-1-1:2004(E) , Section 7.3.4

$M_D + 0.3M_L + M_{PT}$ = (621.8 + 0. 3 x 186.60 – 434.53)
= 243.25 kN-m (179.41 k-ft)
Top:
σ = -243.25 x 1000^2 / 1.307e+8 – 1.51= -3.37 MPa (-489 psi) Compression < -12.41 MPa (1800psi) OK
Bottom:
σ = 243.25 x 1000^2 / 6.231e+7 – 1.51 = 2.39 MPa (347psi) Tension < 2.74 MPa (397psi) OK

Since the tensile stresses at one or more locations exceed the threshold for uncracked sections, rebar has to provide in order to limit the crack width.

7.3 Crack Width Control

A. Based on ACI 318-11/IBC 2012:
None required, since the design stresses were kept in the transition regime.

B. Based on EC2:[23]
Point A:
Crack width, $W_k = S_{r,\ max}\ (\varepsilon_{sm} - \varepsilon_{cm})$[24]
$\varepsilon_{sm} - \varepsilon_{cm} = [\sigma_s - k_t\ (f_{ct,eff}/\rho_{p,eff})(1 + \alpha_e\ \rho_{p,eff})] / E_s \geq 0.6\ \sigma_s / E_s$
Where,
$\alpha_e = E_s / E_{cm}$ = 199,949.20 / 32194.71 = 6.21
$\rho_{p,eff} = (A_s + \xi_1^2\ A'_p)/ A_{c,eff}$
A'_p = area of tendons within $A_{c,eff}$ = 12 x 99 = 1188 mm^2 (1.84 in^2)
A_s = 0 mm^2
$\xi_1 = \sqrt{(\xi\ \varphi_s / \varphi_p)}$
ξ = 0.5 (From Table 6.2)
φ_s = largest diameter of bar =22 mm (# 7)
φ_p = 1.75 x 13 = 23 mm (0.90")
$\xi_1 = \sqrt{(0.5 \times 22 / 23)} = 0.70$
$A_{c,eff} = h_{c,eff}\ bw$
$h_{c,eff}$ = lesser of (2.5 (h-d), (h-x) / 3 , (h/2))
x = 2.99 x 762 / (2.99 + 3.66) = 343 mm (13.49 in)
d = 762 – 51 – 22/2 = 700 mm (27.56 in)
$h_{c,eff}$ = lesser of (2.5 x (762 – 700), (762 – 343)/3 , (762/2))
= 140 mm (5.51")
$A_{c,eff}$ = 140 x 457 =63980 mm^2 (99.17 in^2)
$\rho_{p,eff}$ = (0 + 0.70^2 x 1188) / 63980 =0.0091
σ_s = (f/Ec) Es
f = tensile stress due to DL + 0.5LL = $(M_D + 0.5\ M_L) / s$
= (621.8 + 0.5 x 186.6) 1000^2 / 6.23x10^7
= 11.48 MPa (1665 psi)
σ_s = (11.48 / 32194.71) 199,949.20 = 71.30 MPa (10341 psi)
k_t = 0.4 (coefficient for long-term loading)

[23] EN 1992-1-1:2004(E) , Section 7.3.3
[24] EN 1992-1-1:2004(E) , Section 7.3.4

TABLE 7.2-1. Service Extreme Fiber Stresses at Selected Points (T133US)

Load Combination		Point A	Point B	Point C	Point D	Point E
Based on ACI 11/IBC 2012						
Sustained Load	f_t psi	-489	213	173	-336	-157
	f_b psi	347	-1126	-1042	197	-175
	F_t psi	-1800	569	569	-1800	-1800
	F_c psi	759	-1800	-1800	759	-1800
		OK	**OK**	**OK**	**OK**	**OK**
Total Load	f_t psi	-634	424	357	-398	-145
	f_b psi	651	-1570	-1427	326	-193
	F_t psi	-2400	569	569	-2400	-2400
	F_b psi	759	-2400	-2400	759	-2400
		OK	**OK**	**OK**	**OK**	**OK**
Based on EC2						
Frequent Load	f_t (MPa) (psi)	-3.66 (-530)	1.88 (273)	1.55 (225)	-2.44 (-353)	-1.06 (-153)
	f_b (MPa) (psi)	2.99 (434)	-8.64 (-1253)	-7.94 (-1152)	1.62 (234)	-1.24 (-180)
	F_t (MPa) (psi)	-16.55 (-2400)	2.74 (397)	2.74 (397)	-16.55 (-2400)	-16.55 (-2400)
	F_b (MPa) (psi)	2.74 (397)	-16.55 (-2400)	-16.55 (-2400)	2.74 (397)	-16.55 (-2400)
		NG	**OK**	**OK**	**OK**	**OK**
Quasi-Permanent Load	f_t (MPa) (psi)	-3.37 (-489)	1.46 (212)	1.19 (173)	-2.32 (-336)	-1.08 (-157)
	f_b (MPa) (psi)	2.39 (347)	-7.76 (-1126)	-7.19 (-1042)	1.36 (197)	-1.20 (-175)
	F_t (MPa) (psi)	-12.41 (-1800)	2.74 (397)	2.74 (397)	-12.41 (-1800)	-12.41 (-1800)
	F_b (MPa) (psi)	2.74 (397)	-12.41 (-1800)	-12.41 (-1800)	2.74 (397)	-12.41 (-1800)
		OK	**OK**	**OK**	**OK**	**OK**

Note: F_t and F_b are the respective top and bottom allowable stresses. F_c is allowable compressive stress.

$f_{ct,eff} = f_{ctm} = 0.3\ (27.58)^{(2/3)} = 2.74$ MPa (397psi)

$\varepsilon_{sm} - \varepsilon_{cm} = [\sigma_s - k_t\ (f_{ct,eff}/\rho_{p,eff})(1 + \alpha_e\ \rho_{p,eff})]/Es$

$= [71.30 - 0.4\ (2.74/0.0091)(1 + 6.21 \times 0.0091)]\ /\ 199{,}949.20$

$= -0.0003 \leq 0.6 \times 71.30\ /\ 199{,}949.20 = 0.000214$, use 0.000214

$s_{r,max} = 1.3\ (h - x) = 1.3 \times 419 = 545$ mm

Crack width, $W_k = 545 \times 0.000214$

$= 0.12$ mm < 0.2 mm OK

Provide minimum reinforcement for cracking. It is provided with minimum rebar in 7.3.4.

