CHAPTER 7

POST-TENSIONED BEAM DESIGN STEP-BY-STEP CALCULATION

Post-Tensioned Parking Structure Using Beam Frames and One-Way Slabs (P466)

FOREWORD

The example selected represents a frame of a one-way slab and beam construction—typical of parking structures, or floors, where span in one direction is two or more times the span in the orthogonal direction, for which a beam and one-way slab will be appropriate. The beam frame selected has three spans, each with a different length. The third span is purposely selected to be short, compared to the other two. Also, the optimum post-tensioning for the design is one with different amount of post-tensioning along the length of the structure, and variable profile from span to span.

The objective in selecting a somewhat complex structure is to expose you to the different design scenarios that you generally encounter in real life structures, but are not featured in text books—in particular, where span lengths in a continuous member are widely different.

The example walks you through the 10 steps of design of post-tensioned structures. Aspects of design conditions that are not covered in the design of the example selected, but are important to know, are introduced and discussed as comments or inserted examples.

Design operations that are considered common knowledge, such as the calculation of moments and shears, once the geometry of a structure, its material and loading are known, are not detailed. You are referred to your in-house frame programs.

The design example covers side by side both the unbonded and bonded (grouted) post-tensioning systems, thus providing a direct comparison between the design processes of the two options. In addition, in parallel, the design uses the current American building codes (ACI-318[1] and IBC[2]) along with the European Code (EC2[3]). Where applicable, reference is made to the UK's committee report TR43[4].

[1] ACI 318-11
[2] IBC 12; International Building Code 2012
[3] EN 1992-1-1:2004(E)
[4] TR43-2005; Concrete Society, UK

The common method of analysis for beam frames and one-way slabs is the Simple Frame Method (SFM). While it is practical to use SFM in the environment of consulting firms for design of one-way slabs and beam frames, it becomes laborious if an optimum design for the post-tensioning is sought. The iterative nature of optimization for post-tensioning lends itself well to the application of computer programs, such as ADAPT-PT for expediency in design.

The hand calculations are supplemented by a computer run from ADAPT-PT for verification.

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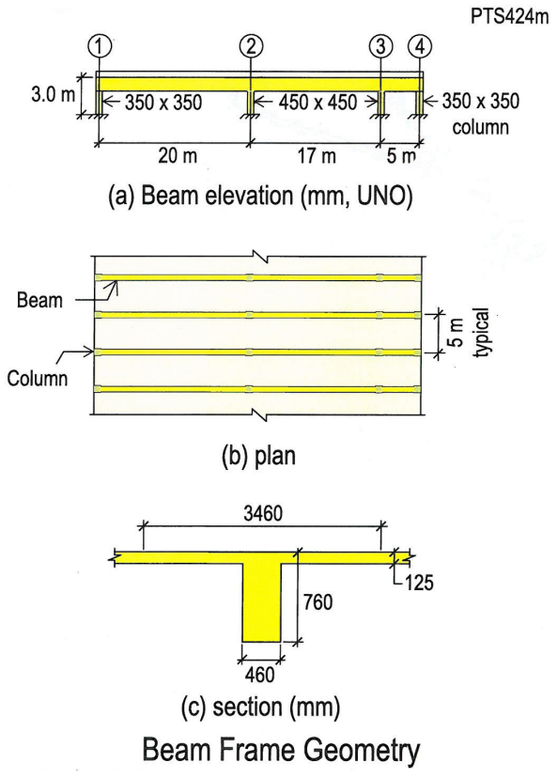
1 - GEOMETRY AND STRUCTURAL SYSTEM

The floor consists of a one-way slab supported on parallel beams as shown in Fig. 1-1.

1.1 Dimensions and Support Conditions

- Geometry is as shown in Fig. 1-1(a) and (b)
- Beam cross section as shown in Fig. 1-1(c)
- Total tributary width = 5 m typical
- Columns extend below the deck only; first and last columns are assumed hinged at the bottom.



Beam Frame Geometry

FIGURE 1-1

TABLE 1.3-1 Section Properties (T131SI)

	Spans 1 and 2		Span 3	
	Axial effects	Bending effects	Axial effects	Bending effects
Area (mm^2)	9.171e+5	5.996e+5	9.171e+5	4.484e+5
I in.^{4} (mm^4)	-----	3.185e+10	-----	2.472e+10
Y_t in. (mm)	184	248	184	310
Y_b in. (mm)	576	512	576	450
S_{top} (mm^3)	-----	1.28e+8	------	7.97e+7
S_{bot} (mm^3)	------	6.22e+7	-------	5.49e+7

I = Second moment of area (moment of inertia);
Y_t = distance of centroid to top fiber of section;
Y_b = distance of centroid to bottom fiber of section;
S_{top} = section modulus for top fiber; (I/Y_t); and
S_{bot} = section modulus for bottom fiber; (I/Y_{bot}).

End columns are assumed hinged and detailed as hinged at the connection to the footing, in order to reduce stresses and potential of cracking due to shrinkage and creep of concrete for the first elevated deck.

1.2 Effective Width of Flanges

When hand calculation is used in analysis of flanged beams, an effective width is selected to account for the bending effects of the structure. ACI-318-11[5] explicitly states that the effective width used for analysis of conventionally reinforced flanged beams does not apply when the same is post-tensioned, but does not clarify the alternative. Section 4.8.3 outlines the reason behind ACI-318's standing and explains the applicable procedure. Briefly, for axial forces (post-tensioning) the entire cross-sectional area is effective. But, for computation of flexural stresses in hand calculation a reduced flange width is applicable.

- For axial effects (precompression) use the entire tributary of the structure
- For bending effects use the "effective width" value associated with the bending of the flanged beam.

Also, note that the effective width concept is associated with the distribution of elastic stresses in the flange of a beam. It is applicable for "serviceability limit" design (SLS) of a post-tensioned member. For safety checks (ULS) the effective width does not apply.

Other codes and TR43 covered herein are also mute on the effective width of a post-tensioned flanged beam. For conventionally reinforced concrete, ACI 318-11[6] recommends the least of the following values for effective width of an interior span in bending:

(i) eight times the flange thickness on each side of the stem,
(ii) one quarter of the span, or
(iii) the beam's tributary.

Tributary width = 5000 mm
(i) Sixteen times flange thickness plus stem width
= 16*125 + 460 = 2460 mm
(ii) One quarter of span
For span 1 = (20*1000)/4 = 5000mm
For span 2 = (17*1000)/4 = 4250mm
For span 3 = (5*1000)/4 = 1250mm
(iii) Tributary width = 5000 mm
Assume the following:
Spans 1 and 2: 2460 mm
Span 3: 1250 mm

1.3 Section Properties

The section properties for the axial effects are the same for all spans. For bending effects, however, due to different effective widths, the section properties differ. The section properties calculated are listed in Table 1.3-1.

I = Second moment of area (moment of inertia);
Y_t = distance of centroid to top fiber of section;
Y_b = distance of centroid to bottom fiber of section;
S_{top}= section modulus for top fiber; (I/Y_t); and
S_{bot}= section modulus for bottom fiber; (I/Y_{bot}).

2 - MATERIAL PROPERTIES

2.1 Concrete

Cylinder strength f'_c ,f_{ck} (28 day) = 28 MPa

[5] ACI 318-11, Section 18.1.3
[6] ACI 318-11, Section 8.12.2

Weight = 24 kN/m^3
Modulus of Elasticity = 4700 $\sqrt{f'c}$ = 24870 MPa [ACI]
= 22* 10^3* [(f_{ck} +8)/ 10] $^{0.3}$ [7] [EC2, TR-43];
= 32308 MPa
Creep coefficient t = 2
Material factor, γ_c = 1 – ACI, 1.50 [EC2, TR-43]
Strength at transfer, f_{ci} = 20 MPa

The creep coefficient is used to estimate the long-term deflection of the slab.

2.2 Nonprestressed (Passive) Reinforcement

f_y = 460 MPa
Elastic Modulus = 200000 MPa
Material factor, γ_c = 1 [ACI]; 1.15 – [EC2, TR-43]
Strength reduction factor (bending), φ = 0.9 [ACI];
= 1 [EC2, TR-43]

2.3 Prestressing: (Figs 2.3-1 through 2.3-4)

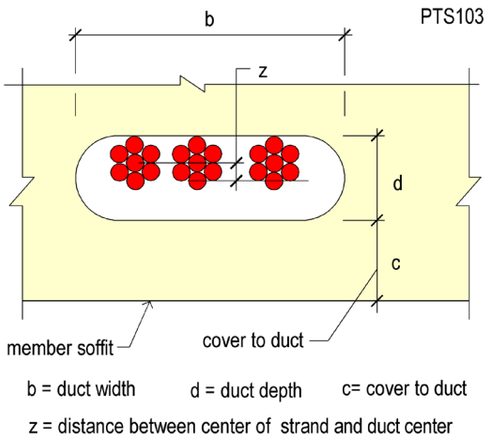
Material—low relaxation, seven wire ASTM 416 strand
Nominal strand diameter = 13 mm
Strand area = 99 mm^2
Elastic Modulus = 200000 MPa
Ultimate strength of strand (f_{pu}) = 1860 MPa
Material factor, γ_c = 1 [ACI]; 1.15 [EC2, TR-43]

System
Unbonded System
Angular coefficient of friction (μ) = 0.07
Wobble coefficient of friction (K) = 0.003 rad/m
Anchor set (wedge draw-in) = 6 mm
Stressing force = 80% of specified ultimate strength
Effective stress after all losses[8] = 1200 MPa

Bonded System
Use flat ducts 20x80mm; 0.35 mm thick metal sheet housing up to five strands
Angular coefficient of Friction (μ) = 0.2
Wobble coefficient of Friction (K) = 0.003 rad/m
Anchor set (Wedge Draw-in) = 6 mm
Offset of strand to duct centroid (z) = 3 mm
Effective stress after all losses = 1100 MPa

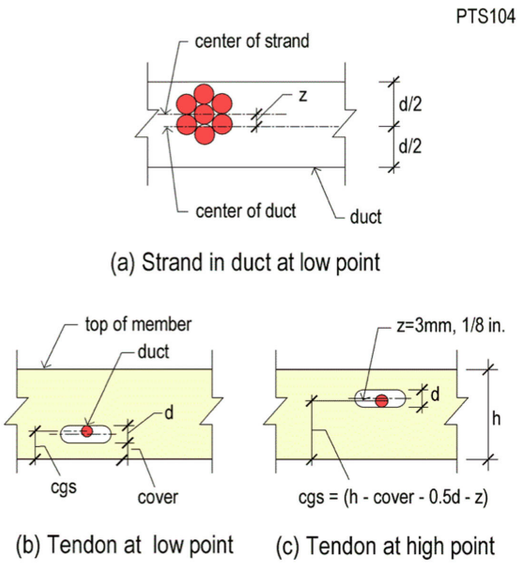
Section through the bonded tendon duct in place is shown in Fig. 2.3-1 and 2.3-2

[7] EN 1992-1-1:2004(E) Table 3.1
[8] For hand calculation, an effective stress of tendon is used. The effective stress is the average stress along the length of a tendon after all immediate and long-term losses. The value selected for effective stresses is a conservative estimate. When "effective stress" is used in design, the stressed lengths of tendons are kept short, as it is described later in the calculations.