EXAMPLE

To illustrate the procedure for crack control by way of addition of reinforcement, as recommended in EC2, as an example let the maximum tensile stress exceed the threshold value by a large margin.

Given: computed hypothetical farthest fiber tensile stress in concrete, f = 30MPa
Required: reinforcement for crack control

Calculate stress in steel at location of maximum concrete stress: $\sigma_s = (f/E_c)\ E_s$
Where f is the hypothetical tensile stress in concrete under service condition
$\sigma_s = (f/E_c)\ E_s = (30/32194.71)\ 199{,}949.20 = 186.32$ MPa(this is a hypothetical value)

Crack spacing can be limited by either restricting the bar diameter and/or bar spacing. Use the maximum bar spacing from Table 7.3 N for the σ_s of 186.32 MPa.
From Table, for 160 MPa - 300 mm
200 MPa - 250 mm
By interpolation, maximum spacing for 186.32 MPa is 267 mm.
Limit the spacing of reinforcement to 267 mm or less in order to control cracking. Select bar spacing at 250mm. Note that based on the magnitude of the computed tensile stress in concrete the required area of the reinforcement is calculated separately,

7.4 Minimum Reinforcement

There are several reasons why the building codes specify a minimum reinforcement for prestressed members.

❖ Crack control, where potential of cracking exists: Bonded reinforcement contributes in mitigating local cracks. The contribution of bonded reinforcement to crack control is gauged by the stress it develops under service load. Force in bonded reinforcement from applied strain is a function of the reinforcement's modulus of elasticity and its cross-sectional area. Hence, the area of reinforcement considered available for crack control is $(A_s + A_{ps})$, where A_{ps} is the area of bonded tendons. It is recognized that both bonded and unbonded prestressing provide precompression. While the physical presence of an unbonded tendon may not contribute to crack control, the contribution through the precompression it provides does. However, for code compliance and conformance with practice, the contribution of unbonded tendons is not included in the aforementioned sum.

❖ Ductility: One reason of ACI 318's stipulation for a minimum bonded reinforcement for members reinforced with unbonded tendons is to enhance ductility. No minimum rebar is specified, when using grouted tendons.

Use # 7 (22 mm) bars (Area = 0.60 in2 (387 mm2); Diameter = 0.875 in. (22 mm))

A. Based on ACI 318-11/IBC 2012:[25]

❖ Unbonded Tendon:
Minimum Required, Top:
$A_s = 0.004\ A_{tens}$

A_{tens} is the area of the section between the tension fiber and the section centroid. The minimum rebar is to mitigate cracking, and enhance the ductility. Since the minimum rebar is intended to address the flexural performance of the member, the cross-sectional properties associated with the flexure are used for the determination of its area.

Top bars at supports 1, 2 and 3
$= 0.004\ [5" \times 98" + (9.68" - 5")\ 18"] = 2.30\ in^2$
No.of bars = 2.30/0.6 = 3.83; Use 4 - #7 bars
$A_s = 4 \times 0.6 = 2.40\ in^2$
Top bar at support 4
$= 0.004\ [5" \times 51" + (12.07" - 5")\ 18"] = 1.53\ in^2$
No.of bars = 1.53/0.6 =2.55; Use 3 - #7 bars
$A_s = 3 \times 0.6 = 1.80\ in^2$

Minimum required at bottom for spans 1 and 2:
$A_s = 0.004\ A_{Tens} = 0.004\ (18" \times 20.32") = 1.46\ in^2$
No.of bars = 1.46 / 0.6 = 2.44; Use 3-#7 bars
$A_s = 3 \times 0.6 = 1.80\ in^2$

Minimum required at bottom for span 3
$A_s = 0.004\ (18" \times 17.93") = 1.29\ in^2$
No.of bars = 1.29 / 0.6 = 2.15; Use 3#7
$A_s = 3 \times 0.6 = 1.80\ in^2$

Since at midspan, the tension is at the top, the area of minimum reinforcement calculated for the top will be used. In this case, 3 #7 will be adequate. Hence
$A_s = 1.80\ in^2$; use 3-#7 bars at top of midspan

❖ Bonded (grouted) Tendons

There is no requirement for minimum reinforcement based on either geometry of the design strip, nor its hypothetical tensile stresses. The minimum requirement is handled through the relationship between the cracking moment of a section and its nominal strength in bending. This is handled in the "strength" check of the member (section 8 of this example).

[25] ACI 318-11, Section 18.9

B. Based on EC2:[26]

❖ Unbonded and bonded tendons

Spans (Fig. 7.2-1):

$A_{smin} \geq (0.26\ f_{ctm}\ b_t\ d\ /\ f_{yk}) \geq 0.0013\ b_t\ d$

b_t = 457 mm (18 in)

d = 762 – 51 – 22/2 = 700 mm (27.56 in)

$f_{ctm} = 0.3 \times 27.58^{(2/3)}$ = 2.74 MPa (397psi)

(i) $A_s = 0.26\ f_{ctm}\ b_t\ d/f_{yk}$ = 0.26 x2.74 x457 x 700 / 413.69 = 551 mm^2 (0.85 in^2)

(ii) $A_s = 0.0013\ b_t\ d$ = 0.0013 x 457 x 700 = 416 mm^2 (0.64 in^2)

Therefore, A_s = 551 mm^2 (0.85 in^2)

Contribution of reinforcement from bonded prestressing:

Point A:

$A_{ps}\ (f_{pk}/f_{yk})$ = 12 x 99 x 1861.60 / 413.69 = 5346 mm^2 (8.29 in^2)> 551 mm^2 (0.85 in^2)

Points D & E:,

$A_{ps}\ (f_{pk}/f_{yk})$ = 9 x 99 x 1861.60 / 413.69 = 4010 mm^2 (6.21 in^2) > 551 mm^2 (0.85 in^2)

Hence, no additional bonded reinforcement is required.