Section through a Flat Duct at Low Point

FIGURE 2.3-1 Bonded Tendon Section



Position of Center of Gravity (cgs) of Strand at Extreme Positions in Member

FIGURE 2.3-2

3 - LOADS

3.1 Selfweight

Slab = 0.125 m*2400 kg/m^3*5m*9.806/1000
= 14.71 kN/m
Stem = 0.635*0.460*2400*9.806/1000
= 6.87 kN/m
Total selfweight = 14.71 + 6.87 = 21.58 kN/m

3.2 Superimposed Dead Load

From mechanical, sealant and overlay 0.5 kN/m^2
= 0.5 kN/m^2*5 m= 2.5 kN/m
Total Dead Load = 21.58 + 2.5 = 24.08 kN/m^2

3.3 Live Load:[9] 2.5 kN/m^2

Total live load = 2.5*5 = 12.5 kN/m
MaxLL/DL ratio = 12.5/24.08 = 0.52 < 0.75 ∴ Do not skip live loading

Strictly speaking, live loads must be skipped (patterned) to maximize the design values. But, when the ratio of live to dead load is small (less than 0.75), it is adequate to determine the design actions based on full value of live loads on all spans (ACI-318-11[10]). This is specified for slab construction, but it is also used for beams.

4 - DESIGN PARAMETERS

4.1 Applicable Codes

The design is carried out according to each of the following codes. Further, reference is made to the Committee Report TR43, where appropriate.

- ACI 318-2011; IBC-2012
- EC2 (EN 1992-1-1:2004)

4.2 Cover to rebar and prestressing strands

Unbonded and bonded system
Minimum rebar cover = 40 mm top and bottom

The cover selected is higher than the minimum code requirement to allow for installation of top slab bars over the beam cage in the transverse direction to the beam.

Minimum prestressing CGS =70 mm

The cover and hence distance to the CGS (Center of Gravity of Strand) is determined by the requirements for fire resistivity and positioning of tendons within the beam cage. The distance 70 mm selected is slightly higher than the minimum required. Its selection is based on ease of placement.

[9] The live load assumed is somewhat high. The common value in the US, based on ASCE 07 is 2 kN/m2. Also, US codes allow reduction of live load under certain conditions. In this example, the higher value common in a number of world regions is used and the value is not reduced.
[10] ACI 318-11, Section 13.7.6

4.3 Allowable Stresses

A. Based on ACI 318-11/IBC 2012[11]

Allowable stresses in concrete are the same for bonded and unbonded PT systems

- For sustained load condition

Compression = 0.45*f'_c = 12.60 MPa

- For total load condition

Compression = 0.60*f'_c = 16.80 MPa

Tension: (Transition condition of design is targeted)
The range for transition (moderate cracking) is as follows:
= 0.62*$\sqrt{f'_c}$ < stress ≤ 1.00*$\sqrt{f'_c}$
= 3.28 MPa < stress ≤ 5.29 MPa

For top fibers the lower value will be targeted, in order to limit crack width and improve durability. For the bottom fiber the higher value will be used, allowing for a wider crack width

- For initial condition

Compression = 0.60 f'_{ci} = 0.6* 20 = 12 MPa
Tension = 0.25 $\sqrt{f'_c}$ = 1.12 MPa

For one-way systems, ACI 318-11 defines three conditions of design, namely uncracked (*U*), transition (*T*) and cracked (*C*). The three conditions are distinguished by the magnitude of the maximum hypothetical tension stress in concrete at the farthest tension fiber. For the current design example the transition (*T*) condition is selected. For this condition, hypothetical tension stresses can exceed $0.62\sqrt{f'_c}$ but not larger than $1.00\sqrt{f'_c}$ However, since the surface of the parking structure being designed is exposed, the design example uses a stress limit of $0.75\sqrt{f'_c}$ for the top surface and the maximum value allowed by the code for the bottom surface. This is not a code requirement. Based on code, $1.00\sqrt{f'_c}$ would have been acceptable. The selection of a lower value for the top surface is based on good engineering practice.

B. Based on EC2[12]

EC2 does not specify "limiting" allowable stresses in the strict sense of the word. There are stress thresholds that trigger crack control. These are the same for both bonded and unbonded systems. For computed stressed below the code thresholds, the minimum reinforcement requirement provisions of EC2 suffices.

- For "frequent" load condition

[11] ACI 318-11, Sections 18.3 and 18.4
[12] EN 1992-1-1:2004(E), section 7.2

Concrete:
Compression = $0.60*f_{ck}$ = 0.6*28 = 16.80 MPa
Tension (concrete) $F_t = f_{ct,eff} = f_{ctm}$[13]
$F_t = 0.30*f_{ck}^{(2/3)} = 0.30*28^{(2/3)}$
= 2.77 MPa (Table 3.1, EC2)
Tension (nonstressed steel) = $0.80*f_{yk}$ = 0.8 *460
= 368 MPa
Tension (prestressing steel) = $0.75*f_{pk}$
= 0.75*1860 = 1395 MPa

❖ For "quasi-permanent" load condition
Compression = $0.45*f_{ck}$ =0.45*28 = 12.60 MPa
Tension (concrete) = 2.77 MPa
same as frequent load combination

Unlike ACI-318/IBC, provisions in EC2 permit[14] overriding the allowable hypothetical tension stress in concrete, provided cracking is controlled not to exceed the allowable values.

❖ For "initial" load condition (Table 3.1; EC2)
Tension (Unbonded) = $f_{ct,eff} = f_{ctm}$
$0.30*f_{ci}^{(2/3)} = 0.30*20^{(2/3)}$ = 2.21 Mpa
Compression[15] = $0.60*f_{ci}$ = 0.6*20 = 12 MPa

C. Based on TR-43[16]
Unbonded tendons
For "frequent" load combination
Tension = 1.35 $f_{ctm,fl}$
$f_{ctm,fl}$ = larger of (1.6- h/1000) f_{ctm} or f_{ctm}[17]
= larger of (1.6- 0.760) f_{ctm} or f_{ctm}
= larger of $0.84*f_{ctm}$ or f_{ctm}
$f_{ctm} = 0.30*f_{ck}^{(2/3)}$ (Table 3.1, EC2)
$= 0.30*28^{(2/3)}$ = 2.77 MPa
Allowable tension stress = 1.35* 2.77 = 3.74 MPa

Bonded tendons
For "frequent" load combination
For the members with 0.2 mm allowable crack width, allowable tension stress without bonded reinforcement is:
Tension = 1.65 $f_{ctm,fl}$ = 1.65* 2.77 = 4.57 MPa
For tension (with bonded reinforcement)
Tension = $0.3f_{ck}$ = 0.30*28 = 8.40 MPa
Compression = $0.6*f_{ck}$ = 0.6*28 = 16.80 Mpa
TR-43 specifies allowable concrete compressive stresses for bonded PT systems, but is mute for unbonded systems. In practice, the same values are used for both systems.

For "quasi-permanent" load combination:
Allowable tension stresses are the same as "frequent" load condition.
Compression = $0.45*f_{ck}$ = 0.45*28 = 12.60 MPa

❖ For "initial" load condition[18]
Tension = $0.72\ f_{ctm}$
$f_{ctm} = 0.30*f_{ci}^{(2/3)}$ (Table 3.1, EC2)
$= 0.30*20^{(2/3)}$ = 2.21 MPa
Allowable tension stress = 0.72*2.21 = 1.59 MPa
Compression = $0.50*f_{ci}$ = -10 MPa

4.4 Crack Width Limitation

A. Based on ACI 318-11/IBC 2012
Crack width control and limitation applies when member is designed for the "cracked" regime. No requirements are stipulated, if as in this example, the stresses are kept within the uncracked (U) or transition (T) regime.

B. Based on EC2[19]
In EC2, the allowable crack width depends on whether the post-tensioning system used is "bonded," or "unbonded," and the load combination being considered.
Frequent load condition:

❖ Prestressed members with bonded tendons
0.2 mm; to be checked for frequent load case
❖ Prestressed members with unbonded tendons
0.3 mm; to be checked at quasi-permanent load case

C. Based on TR-43[20]
For all members = 0.2 mm

4.5 Allowable Deflection

A. Based on ACI 318-11/IBC 2012[21]
In all major codes, the allowable deflection is tied to (i) the impact of the vertical displacement on occupants; (ii) the possible damage to installed nonstructural objects such as partitions, glass, or floor covering; and (iii) functional impairment, such as proper drainage. Details of the allowable values, their measurement and evaluation are given in Chapter 4. For perception of displacement by sensitive persons, consensus is limit of L/240, where L is the deflection span. It is important to note that this is the displacement that can be observed by a viewer.

13 EN 1992-1-1:2004(E) , section 7.3.2(4)
14 EN 1992-1-1:2004(E) , section 7.3.2(4)
15 EN 1992-1-1:2004(E) , section 5.10.2.2(5)
16 TR-43 Second Edition, Table 3. For tensile stress, stress limit without bonded reinforcement is considered.
17 EN 1992-1-1:2004(E) , Eqn.3-23
18 TR-43 Second Edition, Section 5.8.2.
19 EN 1992-1-1:2004(E), Table 7.1N
20 TR-43 Second Edition, Section 5.8.3.
21 ACI 318-11, Section 18.3.5

❖ Since in this design example there is no topping on the finished slab, the applicable vertical displacement is the total deflection subsequent to the removal of forms.
❖ The deflection check for potential damage to non-structural elements is not applicable in this case, since the structure is a frame of an open parking structure.
❖ The drainage and ponding of water will be controlled through proper sloping of the floors.
Total allowable deflection: L/240
The frame will be provided with a camber to minimize the impact of deflection.

B. Based on EC2[22]
The interpretation and the magnitude of allowable deflections in EC2 are essentially the same as that of ACI-318. The impact of vertical displacement on the function of the installed members and the visual impact on occupants determine the allowable values. The following are suggested values:

Deflection subsequent to finishing of floors from quasi-permanent combination: L/250
The frame will be provided with a camber to minimize the impact of deflection.

C. Based on TR-43[23]
TR43 refers to EC2 for allowable deflections.

In summary, the allowable deflection from the two codes and the committee report are essentially the same. Conservatively, it can be summarized as follows:

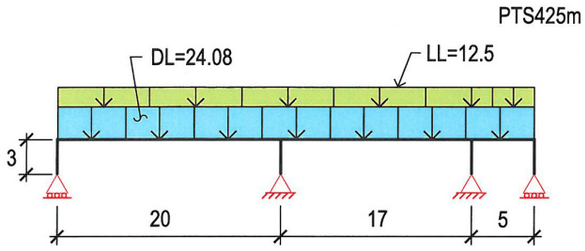
Total deflection from quasi-permanent load combination - L/250
Where, L is the length of the span.

5. ACTIONS DUE TO DEAD AND LIVE LOADING

The structural system of the frame and its dead and live loading are shown in Figs. 5-1 through 5-3.

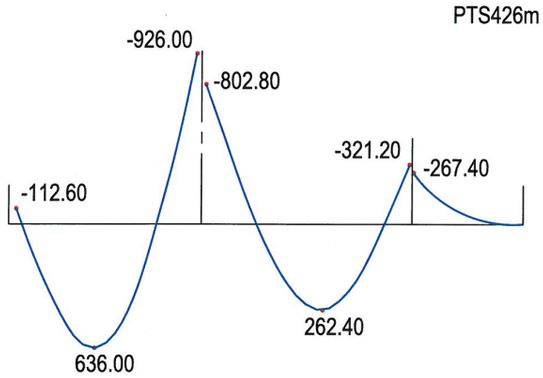
Actions due to dead and live loads are calculated for this example using a generic frame analysis program. The members are assumed prismatic and of uniform cross section throughout the length of each span. Spans 1 and 2 have the same geometry. Centerline to centerline distances are used for span lengths. No allowance is made in the hand calculation for stiffening of members over support. Some software accounts for

[22] EN 1992-1-1:2004(E), Section 7.4.1
[23] TR-43 Second Edition, Section 5.8.4.



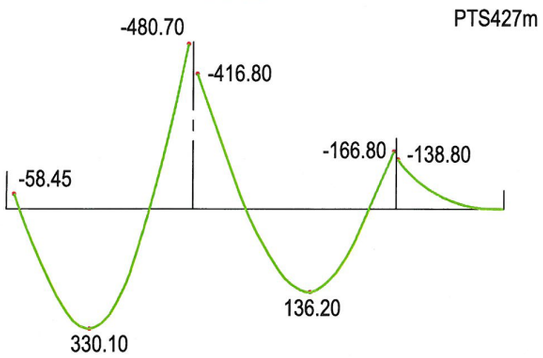
Structural Frame and its Dead and Live Loading (kNm;m)

FIGURE 5-1



Dead Load Moment Distribution (kN-m)

FIGURE 5-2



Live Load Moment Distribution (kNm)

FIGURE 5-3

TABLE 5-1 Moments at Face-of-Supports and Midspans (T132)

	Span 1			Span 2			Span 3		
	Left	Mid	Right	Left	Mid	Right	Left	Mid	Right
M_D (kN-m)	-112.60	636.00	-926.00	-802.80	262.40	-321.20	-267.40	-68.88	5.27
M_L (kN-m)	-58.45	330.10	-480.70	-416.80	136.20	-166.80	-138.80	-35.75	2.73
$M_D + 0.3M_L$ Sustained load (kN-m))	-130.14	735.03	-1070.21	-927.84	303.26	-371.24	-309.04	-79.61	6.09
$M_D + M_L$ Total load (kN-m)	-171.05	966.1	-1406.7	-1219.6	398.6	-488	-406.2	-104.63	8

this stiffening and increase the moment of inertia of the beam over the support region [ADAPT-PT, 2012]. The centerline moments calculated are reduced to the face-of-support using the static equilibrium of each span.

The critical design moments are not generally at midspan. But, for hand calculation, the midspan location is selected. The approximation is acceptable when spans and loads are essentially uniform.

The computed moments from the frame analysis are reduced to the face of each support using statics of respective span. The face-of-support moments and the moments at midspan are summarized in Table 5-1.

6. POST-TENSIONING

6.1 Selection of Design Parameters

Unlike conventionally reinforced members, where given geometry, boundary conditions, material properties and loads result in a unique design, for post-tensioned members in addition to the above a minimum of two other input assumptions are required, before a design can be concluded. A common practice is (i) to assume a level of precompression and (ii) target to balance a percentage of the structure's dead load. In this example, based on experience the level of precompression suggested is larger than the minimum required by ACI-318 code (0.86 MPa). Other major building codes do not specify a minimum precompression. Rather, they specify a minimum reinforcement. Use the following assumption to initiate the calculations.

Minimum average precompression = 1.0 MPa
Maximum average precompression = 2.0 MPa
Target balanced loading =60 % of total dead load

Based on experience for economy of design, a minimum precompression of 1.0 MPa over the entire section is assumed. ACI 318 stipulates a minimum of 0.86 MPa. In the manual calculation, the minimum precompression is used as an entry value (first trial) for design. The stipulation for a maximum precompression does not enter the hand calculation directly. It is stated as a guide for a not-to-exceed upper value. For deflection control the selfweight of the critical span is recommended to be balanced to a minimum of 60% of its selfweight [Aalami, et al, 2003]. Other spans need not be balanced to the same extent. As it will become apparent further in the calculations, for the current beam frame it is beneficial if the tendon exerts a downward force on the third span, as opposed to an upward force in the critical (first) span.

Effective stress in prestressing strand:
For unbonded tendons: f_{se} = 1200 MPa
For bonded tendons: f_{se} = 1100 MPa

The design of a post-tensioned member can be based either on the "effective force", or the "tendon selection" procedure. In the effective force procedure, the average stress in a tendon after all losses is used in design. In this case, the design concludes with the total effective post-tensioning force required at each location. The total force arrived at the conclusion of design is then used to determine the number of strands required, with due allowance for friction and long-term losses. This provides an expeditious and

simple design procedure for hand calculations. In the "tendon selection" procedure, the design is based on the number of strands with due allowance for the immediate and long-term losses. In the following, the "effective force" method is used to initiate the design. Once the design force is determined, it is converted to the number of strands required.

The effective stress assumed in a strand is based on the statistical analysis of common floor slab dimensions for the following conditions (Fig. C6.1-1):
(i) Members have dimensions common in building construction;
(ii) Tendons equal or less than 38 m long stressed at one end. Tendons longer than 38m, but not exceeding 76m are stressed at both ends. Tendons longer than 76m are stressed at intermediate points to limit the unstressed lengths to 38m for one-end stressing or 76m for two-end stressing, whichever be applicable;
(iii) Strands used are the commonly available 13 or 15 mm nominal diameter with industry common friction coefficients as stated in material properties section of this design example; and
(iv) Tendons are stressed to 0.8fpu.

For other conditions, a lower effective stress is assumed, or tendons are stressed at intermediate points. In the current design, the total length of the tendon is 41 m. It is stressed at both ends. Detailed stress loss calculations, not included herein, indicate that the effective tendon stress is 1250 MPa for the unbonded system and also larger than assumed for the grouted system.

6.2 Selection of Post-Tensioning Tendon Force and Profile

The design prestressing force in each span will be chosen to match a whole number of prestressing strands. The following values are used:

1. The effective force along the length of each tendon is assumed to be constant. It is the average of force distribution along a tendon.

Unbonded tendons

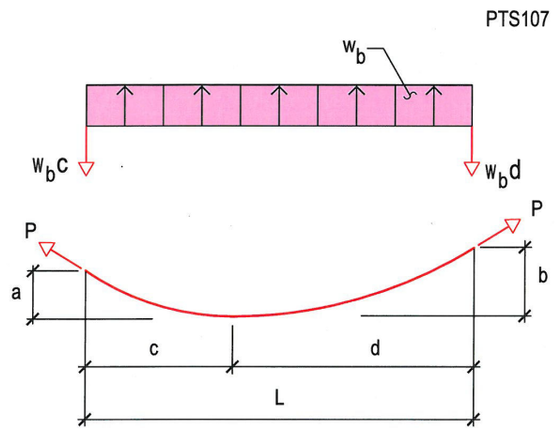
Force per tendon = 1200*99 mm^2/1000
= 118.8 ≈ 119.0 kN/ tendon
Use multiples of 119 kN when selecting the post-tensioning forces for design.

Bonded tendons

Force per tendons = 1100*99 mm^2/1000
= 108.9 ≈ 109.0 kN/ tendon
Use multiples of 109 kN when selecting the post-tensioning forces for design.

2. Tendon profiles are chosen to be simple parabola. These produce a uniform upward force in each span.

For ease of calculation the tendon profile in each span is chosen to be concave upward, simple parabola from centerline to centerline of supports (Fig. C6.2-1). The position of the low point is selected such as to generate a uniform upward force in each span. The relationship given in Fig. C6.2-1 defines the profile. For exterior spans, where the tendon high points are not generally the same, the resulting low point will not be at midspan. For interior spans, where tendon high points are the same, the low point will coincide with midspan. Obviously, the chosen profile is an approximation of the actual tendon profile used in construction. Sharp changes in curvature associated with the simple parabola profile assumed are impractical to achieve on site. The tendon profile at construction is likely to be closer to reversed parabola, for which the distribution of lateral tendon forces will be somewhat different as discussed henceforth. Tendon profiles in construction and the associated tendon forces are closer to the diagrams shown in Fig. C6.2-2.



$c/L = [\sqrt{a/b} \,/\, (1 + \sqrt{a/b})]$

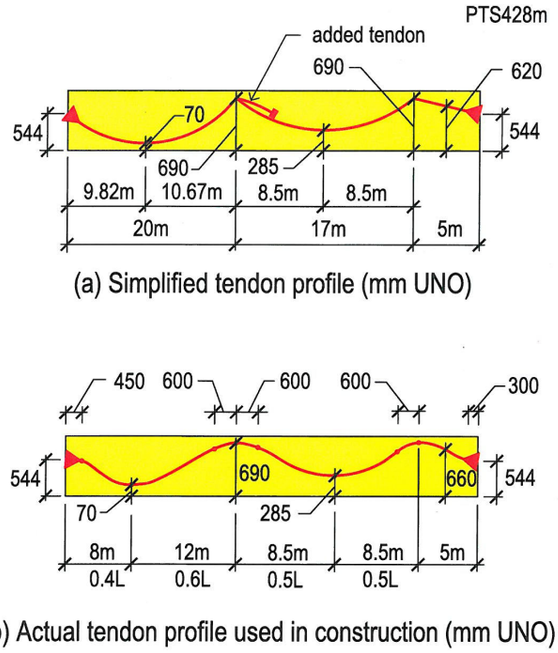
$w_b = 2aP/c^2$

Geometry and Actions of a Parabolic Tendon

FIGURE 6.2-1

6.3 Selection of Number of Strands

Determine the initial selection of number of strands for each span based on the assumed average precompres-



Comparison of Simplified and Actual Tendon Profiles

FIGURE 6.2-2

sion and the associated cross-sectional area of each span's tributary. Then, adjust the number of strands selected, based on the uplift they provide.

Unbonded tendons

1.0N/mm^2*9.171e+5 mm^2/1000 = 917.1 kN
Number of strands = 917.1 kN/119 kN = 7.71
9 strands selected (8 strands would work too)
Force in 9 strands = 9*119 = 1071 kN

Bonded tendons

Number of strands = 917.1 kN/109 kN = 8.4
9 strands selected.

It is noted that the number of strands required to satisfy the same criterion differs between the unbonded and bonded systems. Due to higher friction losses, when using the bonded system, generally more strands are needed to satisfy the in-service condition of design. For brevity, without compromising the process of calculation, in the following the same number of strands is selected for both systems.

6.4 Calculation of Balanced Loads

Balanced loads are the forces that a tendon exerts to its concrete container. It is generally broken down to forces normal to the centerline of the member (causing bending) and axial to it (causing uniform precompression) and added moments at locations of change in centroidal axis. Fig. C6.2-2 shows two examples of balanced loading for members of uniform thickness.