❖ Supports:

$A_{smin} \geq (0.26\ f_{ctm}\ b_t\ d\ /\ f_{yk}) \geq 0.0013\ b_t\ d$

b_t = mean width of the tension zone

depth of tension zone, (h-c) (Refer Fig.7.4-1)

= 1.88 x 762 / (1.88 + 8.64) = 136 mm Considered the point B)

b_t = [2489 x 127 + 457 (136 – 127)]/136 = 2355 mm

d = 762 – 51-22/2 = 700 mm (27.56 in)

$f_{ctm} = 0.3 \times 27.58^{(2/3)}$ = 2.74 Mpa (397psi)

(i) $A_s = 0.26\ f_{ctm}\ b_t\ d\ /\ f_{yk}$ = 0.26 x 2.74 x 2355 x 700 / 413.69 = 2839 mm^2 (4.40 in^2)

(ii) $A_s = 0.0013\ b_t\ d$ = 0.0013 x 2355 x 700 = 2143 mm^2 (3.32 in^2)

Therefore, As = 2839 mm^2 (4.40 in^2)

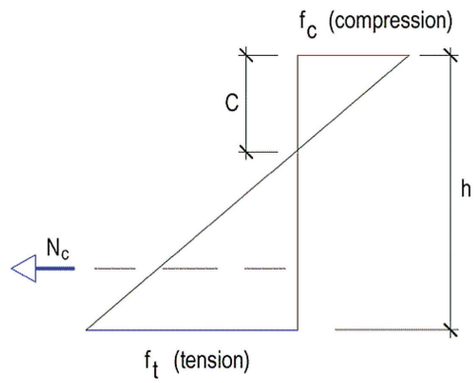


FIGURE 7.4-1 Hypothetical Distribution of Stress over Section

Contribution of reinforcement from bonded prestressing:

$A_{ps}\ (f_{pk}/f_{yk})$ = 12 x 99 x 1861.60 /413.69 = 5346 mm^2 (8.29 in^2)> 2839 mm^2 (4.40 in^2)

Hence, no additional bonded reinforcement is required.

❖ Minimum reinforcement for crack control:

Since the hypothetical tensile stress of concrete exceeds the threshold for crack control at point A, cracking reinforcement need to be provided.

❖ At point A:

$A_{smin} = k_c\ k\ f_{ct,eff}\ A_{ct}\ /\sigma_s$

A_{ct} is the area of the concrete section in tension zone.

f_t = -3.66 MPa (-530psi) (compression at top)

f_b = 2.99 MPa (434 psi) (tension at bottom)

$\sigma_s = f_{yk}$ = 413.69 MPa (60 ksi)

$f_{ct,eff} = f_{ctm} = 0.3\ (27.58)^{(2/3)}$ = 2.74 MPa (397 psi)

k = 0.677 (interpolated for h = 762 mm)

Distance of neutral axis from bottom, using Fig. 7.4-1 = 2.99 x 762 / (2.99 + 3.66)

= 343 mm (13.49 in)

A_{ct} = 343 x 457 = 156751 mm^2 (242.96 in^2)

$k_c = 0.4 \times [1 - (\sigma_c\ /(\ k1\ (h\ /\ h)\ f_{ct,eff})]$

$\sigma_c = N_{ED}\ /bh$ = 1.51 MPa

TABLE 7.4-1 Summary of Minimum Rebar (in^2) (T134US)

Code	Unbonded				Bonded			
	Point A	Point B & C	Point D	Point E	Point A	Point B & C	Point D	Point E
ACI/IBC	1.46	2.30	1.46	1.53	0	0	0	0
EC2	0.27	0	0	0	0.27	0	0	0

[26] EN 1992-1-1:2004(E) , Section 9.2.1 and 7.3.2

h = 762 mm
k_1 = 1.5
k_c = 0.4 [1 - (1.51 / (1.5 (762 / 762) 2.74)]
= 0.25
A_{smin} = 0.25 x 0.677 x 2.74 x 156751 /413.69
= 176 mm^2 (0.27 in^2)
Provide one 22 mm bar ($A_{s,prov}$ = 1 x 387 = 387 mm^2)
$A_{smin,crack}$ = 387 mm^2

The minimum rebar required from different codes is summarized in TABLE 7.4-1.

7.5 Deflection Check

The deflections are calculated for each of the load cases: dead, live, and post-tensioning using a frame analysis program. Gross cross-sectional area and linear elastic relationships are used. Since the stress level for which the design was carried out falls in the transition zone, the elastically calculated stresses must be adjusted to allow for cracking. Strictly speaking, a cracked deflection calculation has to be performed[27]. However, for hand calculation, recognizing that the locations of probable cracks are few, the option of "magnifying" elastic deformation by a factor that allows for cracking is used.

The critical location is span 1. The values for span 1 are as follows:
Dead Load = 1.01"
Post-Tensioning = -0.71"
Dead Load + PT = 0.30"
Live load deflection = 0.30" from frame analysis

The maximum stress under total loading at midspan is 651 psi. Since this is greater than 6 $\sqrt{f'_c}$ = 379 psi, adjustment to the calculated deflection is necessary.[28]

There are several options available to adjust elastically calculated deflection values, if the computed tensile stresses exceed the cracking threshold. One is the substitution of the gross moment of inertia (I_g), by an equivalent moment of inertia (I_e) that accounts for cracking. Another method is the magnification of the elastically calculated deflection by the ratio of (I_g/I_e). Commercially available computer programs, such as ADAPT-Floor Pro can determine the deflection of a member based on rigorous formulations and local cracking. But, for hand calculation, approximate methods are more appropriate. For prestressed sections the equivalent moment of inertia is calculated using the following relationship [PTI, 1990].

$$I_e = [1 - 0.30\,(f_{max} - 6\sqrt{f'_c})/\,6\sqrt{f'_c}\,]\,I_g \text{ (US)}$$

f'_c is in psi

Where, I_e is the equivalent moment of inertia, and I_g is the moment of inertia based on the gross cross-sectional area.