Span 1

The profile of the first span is chosen such that the upward force on the structure due to the tendon is uniform. This is done by choosing the location of the low point so that in each span the profile is a continuous simple parabola (Fig. C6.2-1). Span 1 is the longest span and it is considered the critical span. It is designed with maximum drape, in order to utilize the maximum amount of balanced loading in the most critical span. If the low point of the tendon is not selected at the location determined by "c", two distinct parabolas result. The upward force from a single parabolic profile selected is shown in Fig.C 6.2-1.

Refer to Fig. C6.2-1 and C6.2-2
a = 576 – 70 = 506 mm
b = 690 – 70 = 620 mm
L = 20.0 m
c = $[506/620]^{0.5}/[1 + (506/620)^{0.5}]\}$*20.0
= 9.49 m
W_b = 1071 kN*2*0.506/9.49^2 = 1071*0.01124/m
= 12.04 kN/m
% DL balanced = 12.04/24.08 = 50% < 60% NG
Prorated number of strands = (60%/50%)*9
=10.8 strands; use 12 strands
Force = 12*119 kN =1428 kN
W_b = 1428*0.01124/m = 16.05 kN/m ↑
% DL balanced = 16.05/24.08 = 67% OK
Balanced load reaction, left = 16.05 kN/m*9.49
= 152.31 kN↓
Balanced load reaction, right= 16.05 kN/m*10.51
= 168.69kN↓
Fig. 6.4-1 shows the distribution of balanced loading for span 1.

Span 2
Continuous Tendons

This span is shorter and not the critical span. Therefore, the 9 tendons necessary for the assumed minimum precompression of 1.0 MPa is used. In addition, recognizing that balancing a lower percentage of selfweight will be beneficial to the critical span (span 1); the minimum 60% used as guideline is waived for this span. A smaller percentage for balanced loading is preferred. The dead load in span 2 reduces the design moment of span 1. Balancing more dead load in

span 2 will not be beneficial to the design. Also note that the tendon low point is located at midspan.

W_b = 50%*24.08 kN/m =12.04 kN/m ↑
$a = W_b*L^2/8*P = [(12.04*17^2)/(8*1071)]*1000$
= 405 mm
CGS =690 – 405 = 285 mm
Balanced load reactions = 12.04 kN/m*8.5 m
=102.34 kN ↓ (Left and Right)

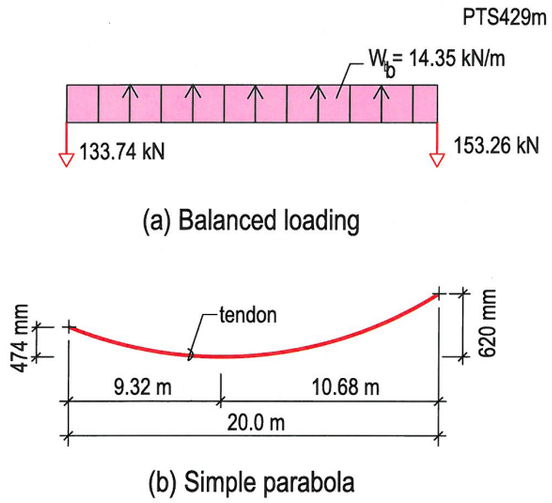
Added Tendons

Reduction of tendons from 12 in span 1 to 9 in span 2 means that 3 tendons from span 1 terminate in span 2. The terminated three tendons are dead-ended in span 2. The dead-end is located at a distance *0.20L* from the left support at the centroid of the beam section. The tails of the terminated tendons are assumed to be in the shape of a half parabola with its apex horizontal over the support and concave downward to the dead end. Hence, the vertical balanced loading of these tendons will be downward, with a concentrated upward force at the dead end (Fig. 6.4-2). The magnitude of the downward vertical force W_b is:

$W_b = (2aP)/c^2$
a = 690 – 576 = 114 mm
c = 0.20*17 = 3.4 m
$W_b = (2aP)/c^2 = (2*0.114*3*119)/3.4^2)$
= 3*119*0.0197 = 7.04 kN/m ↓
Concentrated force at dead end = 7.04*3.4 m
= 23.94 kN ↑

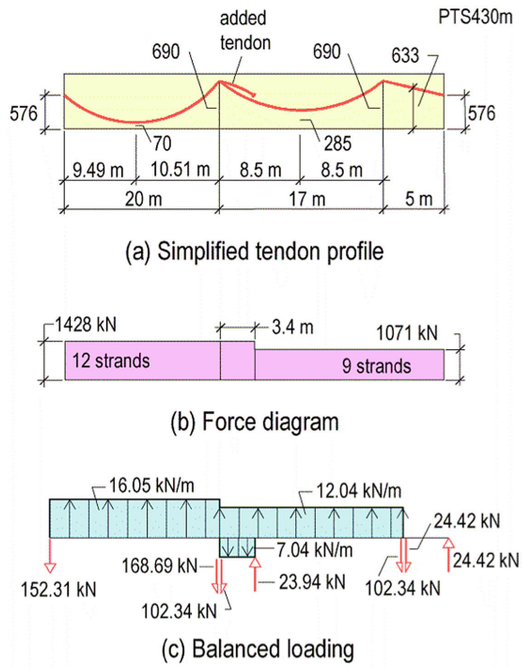
Span 3
The tendon profile in this span is chosen to be straight from the high point at the interior support, to the centroid of the section at the exterior support. The objective is to avoid uplift in the short span. As a matter of fact, for this beam a downward force in the third span would be beneficial to the design of the interior span, since in this span the distribution of dead load moment is all negative.

CGS left = 690 mm
CGS right = 576 mm
CGS center = (690 + 576)/2 = 633 mm
Vertical balanced loading forces are concentrated forces acting at the supports only, they are equal and opposite. Force is calculated using the tangent of the tendon slope for the small angle.
W_b = 1071 kN*(690 – 576)/(5*1000)
= 24.42 kN ↑(right); ↓ (left)



Tendon and Balanced Loading for Span 1

FIGURE 6.4-1



Tendon, Force and Balanced Loading

FIGURE 6.4-2

The complete tendon profile, effective force and balanced loading diagram is shown in Fig. 6.4-2.

Verify the computed balanced loading
(i) Sum of vertical forces must add up to zero:

-152.31 - 168.69 + 16.05 *20 - 102.34 + 23.94 - 7.04 *3.4 + 12.04*17 – 102.34 - 24.42 + 24.42
= 0.004 OK
(ii) Sum of moments of the forces must be zero. Taking moments about the first support gives:
-168.69*20 + 16.05 *20^2/2 – 102.34*20 –7.04 *3.4*(20+3.4/2) + 23.94 *23.4 + 12.04*17*(20+17/2) – 102.34*37 – 24.42*37 + 24.42*42 = 0.92 kN-m OK

The forces exerted by a tendon to its container (beam frame in this case) are always in static equilibrium, regardless of the geometry of tendon and the configuration of the member that contains the tendon. To guarantee a correct solution, it is critical to perform an equilibrium check for the balanced loads calculated (Fig. C6.4-2) before proceeding to the next step. And, that the concentrated forces over the supports are correctly computed and accounted for. In particular, the force due to the short length of the terminated strands in the second span must be included to satisfy equilibrium. If equilibrium is not satisfied, it becomes imperative to ensure that the results err on the conservative side.

6.5 Determination of Actions due to Balanced (Post-Tensioning) Loads

The distributions of post-tensioning moments due to balanced loading are shown in Fig. 6.5-1. These actions are obtained by applying the balanced loads shown in Fig. 6.4-2(c) to the frame shown in Fig. 5-1. The moments shown in the figure are those reduced to the face-of-support. Midspan moments are also shown in the figure.

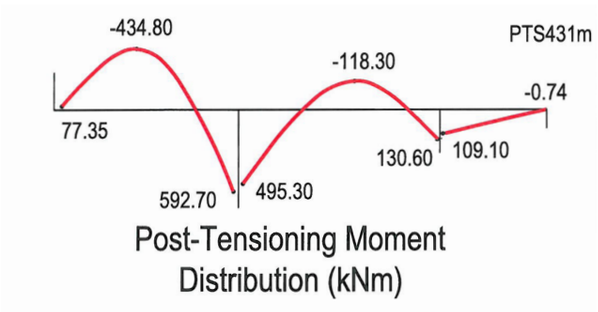


FIGURE 6.5-1

Actions due to post-tensioning are calculated using a standard frame program. The input geometry and boundary conditions to the standard frame program are the same as used for the dead and live loads.

7 - CODE CHECK FOR SERVICEABILITY

7.1 Load Combinations

The following lists the recommended load combinations of the building codes covered for serviceability limit state (SLS).

❖ [ACI, IBC]
Total load condition: 1*DL + 1*LL + 1*PT
Sustained load condition: 1*DL + 0.3*LL + 1*PT[24]

❖ [EC2, TR43]
Frequent load condition: 1*DL + 0.5*LL + 1*PT
Quasi-permanent load condition: 1*DL + 0.3*LL + 1*PT

For serviceability, the actions from the balanced loads from post-tensioning (PT) are used. These are due to "balanced loading." The background for this is explained in detail in reference [Aalami, 1990].

7.2 Stress Check

Critical Locations for Stress Check:

For hand calculation, the critical locations for stress check are selected based on engineering judgment. The selected locations may or may not coincide with the locations of maximum stress levels. This will introduce a certain degree of approximation in design, which reflects the common practice for hand calculations. Computer solutions generally calculate stresses at multiple locations along a span, thus providing greater accuracy.

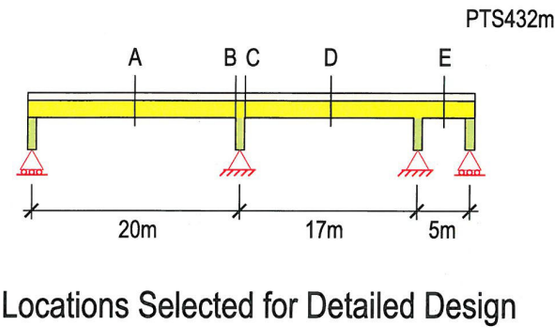


FIGURE 7.2-1

By inspection, locations marked in Fig. 7.2-1 as sections A through E are considered critical for design. These

[24] ACI-318 specifies a "sustained" load case, but does not stipulate the fraction of live load to be considered "sustained." It is left to the judgment of the design engineer to determine the applicable fraction. The fraction selected varies between 0.2 and 0.5. The most commonly used fraction is 0.3, as it is adopted in this design example.

are the midspan locations and the face-of-support locations of the first interior column.

The moment diagrams due to the combined action of dead and live loading (Fig. 5-2 and 5-3) and the moment distribution due to post-tensioning (Fig. 6-5-1) are used to determine the design values at the selected locations.

Stresses:
$\sigma = (M_D + M_L + M_{PT})/S + P/A$
$S = I/Y_c$

Where, M_D, M_L, and M_{PT} are the moments across the entire tributary of the design strip. S is the section modulus of the cross-sectional area reduced through effective width defined for bending action; A is the area of the entire tributary; I is the second moment of area of the portion of the cross-section that is defined by the effective width for bending; and Y_c is the distance of the centroid of the reduced section (defined for bending) to the farthest tension fiber of the section.

Y_T = 248 mm ; Y_B = 512 mm
S_{top} = 3.185e+10/248 = 1.284e+8 mm^3
S_{bot} = 3.185e+10/512 =6.221e+7 mm^3
A = 917100 mm^2
P/A = -1428 *1000/917100 = -1.56 MPa

A. Based on ACI 318-11/IBC 2012
Stress checks are performed for the two load conditions of total load and sustained load. For allowable values see Section 4.3(A).

Point A
❖ Total load combination
$\sigma = (M_D + M_L + M_{PT})/S + P/A$
$M_D + M_L + M_{PT}$ = (636+330.10 – 434.80)
= 531.30 kN-m
Top
σ = -531.30 *1000^2/1.284e+8 – 1.56 = -5.70 MPa Compression < -16.80 MPa OK
Bottom
σ = 531.30 *1000^2/6.221e+7 – 1.56 = 6.98 MPa Tension > 5.29 MPa NG
The stress check is considered acceptable, since it refers to "total" load condition. The member will be considered in "cracked" regime. Deflections have to be calculated using cracked sections

❖ Sustained load combination
$\sigma = (M_D + 0.3M_L + M_{PT})/S + P/A$
$M_D + 0.3\ M_L + M_{PT}$¬ = (636+0.3*330.10 – 434.80)
= 300.23 kN-m
Top
σ = -300.23*1000^2/1.284e+8 – 1.56
= -3.90 MPa Compression < -12.60 MPa OK
Bottom
σ = 300.23 *1000^2/6.221e+7 – 1.56
= 3.27 MPa Tension < 3.28 MPa OK
Since the tensile stress does not exceed the threshold of "Transition," the section is treated as uncracked. Otherwise deflections have to be calculated using cracked sections

B. Based on EC2
Stress checks are performed for the two load conditions of frequent load and quasi-permanent load. The outcome will determine whether crack width needs to be controlled or not. See section on "Allowable Stresses."

Point A
❖ Frequent load condition
$\sigma = (M_D + 0.5\ M_L + M_{PT})/S + P/A$
Stress thresholds:
Compression = -16.80 Mpa ; Tension = 2.77 MPa
$M_D + 0.5M_L + M_{PT}$ = (636 + 0. 5*330.10 – 434.80)
= 366.25 kN-m
Top
σ = -366.25 *1000^2/1.284e+8 – 1.56 = -4.41 MPa Compression < -16.80 MPa OK
Bottom
σ = 366.25 *1000^2/6.221e+7 – 1.56 = 4.32 MPa Tension > 2.77 MPa Control cracking
❖ Quasi-permanent load condition
$\sigma = (M_D + 0.3\ M_L + M_{PT})/S + P/A$
Stress thresholds:
Compression = - 12.60 Mpa; Tension = 2.77 MPa
$M_D + 0.3M_L + M_{PT}$ = (636+0. 3*330.10 – 434.80)
= 300.23 kN-m
Top
σ = -300.23*1000^2/1.284e+8 – 1.56 = -3.90 MPa Compression < -12.60 MPa OK
Bottom
σ = 300.23 *1000^2/6.221e+7 – 1.56
= 3.27 MPa Tension > 2.77 MPa
Hence control cracking[25]

C - Based on TR-43
For stress limits see Section 4.3(C) Design is based on 0.2mm crack width

[25] EN 1992-1-1:2004(E) , Section 7.3.4

TABLE 7.2-1. Sevice Extreme Fiber Stresses at Selected Points (T133)

Load Combination		Point A	Point B	Point C	Point D	Point E
Based on ACI 08/IBC 2009						
Sustained load	f_t (MPa)	-3.90	2.16	1.81	-2.61	-0.86
	f_b (MPa)	3.27	-9.24	-8.51	1.80	-1.62
	F_c (MPa)	-12.60	3.97	3.97	-12.60	-12.60
	F_b (MPa)	5.29	-12.60	-12.60	5.29	-12.60
		OK	OK	OK	OK	OK
Total Load	f_t (MPa)	-5.70	4.78	4.08	-3.35	-0.54
	f_b (MPa)	6.98	-14.64	-13.20	3.34	-2.08
	F_t (MPa)	-16.80	3.97	3.97	-16.80	-16.80
	F_b (MPa)	5.29	-16.80	-16.80	5.29	-16.80
		NG*	NG*	NG*	OK	OK
Based on EC2						
Frequent Load	f_t (MPa)	-4.41	2.91	2.46	-2.82	-0.77
	f_b (MPa)	4.33	-10.78	-9.85	2.24	-1.75
	F_t (MPa)	-16.80	2.77	2.77	-16.80	-16.80
	F_b (MPa)	2.77	-16.80	-16.80	2.77	-16.80
		NG	NG	OK	OK	OK
Quasi-Permanent Load	f_t (MPa)	-3.90	2.16	1.81	-2.61	-0.86
	f_b (MPa)	3.27	-9.24	-8.51	1.80	-1.62
	F_t (MPa)	-12.60	2.77	2.77	-12.60	-12.60
	F_b (MPa)	2.77	-12.60	-12.60	2.77	-12.60
		NG	OK	OK	OK	OK
Based on TR-43						
Frequent Load	f_t (MPa)	-4.41	2.91	2.46	-2.82	-0.77
	f_b (MPa)	4.33	-10.78	-9.85	2.24	-1.75
	F_t – Unbonded	-16.80	3.74	3.74	-16.80	-16.80
	Bonded (MPa)	-16.80	4.57	4.57	-16.80	-16.80
	F_b – Unbonded	3.74	-16.80	-16.80	3.74	-16.80
	Bonded (MPa)	4.57	-16.80	-16.80	4.57	-16.80
		NG	OK	OK	OK	OK
Quasi-Permanent Load	f_t (MPa)	-3.90	NA	NA	-2.61	-0.86
	f_b (MPa)	NA	-9.24	-8.51	NA	-1.62
	(MPa)	-12.60	-12.60	-12.60	-12.60	-12.60
		OK	OK	OK	OK	OK

At point A

❖ Frequent load condition

$\sigma = (M_D + 0.5\ M_L + M_{PT})/S + P/A$

Stress limits

Unbonded tendons

Compression = -16.80 Mpa; Tension = 3.74 MPa

Bonded tendons

Compression = -16.80 Mpa

Tension for 0.2mm crack width; without bonded reinforcement = 4.57 Mpa

0.2mm crack witdth with bonded reinforcement =8.40 MPa

$M_D + 0.5M_L + M_{PT}$

= (636+0. 5*330.10 – 434.80) = 366.25 kN-m

Top

σ = -366.25 *1000^2/1.284e+8 – 1.56 = -4.41 MPa Compression < -16.80 MPa OK

Bottom

σ = 366.25 *1000^2/6.221e+7 – 1.56 = 4.32 MPa Tension > 3.74 MPa for unbonded tendon. Need to add non-prestressed reinforcment and control crack width.

Tension = 4.32 MPa< 4.57 MPa

OK for bonded tendon

❖ Quasi-permanent load condition

$\sigma = (M_D + 0.3\ M_L + M_{PT})/S + P/A$

$M_D + 0.3M_L + M_{PT}$ = (636+0. 3*330.10 – 434.80)

= 300.23 kN-m

Top

σ = -300.23*1000^2/1.284e+8 – 1.56

= -3.90 MPa Compression < -12.60 MPa OK

Since the tensile stresses at one or more locations exceed the threshold for uncracked sections, rebar has to be provided in order to limit the crack width. For ACI-318 code, if the tensile stress exceeds the limit of $1\sqrt{f'c}$, deflections should be calculated using cracked sections.

7.3 Crack Width Control

A. Based on ACI 318-11/IBC 2012

Since the tensile stress exceeds the limit for cracked sections, the section should be treated as cracked and the deflection should be calculated for cracked sections.

B. Based on EC2[26]

The allowable crack width for members reinforced with unbonded tendons (quasi-permanent load combination) is 0.3 mm, and for bonded tendon (frequent load combination) is 0.2 mm. Since in this example the maximum computed tensile stress exceeds the threshold limit, crack width calculation is required based on section 7.3.4 of EC2 code. If the calculated crack width exceeds the threshold, EC2 recommends to limit the bar diameter and bar spacing to the values given in Table 7.2N or 7.3N of EC2 to limit the width of cracks

Using EC2, The crack width calculation for frequent load combination is explained in the following.

Point A

Crack width, $W_k = S_{r,\,max}\ (\varepsilon_{sm} - \varepsilon_{cm})$[27]

$\varepsilon_{sm} - \varepsilon_{cm} = [\sigma_s - k_t\ *(f_{ct,eff}/\rho_{p,eff})(1 + \alpha_e\ \rho_{p,eff})]/E_s$

$\geq 0.6\ \sigma_s/E_s$

Where,

$\alpha_e = E_s/E_{cm}$ = 200000/32308 = 6.19

$\rho_{p,eff} = (A_s + \xi_1^2\ A'_p)/\ A_{c,eff}$

A_s = 0 mm^2

A'_p = area of tendons within $A_{c,eff}$

= 12 *99 = 1188 mm^2

$\xi_1 = \sqrt{(\xi^*\ \varphi_s/\varphi_p)}$

ξ = 0.5 (From Table 6.2)

φ_s = largest diameter of bar =22 mm

φ_p = 1.75* 13 = 23 mm

$\xi_1 = \sqrt{(0.5^*\ 22/23)}$ = 0.70

$A_{c,eff} = h_{c,eff}$*bw

$h_{c,eff}$ = lesser of (2.5*(h-d), (h-x)/3 , (h/2))

= 4.33* 760/(4.33+4.41) = 377 mm

d = 760- 40-22/2 = 709 mm

$h_{c,eff}$ = lesser of (2.5*(760-709), (760-377)/3 , (760/2)) = 128 mm

$A_{c,eff}$ = 128*460 =58880 mm^2

$\rho_{p,eff}$ = (0 + 0.70^2*1188)/ 58880 =0.00989

$\sigma_s = (f/E_c)*E_s$

f = tensile stress due to DL+0.3LL = $(M_D+0.3\ M_L)/S$

= (636+0.5*330.10)*1000^2/6.22*10^7

= 12.88 MPa

σ_s = (12.88/32308)*200000 =79.73 MPa

k_t = 0.4 (coefficient for long-term loading)

$f_{ct,eff} = f_{ctm}$ = 0.3 *$(28)^{(2/3)}$ = 2.77 MPa

$\varepsilon_{sm} - \varepsilon_{cm} = [\sigma_s - k_t\ *(f_{ct,eff}/\rho_{p,eff})(1 + \alpha_e\ \rho_{p,eff})]/Es$

= [79.73– 0.4 *(2.77/0.00989)(1 + 6.19*0.00989)]/200000

= -0. 000196 < 0.6*79.73/200000

= 0.000239 sr,max = 1.3*(h-x) =1.3* 383 = 498 mm

Crack width, W_k = 498*0.000239 = 0.12 mm

< 0.2 mm OK

Provide minimum reinforcement for cracking. It is provided with minimum rebar in 7.3.4.

---

[26] EN 1992-1-1:2004(E) , Section 7.3.3

[27] EN 1992-1-1:2004(E) , Section 7.3.4

Similarly the crack width calculation should be performed at B.

EXAMPLE 1

To illustrate the procedure for crack control by way of addition of reinforcement, as recommended in EC2, as an example let the maximum tensile stress exceed the threshold value by a large margin.