Reduction in moment of inertia due to cracking:
$I_e = [1 - 0.30\,(f_{max} - 6\sqrt{f'_c})\,/\,6\sqrt{f'_c})\,I_g$
$= [1 - 0.30\,(651\text{-}379)\,/\,379]I_g = 0.78\,I_g$

Hence deflection due to dead load and PT = 0.30 / 0.78 = 0.38"
Live load deflection with cracking allowance = 0.30 / 0.78 = 0.38"

❖ Long-term deflection
Multiplier factor assumed for effects of creep and shrinkage on long-term deflection = 2[29]
Load combination for long-term deflection, using a factor of 0.3 for sustained "quasi-permanent" live load:
(1.0 DL + 1.0 PT + 0.3 LL) (1 + 2)
Long-term deflection: (1 + 2) (0.38 + 0.3 x 0.38) = 1.48 in
Deflection ratio = 1.48/ (64 x 12) = 1/519 < 1/250 OK

❖ Instantaneous deflection due to design live load:
Live load deflection = 0.38 in.
Deflection ratio = 0.38 / (64 x 12) = 1/2021 OK

8 CODE CHECK FOR STRENGTH

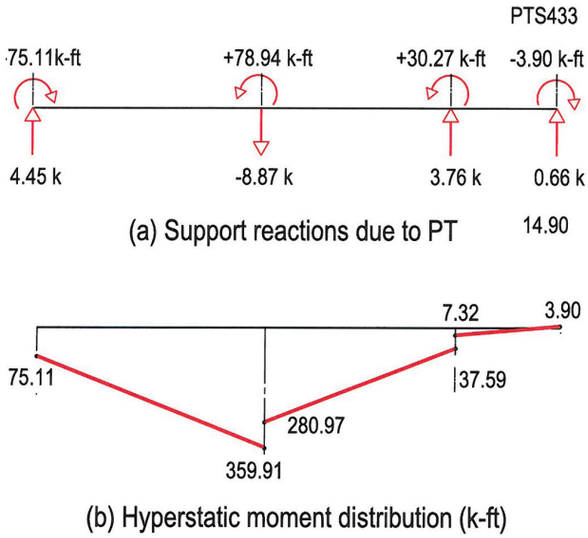
8.1 Load Combinations

❖ ACI 318/IBC
1.2 DL + 1.6 LL + 1 HYP
1.4 DL + 1 HYP

❖ EC2
1.35 DL + 1.5 LL + 1 Hyp

For strength combination, the hyperstatic (Hyp) actions from prestressing are used. The background for this is explained in detail in Chapter 4, Section 4.11.2.

27 Compter programs, such as ADAPT Floor have the option of cracked deflection calculation
28 The stress threshold for "transision" to cracking state is 7.5 $\sqrt{f'_c}$ however the empirical formula given in the reference used is base7.5 $\sqrt{f'_c}$.
29 ACI- 318 multiplier factor, Section 9.5.2.5



Hyperstatic (Secondary) Actions

FIGURE 8.2-1

8.2 Determination of Hyperstatic Actions

The hyperstatic moments are calculated from the reactions of the frame acted upon by the balanced loads (Fig. 6.4-2). The reactions obtained are shown in Fig. 8.2-1(a). The reactions shown produce hyperstatic moments in the frame as shown in Fig. 8.2-1(b).

The hyperstatic (secondary) reactions must be in self-equilibrium, since the applied loading (balanced loads) are in self-equilibrium.

Check the validity of the solution for static equilibrium of the hyperstatic actions, using the reactions shown in Fig. 8.2-1a:

ΣVertical forces = 4.45 – 8.87 + 3.76 + 0.66 = 0 OK

ΣMoments about support 1 = -75.11 + 78.94 + 30.27 – 3.90 – (8.87 x 64)
+ (3.76 x 119) + (0.66 x 136) = -0.28 k-ft ≈ 0 OK

Reduce hyperstatic moments to face-of-support using linear interpolation.

For right face-of-support (FOS) of span 1:

M_{HYP} = 359.91 – [(359.91 – 75.11) / 64'] 9/12
= 356.57 k-ft

Some engineers use the expression given below to compute hyperstatic (secondary) moments due to prestressing. This expression gives acceptable results for articulated members, and only if the balanced loads used in the determination of post-tensioning moments (M_{pt}) satisfy equilibrium.

$M_{hyp} = M_{pt} - P\,e$

Where M_{hyp} is the hyperstatic moment, P is the post-tensioning force, and e is the eccentricity of the post-tensioning.

8.3 Calculation of Design Moments

The design moment (M_u) is the factored combination of dead, live and hyperstatic moments.

Using ACI/IBC

Design moments are :

$M_{U1} = 1.2\ M_D + 1.6\ M_L + 1.0\ M_{HYP}$

$M_{U2} = 1.4\ M_D + 1.0\ M_{HYP}$

The second combination governs, when the values from dead loads are eight times or more of those of from live loading. This is a rare condition. The moments shown in Fig. 8.2-1 are centerline moments. These are reduced to the face-of-support in Table 8.3-1.

By inspection, the second load combination does not govern, and will not be considered in the following.

The factored moment for the codes considered are listed in the following table.

TABLE 8.3-1 Ultimate Design Moments (T135US)

	Point A	Point B	Point C	Point D	Point E
M_D k-ft	458.6	-670.9	-583.5	196.9	-46.2
M_L k-ft	137.6	-200.7	-174.2	58.3	-7.2
M_{HYP} k-ft	217.5	356.6	277.4	159.2	1.71
ACI 318-11/IBC 2012 : 1.2DL + 1.6LL + 1Hyp					
M_U k-ft	987.9	-769.6	-701.5	488.8	-65.3
EC2 : 1.35DL + 1.5LL + 1Hyp					
M_U kN-m (k-ft)	1414.2 (1043.1)	-1152.6 (-850.1)	-1046.2 (-771.6)	694.8 (512.4)	-96.9 (-71.5)

8.4 Strength Design for Bending and Ductility

The strength design for bending consists of two provisions, namely

❖ The design capacity (ΦM_n) shall exceed the demand. A combination of prestressing and non-stressed steel provides the design capacity

❖ The ductility of the section in bending shall not be less than the limit set in the associated building code. The required ductility is deemed satisfied, if failure of a section in bending is initiated in post-elastic response of its reinforcement, as opposed to crushing of concrete. For the codes covered in this example this is achieved through the limitation imposed on the depth of the compression zone (see Fig. C8.4-1 in Chapter 6). The depth of compression zone is generally limited to 50% or less than the distance from the compression fiber to the farthest reinforcement (d_r). Since the concrete strain (ε_c) at crushing is assumed between 0.003 to 0.0035, the increase in steel strain (ε_s) will at minimum be equal to that of concrete at the compression fiber. This will ensure extension of steel beyond its yield point (proof stress) and hence a ductile response.

For expeditious hand calculation, the flexural capacity of a post-tensioned member in common building structures can be approximated by assuming a conservative maximum stress for prestressing tendons. For detailed application of the code-proposed formulas refer to Chapter 12, Section 12.2. Application of strain compatibility for the calculation of section capacity is the preferred option, but its use for hand calculation in the routine work of a consulting office is not warranted, unless a software is used.