Given: computed hypothetical farthest fiber tensile stress in concrete f = 20MPa

Required: reinforcement design for crack control
Calculate stress in steel at location of maximum concrete stress: σ_s = (f/Ec)*Es
Where f is the hypothetical tensile stress in concrete at crack tip

σ_s = (20/32308)*200000 =123.81 MPa
(this is a hypothetical value)
Crack spacing can be limited by either restricting the bar diameter and/or bar spacing. Use the maximum bar spacing from Table 7.3 N for the σ_s of 123.81 MPa.
From Table, for 160 MPa - 300 mm

Since it is less than the minimum steel stress, use the same spacing as 160 MPa. The maximum spacing for 12381 MPa is 300 mm. Note that based on the magnitude of the computed tensile stress in concrete the area of the required reinforcement for crack control is calculated separately.

C. Based on TR-43[28]

The allowable crack width for all members is 0.2 mm. Since in this example the maximum computed tensile stress at A exceeds the threshold limit for unbonded tendon, crack width calculation is required based on EC2 section 7.3.4. From the crack width calculation performed at A for the EC2 code in *B* of this section, it is found that calculated crack width, 0.12 mm, is less than the allowable width of 0.2mm. If the calculated crack width exceeds the threshold, TR43 recommends either revise the design parameters (slab depth, prestress level etc) or add additional bonded reinforcement and recalculate crack width.

For bonded tendons, if the hypothetical tensile stress exceeds the threshold values, rebar needs to be added to limit the cracking as follows:

Add $0.0025A_t$ rebar in tension zone as close to extreme tension fiber as practical for every 1MPa of stress above the threshold up to the stress of $0.30f_{ck}$. The addition of this rebar for overage of stress is deemed to satisfy the intent of crack control.

Since the maximum computed tensile stresses at the selected points are below the threshold for crack control, added rebar is not required. For completeness, the following example illustrates the procedure, should crack control become necessary.

EXAMPLE

To illustrate the procedure for crack control rebar for bonded tendon, as an example let the maximum computed tensile stresses exceed the threshold value.

Given: Concrete strength: 28 MPa; threshold for crack control 3.5 MPa; computed hypothetical stresses:
Top fiber: f_t = -4.41 MPa compression
Bottom fiber: f_b = 4.33 MPa tension
Maximum allowable tension = $0.3f_{ck}$ = 8.4 MPa

Required: Determine (i) depth of neutral; (ii) area of tension zone; (iii) percentage of rebar to be added using the area of tension zone.
Rebar to be added: A_s = 0.0025*A_t*(4.33-3.5)
Where, A_t = area of tension zone
Depth of neutral axis
x = 4.33* 760/(4.33+4.41) = 377 mm
A_t = 377*460 = 1.734e+05 mm^2
A_s = 0.0025*1.734e+05*(4.33-3.5) = 360 mm^2
No.of Bars = 360 /387 = 0.93 Use 1- 22 mm bars
A_s = 1 *387 = 387 mm^2

7.4 Minimum Reinforcement

There are several reasons why the building codes specify a minimum reinforcement for prestressed members. These are:

Crack control: Bonded reinforcement contributes in reducing the width of local cracks. The contribution of bonded reinforcement to crack control is gauged by the strain it develops under service load. The force developed by bonded reinforcement in resisting cracking depends on the area of steel and its modulus of elasticity.

The area of reinforcement considered available for crack control is $(A_s + A_{ps})$, where A_{ps} is the area of bonded tendons. It is recognized that both bonded and unbonded prestressing provide precompression. While the physical presence of an unbonded

[28] TR-43, Second edition, Section 5.8.1 and 5.8.3

tendon may not contribute to crack control, the contribution through the precompression it provides does. However, for code compliance and conformance with practice, the contribution of unbonded tendons is not included in the aforementioned sum.

Ductility: An underlying reason of ACI-318 requirement of minimum bonded reinforcement for members reinforced with unbonded tendons is to enhance ductility at ULS. Current ACI-318/IBC does not specify a minimum of non-stressed bonded reinforcement for members reinforced with bonded tendons.

Use 22 mm bars (Area = 387 mm^2; Diameter = 22 mm) for top and bottom, where required
d = 760 – 40 - 22/2 = 709 mm

A. Based on ACI 318-11/IBC 2012[29]

❖ Unbonded Tendon
Minimum required, top
$A_s = 0.004*A_{tens}$
A_{tens} is the area of the section between the tension fiber and the section centroid. The minimum rebar is required for members reinforced with unbonded tendons. The added rebar is to reduce the in-service crack width and enhance the ductility of the member for ultimate strength condition. Since the minimum rebar is intended to address the flexural performance of the member, the cross-sectional properties associated with the flexure are used for the determination of its area.

Top bars at supports 1, 2 and 3
$A_s = 0.004*[125*2460 + (248 – 125)*460]$
$= 1457$ mm^2
Number of Bars = 1457/387 = 3.76;
Use 4 - 22 mm bars; A_s= 4 *387 = 1548 mm^2 OK
Top bar at support 4
$A_s = 0.004*[125*1250 + (310 – 125)*460]$
$= 966$ mm^2
Number of Bars = 966/387 = 2.49;
Use 3 - 22 mm bars; A_s = 3 *387 =1161 mm^2 OK
Minimum required at bottom for spans 1 and 2:
$A_s = 0.004*A_{Tens}$= 0.004*(460*512) = 942mm^2
Number of Bars = 942/387 =2.43;
Use 3-22 mm bars; A_s = 3*387 = 1161 mm^2 OK
Minimum required at bottom for span 3
$A_s = 0.004*(460*450) = 828$ mm^2
Number of Bars = 828/387 = 2.14;
Use 3-22 mm bars; A_s = 3*387 = 1161 mm^2

Since at midspan, the tension at service condition is at the top fiber, the minimum reinforcement calculated for the top will be used. In this case, 3-22mm will be adequate. Hence
A_s = 1161 mm^2; use 3-22 mm bars at top of midspan

❖ Bonded (grouted) tendons
There is no requirement for minimum reinforcement based on either geometry of the design strip, nor its hypothetical tensile stresses. The minimum requirement is handled through the relationship between the cracking moment of a section and its nominal strength in bending. This is handled in the "strength" check of the member (section 8 of this example). The code check for strength adequacy after the initiation of first crack is handled in the strength design (ULS).

B. Based on EC2[30]

EC2 specifies the same requirement for minimum reinforcement at supports and spans, and also for both unbonded and bonded tendons. Two checks apply. One is based on the cross-sectional geometry of the design strip and its material properties and the other on computed stresses. In the former, the minimum reinforcement applies to the combined contributions of prestressed and non-prestressed reinforcement. Hence, the participation of each is based according to the strength it provides, the prestressing steel is accounted for with higher values. The reinforcement requirement for crack control is handled separately.

❖ Unbonded and bonded tendons
Spans
$A_{smin} \geq (0.26* f_{ctm} *b_t*d/f_{yk}) \geq 0.0013* b_t *d$
b_t = 460 mm
d = 760- 40-22/2 = 709 mm
$f_{ctm} = 0.3 *28^{(2/3)} = 2.77$ MPa
(i) $A_s = 0.26* f_{ctm}*b_t*d/f_{yk}$
$= 0.26*2.77* 460*709/460 = 511$ mm^2
(ii) $A_s = 0.0013*b_t*d = 0.0013 *460*709$
$= 424$ mm^2
Therefore, A_s = 511 mm^2
Contribution of reinforcement from bonded prestressing
Point A
$A_{ps} *(f_{pk}/f_{yk}) = 12* 99*1860/460$
$= 4804$ mm^2 > 511mm^2
Points D & E:,
$A_{ps} *(f_{pk}/f_{yk}) = 9* 99*1860/460$
$= 3603$ mm^2 > 511mm^2
Hence, no additional bonded reinforcement is required.

29 ACI 318-11, Section 18.9

30 EN 1992-1-1:2004(E) , Section 9.2.1 and 7.3.2

Supports
$A_{smin} \geq (0.26^* f_{ctm} {}^*b_t{}^*d/f_{yk}) \geq 0.0013^* b_t {}^*d$
b_t = mean width of the tension zone
depth of tension zone, (h-c) (Refer Fig.7.4-1)

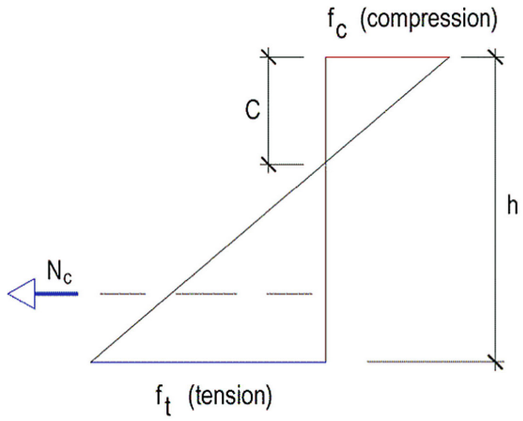


FIGURE 7.4-1 Distribution of Stress over Section

= 2.91*760/ (2.91+10.78)
= 162 mm (Consider point B)
b_t = [2460*125+ 460*(162-125)]/162 = 2003 mm
d = 760- 40-22/2 = 709 mm
(i) $A_s = 0.26^* f_{ctm}{}^*b_t{}^*d/f_{yk}$
$= 0.26^*2.77^* 2003^*709/460 = 2223\ mm^2$
(ii) $A_s = 0.0013^*b_t{}^*d = 0.0013\ ^*2003^*709$
$= 1846\ mm^2$ Therefore, $A_s = 2223\ mm^2$
Contribution of reinforcement from bonded Prestressing:
$A_{ps}\ ^*(f_{pk}/f_{yk}) = 12^*\ 99^*1860/460$
$= 4804\ mm^2 > 2223 mm^2$
Hence, no additional bonded reinforcement is required.

❖ Minimum reinforcement for crack control

In EC2 necessity of reinforcement for crack control is triggered, where computed tensile stresses exceed a code-specified threshold.

Since the hypothetical tensile stress of concrete exceeds the threshold for crack control at point A, cracking reinforcement need to be provided.

At point A
$A_{smin} = k_c\ k\ f_{ct,eff}\ A_{ct} / \sigma_s$
A_{ct} is the area of the concrete section in tension zone.
f_t = -4.41 Mpa (compression at top)
f_b = 4.33 MPa (tension at bottom)
$\sigma_s = f_{yk}$ = 460 MPa
$f_{ct,eff} = f_{ctm} = 0.3\ ^*(28)^{(2/3)} = 2.77$ MPa
k = 0.678 (interpolated for h=760 mm)
(h – c) = 4.33*760/ (4.41+4.33) = 377 mm
$A_{ct} = 377^*460 = 173420\ mm^2$
$k_c = 0.4^*\ [1-(\ \sigma_c /(\ k_1\ (h/h^*)\ f_{ct,eff})]$
$\sigma_c = N_{ED} /bh = 1.56$ MPa
h* = 760 mm
$k_1 = 1.5$
$k_c = 0.4^*\ [1-(\ 1.56 /(\ 1.5\ (760/760^*)\ 2.77)] = 0.25$
$A_{smin} = 0.25^*\ 0.678^*\ 2.77^*\ 173420 /460 = 177\ mm^2$
Provide one 22mm bar ($A_{s,prov} = 1^*\ 387 = 387\ mm^2$)
$A_{smin,crack} = 387\ mm^2$

At point B
$A_{smin} = k_c\ k\ f_{ct,eff}\ A_{ct} / \sigma_s$
A_{ct} is the area of the concrete section in tension zone.
f_t = 2.91 Mpa (tension at top, refer to Fig. 7.4-1)
f_b = -10.78 MPa (compression at bottom)

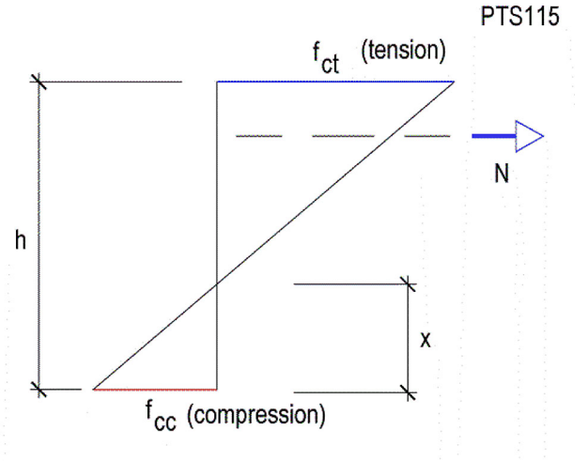


FIGURE 7.4-2 Distribution of Stress over Section

$\sigma_s = f_{yk}$ = 460 MPa
$f_{ct,eff} = f_{ctm} = 0.3\ ^*(28)^{(2/3)} = 2.77$ MPa
k = 0.678 (interpolated for h=760 mm)
Distance of neutral axis from top
= 2.91*760/ (2.91+10.78) = 162 mm
$A_{ct} = 162^*2460 = 398520\ mm^2$
$k = 0.4^*\ [1-(\ \sigma_c /(\ k_1\ (h/h^*)\ f_{ct,eff})]$
$\sigma_c = N_{ED} /bh = 1.56$ MPa
h* = 760 mm
$k_1 = 1.5$
$k_c = 0.4^*\ [1-(\ 1.56 /(\ 1.5\ (760/760^*)\ 2.77)] = 0.25$
$A_{smin} = 0.25^*\ 0.678^*\ 2.77^*\ 398520 /460 = 407\ mm^2$
Provide 2-22mm bar ($A_{s,prov} = 2^*\ 387 = 774\ mm^2$)
$A_{smin,crack} = 774\ mm^2$

C. Based on TR-43

❖ Unbonded Tendon

(i) Flexural un-tensioned reinforcement[31]

[31] TR-43 2nd Edition, Section 5.8.7

TABLE 7.4-2 Summary of Minimum Rebar (mm^2) (T134SI)

Code	Unbonded				Bonded			
	Point A	Point B & C	Point D	Point E	Point A	Point B & C	Point D	Point E
ACI/IBC	942	1457	942	966	0	0	0	0
EC2	177	407	0	0	177	407	0	0
TR43	1303	2017	603	0	0	0	0	0

TR-43 stipulates that a minimum amount of non-prestressed bonded reinforcement be present at all locations for the full tension force generated by the computed tensile stresses in the concrete under service load combination.

At point A:(Frequent load combination)
Depth of tension zone: $h-x = -f_{ct}*h/(f_{cc}-f_{ct})$
Refer to Fig. 7.4-2 where f_{cc} is the concrete fiber stress in compression; f_{ct} is the extreme concrete fiber stress in tension.
f_{ct} = tensile stress (-ve) = -4.33 MPa
f_{cc} = compressive stress = 4.41 MPa
h = depth of the section = 760 mm
b = width of the section = 460 mm
x = depth of the compression zone
h-x = -(-4.33)*760/(4.41+4.33) = 376 mm
$A_s = F_t /(5*f_{yk} /8)$
Where F_t is the total tensile force over the tensile zone of the entire section
$F_t = -f_{ct}*b*(h-x)/2$ = -(-4.33)*460 *376/(2*1000)
= 374.46 kN
A_s = 374.46*1000/(5*460/8) =1303 mm^2
Provide 4- 22mm bar ($A_{s,prov}$ = 4* 387 =1548 mm^2)

At points B and C
Depth of tension zone: $h-x= -f_{ct}*h/(f_{cc}-f_{ct})$
Refer to Fig. 7.4-2 where f_{cc} is the concrete fiber stress in compression; f_{ct} is the extreme concrete fiber stress in tension.
f_{ct} = tensile stress (-ve) = -2.91
f_{cc} = compressive stress= 10.78 MPa
h = depth of the section = 760 mm
b = width of the section = 2460 mm
x = depth of the compression zone
h-x= -(-2.91)*760/(2.91+10.78) = 162 mm
$A_s = F_t /(5*f_{yk} /8)$
Where F_t is the total tensile force over the tensile zone of the entire section
$F_t = -f_{ct}*b*(h-x)/2$ = -(-2.91)*2460 *162/(2*1000) = 579.85 kN
A_s = 579.85*1000/(5*460/8) =2017 mm^2
Provide 6- 22mm bar ($A_{s,prov}$ = 6* 387 =2322 mm^2)

At point D
f_{ct} = tensile stress (-ve) = -2.24 MPa f_{cc}
= compressive stress=2.82 MPa
h = 760 mm ; b =460 mm
h-x = -(-2.24)*760/(2.24+2.82) = 336 mm
$A_s = F_t /(5*f_{yk} /8)$
$F_t = -f_{ct}*b*(h-x)/2$ = -(-2.24)*460 *336/(2*1000)
= 173.11 kN
A_s = 173.11 *1000/(5*460/8) =603 mm^2

Provide 2- 22mm bar (As,prov = 2* 387 =774 mm^2)

❖ Other minimum reinforcement for unbonded tendons.

In addition to the preceding that was based on the value of hypothetical tensile stress, TR-43 requires that the bonded reinforcement at each section not to be less that that specified in EC2[32]. The computation of the minimum rebar based on EC2 is carried out in the above section under EC2.

The second check for minimum bonded reinforcement is the same as carried out for EC2 code in the preceding section.

❖ Bonded Tendon

There are no minimum rebar requirements for one-way spanning members reinforced with bonded tendons. Any additional SLS reinforcement will be related to the design crack width, and the potential of the cracks exceeding the design value. This check was performed in Section 7.3(B).

The minimum area of rebar required using the codes covered is summarized in Table 7.4-1

[32] TR-43 2nd Edition, Section 5.8.8

7.5 Deflection Check

Recognizing that (i) the accurate determination of probable deflection is complex [see Chapter 4, Section 4.10.6]; and (ii) once a value is determined, the judgment on its adequacy at design time is subjective, and depends on unknown, yet important, parameters such as age of concrete at time of installation of nonstructural members that are likely to be damaged from large displacement. For hand calculation deflection checks are generally based on simplified procedures. A rigorous analysis is initiated, only where the parameters of design and applied loads are more reliably known. In most cases, post-tensioned members are sized according to recommended span/depth ratios proven to perform well in deflection.[33]

The simplified procedure includes:
(i) For visual and functional effects, total long-term deflection from the day the supports are removed not to exceed span/250 for EC2 or span/240 ACI-318. Camber can be used to offset the impact of displacement.
(ii) Immediate deflection under design live load not to exceed span/500 for EC2 or span/480 for ACI-318. Both ACI 318/IBC and EC2, tie the deflection adequacy to displacement subsequent to the installation of members that are likely to be damaged. This requires knowledge of construction schedule and release of structure for service. In the following the common design practice is followed.

For assessment of long-term displacement in the context of foregoing, ACI-318 recommends a multiplier factor of 2[34].

The deflections are calculated using a frame analysis program for each of the load cases: dead, live, and post-tensioning. Gross cross-sectional area and linear elastic relationships are used. Since the stress level for which the design was carried out falls essentially in the transition regime, the elastically calculated deflections must be adjusted to allow for cracking at locations where cracking stresses are exceed the threshold of "uncracked" regime. Strictly speaking, a cracked deflection calculation has to be performed,[35] where stresses exceed the "transition" regime. However, for hand calculation, recognizing that the locations of probable cracks, as in this example are few, the option of "magnifying" elastic deformation by a factor that allows for cracking is used.

The critical location is in span 1. The values for span 1 from the frame analysis are:

Span 1 Deflection	
Dead Load	27.8 mm
Post-Tensioning	-19.2 mm
Dead Load + PT	8.6 mm
Live load deflection	14.5 mm

The maximum stress under total loading at midspan is 6.98 MPa. Since this is greater than 0.62 $\sqrt{f'_c}$, adjustment to the calculated deflection is required.

There are several options available to adjust elastically calculated deflection values, if the computed tensile stresses exceed cracking. Among the most commonly used are: (i) substitution of the gross moment of inertia (I_g), by an equivalent moment of inertia (I_e), followed by the magnification of the elastically calculated deflection by the ratio of (I_g/I_e); and (ii) use of a bilinear deflection calculation, in which the amount of deflection prior to cracking is calculated using Ig and the elastic solution, the deflection after the initiation of crack is calculated for the overage of load, using the cracked moment of inertia (I_{cr}).

For prestressed sections the equivalent moment of inertia is calculated using the following relationship [PTI design manual, 1990].

$I_e = [1 - 0.30*(f_{max} - 0.5\sqrt{f'_c})/ 0.5\sqrt{f'_c}]*I_g$
f'_c is in MPa

Where, I_e is the effective moment of inertia; I_g is the moment of inertia based on the gross cross-sectional area. Initially, the relationship was proposed for f_{max} not exceeding $\sqrt{f'_c}$. But, it is now used for values above $\sqrt{f'_c}$.

The calculated maximum tensile stress
f_{max} = 6.98 MPa.
Reduction in moment of inertia due to cracking:
$I_e = [1 - 0.30*(f_{max} - 0.5\sqrt{f'_c})/0.5\sqrt{f'_c}]*I_g$
$= [1 - 0.30*(6.98-2.65)/2.65] *I_g$
$= 0.51*I_g$
Hence deflection due to dead load and PT
= 8.6/0.51 = 16.86 mm
Live load deflection with cracking allowance
= 14.5/0.51 = 28.43 mm

33 TR-43 2nd Edition, Section 5.8.4; ADAPT-TN292
34 ACI multiplier 2
35 Compter programs, such as ADAPT Floor do cracked deflection calculation

❖ Long-term deflection

Multiplier factor assumed for effects of creep and shrinkage on long-term deflection = 2[36]

Load combination for long-term deflection, using a factor of 0.3 for sustained "quasi-permanent" live load :

(1.0*DL + 1.0*PT + 0.3*LL)*(1 + 2)

Long-term deflection: (1 + 2)*(16.86 + 0.3*28.43) = 76.17 mm

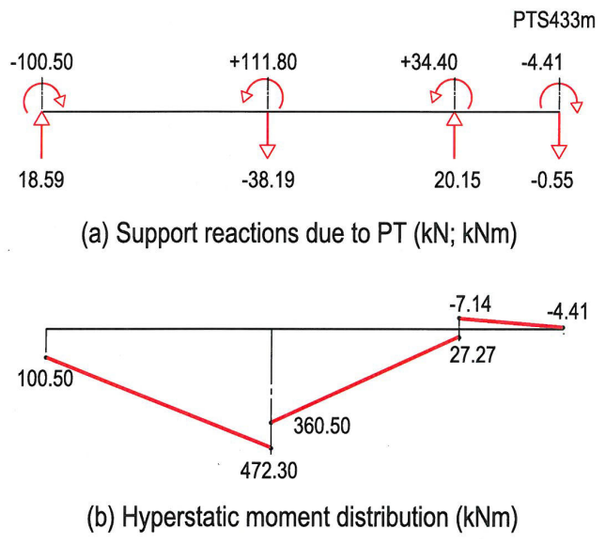
Deflection ratio = 76.17/(20000) = 1/263 < 1/250 OK

Instantaneous deflection due to design live load

Live load deflection = 28.43 mm.