There are two justifications, why a simplified method for ULS design of post-tensioned sections in daily design work are recommended. These are:

❖ Unlike conventionally reinforced concrete, where at each section along a member non-prestressed reinforcement must be provided to resist the design moment, in prestressed members this may not be necessary, since prestressed members possess a base capacity along the entire length of prestressing tendons (Fig. C8.4-2b in Chapter 6). Non-prestressed reinforcement is needed at sections, where the moment demand exceeds the base capacity of the section.

❖ In conventionally reinforced concrete, the stress used for rebar at ULS is well-defined.. For prestressed sections, however, the stress in tendon at ULS is mostly expressed in terms of involved relationships – hence the tendency to use a simplified, but conservative scheme for everyday hand calculation. For repetitive work, computer programs are recommended.

The basics of the simplified procedure and numerical examples for it are given in Chapter 6, Section 8.4.

A-strength design

Figure C8.4-3 in Chapter 6 illustrates the forces and dimensional parameters used in the calculations for rectangular sections. Similar force configuration will apply to T-sections. Using strain compatibility procedure[30] the required reinforcement for each of the two codes are calculated. The outcome is given in the Table 8.4-3 including the following cracking moment requirement.

B-cracking moment

❖ Cracking moment larger than moment capacity: Where cracking moment of a section is likely to exceed its design capacity in flexure, reinforcement is added to raise the moment capacity. In such cases, the contribution of each reinforcement is based on the strength it provides. If the value of reinforcement required is expressed in terms of cross-sectional area of nonprestressed reinforcement, the applicable value for compliance will be ($A_s+A_{ps}*f_{py}/f_y$).

A. Based on ACI 318-11/IBC 2012:

❖ Bonded Tendon:

ACI 318[31]/IBC requires that for members reinforced with bonded tendons the total amount of prestressed and nonprestressed shall be adequate to develop a factored load at least 1.2 times the cracking load computed on the basis of the modulus of rupture of the section. In practice, this is taken as cracking moment of the section M_{cr}.

The necessity and amount of rebar is defined as a function of cracking moment of a section (M_{cr}). For prestressed members

$M_{cr} = (f_r + P/A)\ S$

Where, f_r is the modulus of rupture defined.[32]

$f_r = 7.5\sqrt{f'_c} = 7.5\sqrt{4000} = 474$ psi

P/A is the average precompression, and S the section modulus. The Table 8.4-1 summarizes the leading values and the outcome.

30 ADAPT-TN178
31 ACI 318-11, Section 18.8.2
32 ACI-318-11, Section 9.5.2.3

TABLE 8.4-1 Cracking Moment Values and Parameters for ACI (T138US)

		Section A	Section B&C	Section D
Basic parameters and results	S_{top} (in^3)		7979	
	S_{bot} (in^3)	3801		3801
	P (kips)	321.24	321.24	240.93
	P / A (psi)	-219	-219	-164
	f_r + (P/A)	693	693	638
	M_{cr} (k-ft)	219.51	460.79	202.09
	1.2 M_{cr} (k-ft)	263.41	552.94	242.50
	Φ M_n (k-ft)	911.81	911.81	501.92
	Status	OK	OK	OK

TABLE 8.4-2 Cracking Moment Values and Parameters for EC2 (T138SI)

		Section A	Section B&C	Section D
Basic parameters and results	S_{top} (mm^3)		1.31e+8	
	S_{bot} (mm^3)	6.23e+7		6.23e+7
	P (kN)	1429	1429	1072
	P / A (MPa)	-1.51	-1.51	-1.13
	f_r + (P/A)	4.25	4.25	3.87
	M_{cr} (kNm)	264.78	556.75	241.10
	1.15 M_{cr} (kNm)	304.50	640.26	277.27
	Φ M_n (kNm)	1196.69	1196.69	658.87
	Status	OK	OK	OK

Since at the selected sections, the design capacity of the section with prestressing alone exceeds 1.2M_{cr}, no additional rebar is required from this provision.

In design situations like above, where the design is initiated by determination of whether a value is less or more than a threshold, it is expeditious to start using a simplified, but conservative procedure. If the computed value is close to the target, design check can be followed with a more rigorous computation.
Assume the following:
Cover to strand CGS = 2.75 in; hence $d = h$ (thickness) – 2.75
Moment arm = 0.9d
Design force in strand = 270 A_{ps} ksi; Φ = 0.9
At midspan, with 12 strands, 270 ksi strength
$\Phi\ M_n$ = 0.9 x 12 x 0.153 x 270 (30 – 2.75) 0.9 / 12 = 911.81 k-ft
Design moment at other locations are calculated in a similar manner.

B. Based on EC2:

❖ Unbonded tendons

EC2[33] requires that for members reinforced with unbonded tendons the total amount of prestressed and nonprestressed shall be adequate to develop a factored load at least 1.15 times the cracking load computed on the basis of the modulus of rupture of the section. In practice, this is taken as cracking moment of the section M_{cr}.

The necessity and amount of rebar is defined as a function of cracking moment of a section (M_{cr}). For prestressed members
$M_{cr} = (f_r + P/A)\ S$
Where, f_r is the modulus of rupture[34]
$f_r = f_{ctm} = 0.3 f_{ck}^{(2/3)}$ = 2.74 MPa (397 psi)

33 EN 1992-1-1:2004 (E), Section 9.2.1.1(4)
34 EN 1992-1-1:2004 (E), Section 7.1(3). Here tensile stress limit for uncracked section is used.

TABLE 8.4-3 Summary of Required Reinforcement for Strength Limit State (T136US)

Code	Unbonded				Bonded			
	Point A	Point B & C	Point D	Point E	Point A	Point B & C	Point D	Point E
ACI/IBC	1.57	2.3	1.46	0	0.35	0	0	0
EC2	2.35	2.07	0.75	0	1.49	1.22	0.29	0

TABLE 8.4-4 Envelope of Reinforcement for Serviceability (SLS) and Strength (ULS)Conditions (T137)

Code	Unbonded				Bonded			
	Point A	Point B & C	Point D	Point E	Point A	Point B & C	Point D	Point E
ACI/IBC	1.57	2.3	1.46	1.53	0.35	0	0	0
EC2	2.35	2.07	0.75	0	1.49	1.22	0.29	0

P/A is the average precompression, and S the section modulus. The Table 8.4-2 summarizes the leading values and the outcome.
Since at the selected sections, the design capacity of the section with prestressing alone exceeds $1.15M_{cr}$, no additional rebar is required from this provision.

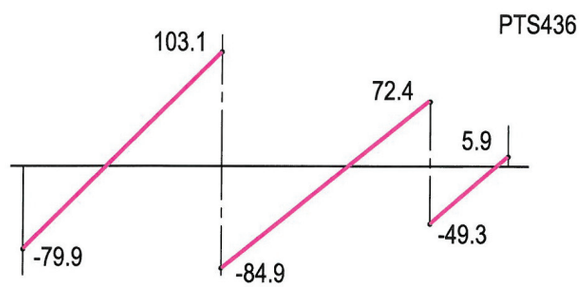
In design situations like above, where the design is initiated by determination of whether a value is less or more than a target, it is advisable to start the check using a simplified, but conservative procedure. If the computed value is close to the target, design check can be followed with a more rigorous computation.
Assume the following:
Cover to strand CGS = 70 mm ; hence $d = h$ (thickness) - 70
Moment arm = $0.9d$
Design force in strand = A_{ps} x 1860 MPa/1.15;
At midspan, with 12 strands, 1860 MPa strength
$\Phi\ M_n$ = 12 x 99(1860/1.15) 0.9(762 – 70) /10^6 = 1196.69 kNm
Design moment at other locations are calculated in a similar manner.

The summary of the strength reinforcement is given in the above table.

Strength computations performed herein were limited to points considered critical by inspection. When spans and loading are not regular, the selection of critical points by inspection becomes difficult. In such cases, stress and strength checks must be performed at a greater number of locations. Also, note that due to the contribution of tendon to ultimate strength, and change in drape of tendon along the length of a member, the most critical location for design is not necessarily the location of maximum moment.

8.5 One Way Shear Design

Distribution of design shear is shown in Figure 8.5-1. The design shear (V_u) is computed from the results of the standard frame analysis performed for the loading conditions D, L and PT. The following combination was used:



Distribution of Shear (kips)

FIGURE 8.5-1

$V_u = 1.2\ V_D + 1.6\ V_L + 1.0\ V_{HYP}$
Shear check is performed for both codes ussing the same shear force demand V_u based on ACI load factors.

A. Based on ACI 318-11/IBC 2012
Span 1:
Point of zero shear = 79.9 x 64' / (79.9 + 103.1) = 27.94'
Design at distance = (column width + h) / 2

= (14" + 30) / 2 = 22" for exterior column
= (18" + 30) / 2 = 24" for interior column
For the left support:
V_u = -79.9 x (27.94 - 1.83) / 27.94
= -74.67 k
For the right support:
V_u = 103.1 x (64 - 27.94 - 2.00) / (64 - 27.94)
= 97.38 k
Hence, the right support governs

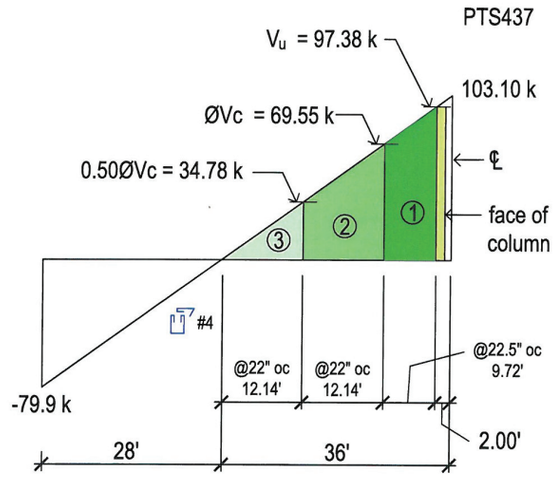
The shear design for the right support of span 1 will be followed in detail, since this is the most critical location. The procedure for the shear design of other locations is identical. The design starts with the calculation of vc, the code allowable shear stress contribution of concrete over the shear area of the section. Depending on the value of φv_c compared to that of the computed average design shear stress v_u, in the general case, up to three regions along the length of a member can be identified for design. These are shown schematically in Fig. 8.5-2 for the right region of span 1.

b_w = 18"
d = 0.8 h = 0.8 x 30" = 24"
d_p = 27.25" > 0.8h = 24"
conservatively assumed 24"
v_{cmin} = 2 √4000 = 127 psi
v_{cmax} = 5 √4000 = 316 psi
v_c = 0.6 √f'c + 700 V_u d / M_u[35]
The term (V_ud/ M_u) must be less than 1 or use 1.
V_ud/ M_u = 97.38 x 24 / (769.6 x 12) = 0.25 < 1 OK
v_c = 0.6 √4000 + 700 x 97.38 x 24 / (769.6 x 12)
= 215 psi > $v_{c\,min}$ = 127 psi < $v_{c\,max}$ = 316 psi
Hence v_c = 215 governs the design.

For this example the ultimate moment was taken at the face-of-support for brevity of the example, even though the shear check is done at a distance *h/2* away from the support. This assumption is conservative and does not have a significant effect on the outcome of the calculation. In the general case, the value of $V_u d/ M_u$ varies along the length of the member. But, it is assumed constant for this example.

v_u = (97.38 x 1000) / (18 x 24) = 225 psi
v_u = 225 psi > φ v_c = 0.75 x 215 = 161 psi
Hence shear reinforcement is required by calculation,

Assume #4 stirrups with two legs: A_v = 2 x 0.2 in^2 = 0.40 in^2
The spacing, s, between the stirrups is given by:



Shear Force and Reinforcement for Right Side of Span 1

FIGURE 8.5-2

s = φ A_v f_v / [b_w (v_u – φv_c)]
= 0.75 x 0.40 x 60000 / [18 x (225 – 161)] = 15.63"
but, s[36] <= 0.75 h = 0.75 x 30 = 22.5"; and
s <= 24"
Select s = 15" for the entire region where stirrups by calculation governs.

Using similar triangles, the three regions for the calculation of shear reinforcement are worked out and shown graphically in Fig. 8.5-2.

For the first region V_u > = φ V_c = 18 x 24 x 161 / 1000 = 69.55 k
Use stirrups at 15" (381 mm) spacing.

For the second region V_u > = 0.5 φ V_c
= 0.5 x 18 x 24 x 161 / 1000 = 34.78 k
Use the minimum value specified by code.

For the third region V_u < 0.5 φ V_c = 34.78 k
No web shear reinforcement required by code. Conservatively, use the same stirrups at 22" spacing (s <= 0.75 h = 22.5")

For the region governed by the minimum rebar, the spacing shall be the smallest of the following:

In the following the three applicable code relationships[37] are re-arranged to express them in terms of

35 ACI 318-11, Section 11.3.2

36 ACI 318-11, Section 11.4.5.1
37 ACI 318-11, Section 11.4.6

s stirrup spacing. The format of the relationships in the code is in terms of A_{min}. In this case, since the stirrups are already selected as two-legged #4 bars, the required spacing is worked out.

For each stirrup, $A_{min} = A_v = 2 \times 0.2 = 0.4$ in^2.

(i) $s = A_v f_v / (50\ b_w)$ (US)
$= 0.4 \times 60000 / (50 \times 18) = 26.67$ "

(ii) $s = 80\ A_v (f_v / f_{pu})\ d\ (b_w / d)^{0.5} / A_{ps}$
$= 80 \times 0.4\ (60 / 270)\ 24\ (18 / 24)^{0.5} / 1.84 = 80.33$"

(iii) $s = A_v f_v / (0.75\ b_w f'_c{}^{0.5})$
$= 0.4 \times 60000 / (0.75 \times 18 \times 4000^{0.5}) = 28.11$ "
At the same time, spacing s shall not be more than 24", nor 0.75h = 22.5"

Use #4 two-legged stirrups at 22" on spacing for this region.

B. Based on EC2

Shear check carried out in the following is based on factored demand shears obtained above, using ACI load factors. For EC2, however, the load factors will be different, leading to a somewhat different demand shear. However, to simplify comparison, the same demand force is used in for both design codes.
Span 1:
$b_w = 457$ mm (18 in)
$d = 760 - 51 - 22/2 = 698$ mm (27.48 in)
Point of zero shear = 355.41 x 19.51 / (355.41 + 458.61) = 8.52 m (27.95 ft)
Design at distance = column width/2 + d
= 356/2 + 698 = 876 mm (34.49 in) from exterior column CL
= 457/2 + 698 = 927 mm (36.48 in) from interior column CL
For the left support:
V_{ED} = -355.41 x (8.52 – 0.876) / 8.52 =-318.87 kN (71.69 k)
For the right support:
V_{ED} = 458.61 x (20 – 8.52 – 0.927) / (20 – 8.52)
= 421.58 kN (94.78 k)
Hence, the right support governs.

The shear design for the right support of span 1 will be followed in detail, since this is the most critical location. The procedure for the shear design of other locations is identical. The design starts with the calculation of $V_{Rd,c}$, the design shear resistance of the member without shear reinforcement.

$V_{Rd,c}$[38] = [$C_{Rd,c}$ k (100 ρ1 f_{ck})1/3+ k 1 σ_{cp}] b_w d
but not less than (v_{min} + k $_1\sigma_{cp}$) b_w d
Where,
f_{ck} = 27.58 MPa (4000 psi)
k = 1+ (200 / d)$^{1/2}$ = 1 + (200 / 698)$^{1/2}$ = 1.54 < 2.0
$\rho_1 = A_{sl} / (b_w d) = 5 \times 387 / (457 \times 698) = 0.0061$
$\sigma_{cp} = N_{ED}/A_C = 1429 \times 10^3 / 94840 = 1.51$ MPa < 0.2 x18.39 = 3.68 MPa
$C_{Rd,c} = 0.18 / \gamma_c = 0.18 / 1.50 = 0.12$
$k_1 = 0.15$
$v_{min} = 0.035\ k^{3/2} f_{ck}{}^{1/2} = 0.035 \times 1.54^{3/2} \times 27.58^{1/2} = 0.34$ MPa
$V_{Rd,c}$ = [0.12 x 1.54 (100 x 0.0061 x 27.58)$^{1/3}$+ 0.15 x 1.51] 457 x 698 / 1000
= 222.97 kN (50.13 k)
$V_{Rd,cmin}$ = (0.34 + 0.15 x 1.51) 457 x 698/1000
= 180.71 kN (40.62 k)
$V_{Rd,c}$ = 222.97 kN (50.13 k)$V_{ED} > V_{Rd,c}$, Shear reinforcement is required by calculation.

Assume 13 mm (#4) stirrups with two legs: A_{sw} = 2 x 129 mm^2 = 258 mm^2 (0.40 in^2)
The spacing,[39] s, between the stirrups is given by:
$s = (A_{sw} / V_{Rd,s})\ z\ f_{ywd} \cot\theta$
Where,
Assume $\theta = 40^0$, $\cot\theta = 1.20$
$V_{Rd,s} = V_{ED} - V_{Rd,c}$ = 421.58 – 222.97 = 198.61 kN (44.65 k)
z = 0.9 d = 0.9 x 698 = 628 mm (24.73 in)
s = (258 /198.61 x 1000) x 628 (413.69 / 1.15) x 1.20
= 352 mm (13.86 in)
$V_{Rd,max}$[40] = $\alpha_{cw}\ b_w\ z\ \nu_1\ f_{cd} / (\cot\theta + \tan\theta)$
Where,
ν_1[41] = 0.6[1 - (f_{ck} / 250)] = 0.53 since $f_{ywd} > 0.8 f_{yk}$
f_{cd} = 18.39 MPa (2667psi)
α_{cw}[42] = (1+ σ_{cp} / f_{cd}) for σ_{cp} = 1.51 MPa < 0.25f_{cd} = 0.25 x 18.39 = 4.60 MPa
= (1 + 1.51 / 18.39) = 1.08
$V_{Rd,max}$[43] = [1.08 x 457 x 628 x 0.53 x 18.39 / (1.20 + 0.84)] / 1000 = 1480.90 kN (332.92 k)
> 198.61 kN (44.65 k) OK

Select s = 350 mm (13.8 in) (s <= 0.75 d (1+cotα) = 0.75 x 698 = 523 mm (20.59 in)) for the entire region where stirrups by calculation governs.

38 EN 1992-1-1:2004 (E), Section 6.2.2
39 EN 1992-1-1:2004 (E), Exp: 6.8
40 EN 1992-1-1:2004 (E), Exp: 6.9
41 EN 1992-1-1:2004 (E) Exp: 6.6N
42 EN 1992-1-1:2004 (E) Exp: 6.11aN
43 EN 1992-1-1:2004 (E) Exp: 6.9

If $V_{ED} < V_{Rd,c}$, use the minimum rebar specified by code.[44] For the region governed by the minimum rebar, the spacing shall be the following:

In the following the applicable code relationship is re-arranged to express them in terms of s spacing. The format of the relationships in the code is in terms of A_{min}. In this case, since the stirrups are already selected as two-legged 13 mm(#4)bars, the required spacing is worked out. Hence, $A_{min} = A_v = 2 \times 129 = 258$ mm^2 (0.40 in^2).

$s = A_{sw} f_{yk} / (0.08 \sqrt{f_{ck}}\, b_w)$
$= 258 \times 413.69 / (0.08 \sqrt{27.58}\, 457)$
$= 556$ mm (21.89 in)

At the same time, spacing s shall not be more than 523 mm (20.59 in).
Use 13 mm two-legged stirrups at 520 mm (20.47 in) spacing for this region.

9 CODE CHECK FOR INITIAL CONDITION

At stressing (i) concrete is at low strength; (ii) prestressing force is at its highest value; and (iii) live load generally envisaged to be counteracted by prestressing is absent. As result, the stresses experienced by a member can fall outside the envelope of the limits envisaged for the in-service condition. Hence, post-tensioned members are checked for both tension and compression stresses at transfer of prestressing. Where computed compression stresses exceed the allowable values, stressing is delayed until either concrete gains adequate strength or the member is loaded. Where computed tension stresses are excessive, ACI/IBC[45] suggest adding non-stressed reinforcement to control cracking.

9.1 Load Combinations

The codes covered are not specific on the applicable load combination at transfer of prestressing. The following is the combination generally assumed among practicing engineers;

Load Case: 1.0 DL + 1.15 PT
$f'_{ci} = ¾ \times 4000 = 3000$ psi

9.2 Stress Check

$\sigma = \pm(M_D + 1.15\, M_{PT}) / S + 1.15\, P/A$
$S = I/Y_c$

9.3 Allowable Stresses

A. Based on ACI-318-11; IBC 2012
Tension = $3 \sqrt{3000} = 164$ psi

TABLE 9-1 Stresses at Transfer of Post-Tensioning (T139US)

	Point A	Point B	Point C	Point D	Point E
M_D k-ft	458.6	-670.9	-583.5	196.9	-46.2
M_{PT} k-ft	-320.5	443.8	375	-99.9	45.3
P kips	321.24	321.24	321.24	240.93	240.93
P / A psi	-219	-219	-219	-164	-164
f_t psi (MPa)	-387 (-2.67)	-10 (0.07)	-23 (-0.16)	-312 (-2.15)	-203 (-1.40)
f_b psi (MPa)	32 (0.22)	-759 (-5.23)	-733 (-5.05)	70 (0.48)	-167 (-1.15)
ACI-11/IBC 2012					
F_t psi	-1800	-1800	-1800	-1800	-1800
F_b psi	164	-1800	-1800	164	-1800
	OK	**OK**	**OK**	**OK**	**OK**
EC2					
F_t MPa (psi)	-12.41 (-1800)	-12.41 (-1800)	-12.41 (-1800)	-12.41 (-1800)	-12.41 (-1800)
F_b MPa (psi)	2.26 (327.79)	-12.41 (-1800)	-12.41 (-1800)	2.26 (327.79)	-12.41 (-1800)
	OK	**OK**	**OK**	**OK**	**OK**

Note: F_t and F_b are allowable stresses at top and bottom respectively.

44 EN 1992-1-1:2004 (E) Exp: 9.4 and 9.5(N)

45 ACI 318-11; Section 18.4

Compression = 0.60 x 3000 = -1800 psi

B. Based on EC2
Tension = f_{cteff} = 2.26 MPa (323 psi)
Compression = 0.60 x 20.69 = -12.41 Mpa (-1800 psi)
Farthest fiber stresses are calculated in a similar manner with to service condition as outlined earlier. The outcome is summarized in table 9-1.

If in any of the above locations the stresses exceeded the allowable values the following would have been done.

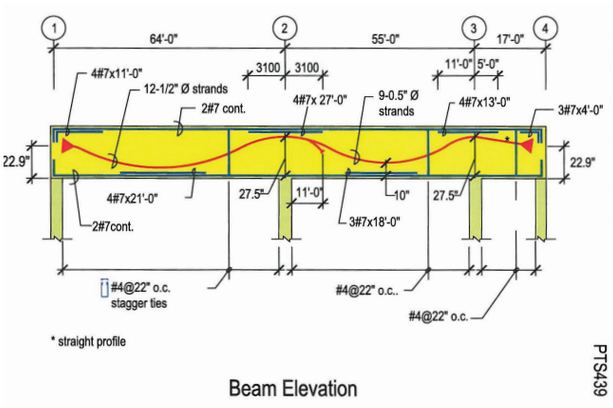


FIGURE 10.1

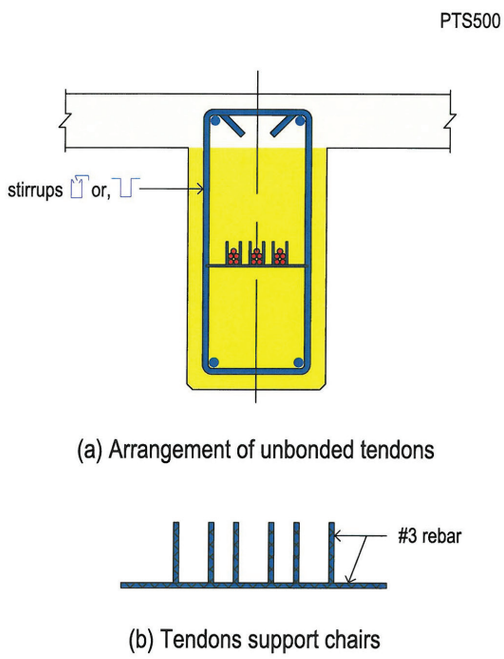


FIGURE 10-2

If compression stresses exceed the allowable value, the design parameters must be modified to bring the stresses within the code limits. If tensile stresses exceed the allowable value, bonded additional reinforcement (nonprestressed, or prestressed) shall be provided in the tensile zone to resist the total tension force in concrete computed with the assumption of an uncracked section.

10 DETAILING

The final tendon and reinforcement layout for the parking structure beam is shown in Figs. 10-1 through 10-2 below.

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