Deflection ratio = 28.43/(20,000) = 1/703 OK

Deflection does not generally govern the design for members dimensioned within the limits of the recommended values in ACI 318 and balanced with post-tensioning tendons within the recommended range [Aalami et al, 2003], and when subject to loading common in building construction. For such cases, deflections are almost always within the permissible code values, when design is performed within U or T stress values.



Hyperstatic (Secondary) Actions

FIGURE 8.2-1

8. CODE CHECK FOR STRENGTH

8.1 Load Combinations

(i) ACI-318/IBC

1.2*DL + 1.6*LL + 1*HYP

1.4*DL + 1*HYP

(ii) EC2

1.35*DL + 1.5*LL + 1*Hyp

(iii) TR43

1.35*DL + 1.5*LL + 0.9*Hyp

For strength combination, the hyperstatic (Hyp) actions (secondary) due to prestressing are used. The background for this is explained in detail in Chapter 4, Section 4.11.2.

8.2 Determination of Hyperstatic Actions

The hyperstatic moments are calculated from the reactions of the frame analysis under balanced loads from prestressing (Loads shown in Fig. 6.4-2). The reactions obtained from a standard frame analysis are shown in Fig. 8.2-1(a). The reactions shown produce hyperstatic moments in the frame as shown in Fig. 8.2-1(b).

The hyperstatic (secondary) reactions must be in self-equilibrium, since the applied loads (balanced loads) are in self-equilibrium.

Check the validity of the solution for static equilibrium of the hyperstatic actions using the reactions shown in Fig. 8.2-1a.

ΣVertical Forces

= 18.59 – 38.19 +20.15 – 0.545 =0.005 OK

ΣMoments about Support 1

= -100.50+ 111.80+34.40 – 4.41 –(38.19*20) + (20.15*37) – (0.55*42) = -0.06 ≈ 0 OK

Support reactions due to post-tensioning are applied to the beam in order to construct the hyperstatic moment diagram shown 8.2-1(b). The support reactions are shown in part (a) of the figure.

Reduce hyperstatic moments to face-of-support using linear interpolation.

For right face of support (FOS) of span 1:

M_{HYP} = 472.30 – [(472.30 – 100.50)/20]*0.45/2

= 468.12 kN-m

A number of engineers use the expression given below to compute hyperstatic (secondary) moments due to prestressing. This expression gives acceptable results for articulated members, only if the balanced loads used in the determination of post-tensioning moments (M_{pt}) satisfy equilibrium.

$$M_{hyp} = M_{pt} - P\,e$$

[36] ACI- 318 multiplier factor

TABLE 8.3-1 Ultimate Design Moments (T135)

	Point A	Point B	Point C	Point D	Point E
M_D (kN-m)	636.00	-926.00	-802.80	262.40	-68.88
M_L (kN-m)	330.10	-480.70	-416.80	136.20	-35.75
M_{HYP} (k-ft)	286.40	468.10	356.10	193.90	-5.77
ACI 318-08/IBC 2009 : 1.2DL+1.6LL+1Hyp					
M_U (k-ft)	1577.76	-1412.22	-1274.14	726.70	-145.63
EC2 : 1.35DL+1.5LL+1Hyp					
M_U (k-ft)	1640.15	-1503.05	-1352.88	752.44	-152.38
TR 43 : 1.35DL+1.5LL+0.9Hyp					
M_U (k-ft)	1611.51	-1549.86	-1388.49	733.05	-151.81

Where M_{hyp} is the secondary moment, P is the post-tensioning force, and e is the eccentricity of the post-tensioning. The expression does not afford a validity check, such as the static equilibrium of all hyperstatic actions.

8.3 Calculation of Design Moments

The design moment (M_u) is the factored combination of dead, live and hyperstatic moments.

❖ Using ACI/IBC

Design moments are:

$M_{U1} = 1.2*M_D + 1.6*M_L + 1.0*M_{HYP}$

$M_{U2} = 1.4*M_D + 1.0*M_{HYP}$

The second combination governs, where dead loads are eight times or larger than live loads. This is rare. The moments shown in Fig. 8.2-1 are centerline moments. These are reduced to the face-of-support in Table 8.3-1.

By inspection, the second load combination does not govern, and will not be considered in the following.

The factored moment computed for EC2 and TR43 are listed in the following table.

8.4 Strength Design for Bending and Ductility

The strength design for bending consists of two provisions, namely

❖ The design capacity ($\Phi*M_n$; R) shall exceed the demand. A combination of prestressing and non-prestressed steel provides the design capacity

❖ The ductility of the section in bending shall not be less than the limit set in the associated building code. The required ductility is deemed satisfied, if failure of a section in bending is initiated in post-elastic response of its tension reinforcement, as opposed to crushing of concrete. For the codes covered in this example this is achieved through the limitation imposed on the depth of the compression zone (see Fig. C-8.4-1 in Chapter 6). The depth of compression zone is generally limited to 50% or less than the distance from the compression fiber to the farthest reinforcement (dr). Since the concrete strain (ϵ_c) at crushing is assumed between 0.003 and 0.0035, the increase in steel strain (ϵ_s) will at minimum be equal to that of concrete at the compression fiber. This will ensure extension of steel beyond its yield point (proof stress) and hence a ductile response.

For expeditious hand calculation, the flexural capacity of a post-tensioned member in common building structures can be approximated by assuming a conservative maximum stress for prestressing tendons. For detailed application of the code-proposed formulas refer to Chapter 12 for the calculation of a section's capacity both on the basis of strain compatibility and approximate code formulas. For daily hand calculation in the a consulting office, unless a software is used use the following simple procedure.

There are two justifications, why the simplified method for ULS design of post-tensioned sections in daily design work are recommended. These are:

(i) Unlike conventionally reinforced concrete, where at each section along a member non-prestressed reinforcement must be provided to resist the design moment, in prestressed members this may not be necessary. Prestressed members possess a base capacity along the entire length of prestressing tendons (Fig. C-8.4-2b in Chapter 6). Non-prestressed reinforcement is needed only at sections, where the moment demand exceeds the base capacity of the section.

(ii) In conventionally reinforced concrete, the stress used for rebar at ULS is well-defined in major building codes. For prestressed sections, however, the

TABLE 8.4-1 Summary of Reinforcement for Strength Limit State (T136SI)

Code	Unbonded				Bonded			
	Point A	Point B & C	Point D	Point E	Point A	Point B & C	Point D	Point E
ACI/IBC	2368	2631	1161	0	2463	2625	926	0
EC2	2915	3399(top) 1180(bot)	1134	0	2915	3399(top) 1180(bot)	1134	0
TR43	2703	3821(top) 1602(bot)	986	0	2836	3708(top) 1489(bot)	1134	0

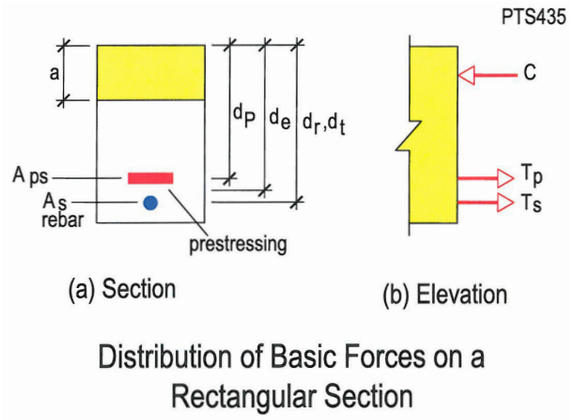


FIGURE C8.4-3

TABLE 8.4-2 Cracking Moment Values and the Respective Data (T138SI)

Basic parameters and analysis		Section A	Section B&C	Section D
	S_{top} (mm³)		1.28e+8	
	S_{bot} (mm³)	6.22e+7		6.22e+7
	P (kN)	1428	1428	1071
	P / A (MPa)	-1.56	-1.56	-1.17
	fr + (P/A)	4.87	4.87	4.48
	Mcr (KNm)	302.91	623.36	278.66
	1.2 Mcr (kNm)	363.50	748.03	334.39
	Φ Mn (kNm)	1234.99	1234.99	637.63
	Status	OK	OK	OK

stress in tendon at ULS is oftentimes expressed in terms of an involved relationship—hence the tendency to use a simplified, but conservative scheme for everyday hand calculation. For repetitive work, computer programs are recommended.

Using strain compatibility procedure [37] the required reinforcement for each of the three codes are calculated. The outcome is listed in Table 8.4-1.

Using strain compatibility procedure[38] the required reinforcement for each of the three codes are calculated. The outcome is as follows:

❖ **Cracking moment larger than moment capacity:** Where cracking moment of a section is likely to exceed its nominal capacity in flexure, reinforcement is added to raise the moment capacity. In such cases, the contribution of each reinforcement is based on the strength it provides. If the minimum value is expressed in terms of cross-sectional area of reinforcement, the applicable value for this requirement is: $(A_s + A_{ps} * f_{py}/f_y)$.

[37] ADAPT-TN178
[38] ADAPT-TN178

A. Based on ACI 318-11/IBC 2012

❖ Bonded (grouted) tendons

ACI 318 [39] /IBC stipulates that beams and one-way slabs reinforced with bonded tendons develop a nominal moment capacity at ULS not less than 1.2 times their cracking moment M_{cr}.

The necessity and amount of rebar is defined as a function of cracking moment of a section (M_{cr}). For pre ACI-318 Section 9.5.2.3 stressed members

$M_{cr} = (f_r + P/A)*S$

Where, fr is the modulus of rupture defined [40]

$f_r = 0.625\sqrt{f'c} = 0.625\sqrt{28} = 3.31$ MPa

P/A is the average precompression, and S the section modulus. The Table 8.4-2 summarizes the leading values and the outcome. The computation of the cracking moment and the nominal capacity are given in the following table.

Since at the selected sections, the design capacity of the section with prestressing alone exceeds $1.2*M_{cr}$, no additional rebar is required from this provision.

[39] ACI 318-11 Section 18.8.2
[40] ACI-318 Section 9.5.2.3

In design situations like above, where the design is initiated by determination of whether a value is less or more than a target, it is advisable to start the check using a simplified, but conservative procedure. If the computed value is closer to the target than the approximations in the simplified method, design check can be followed with a more rigorous computation.

Assume the following for simplified calculation:
Strand CGS = 70 mm
hence $d = h$ (thickness) - 70
Moment arm = 0.9d
Design force in strand = A_{ps}*1860 MPa ; Φ = 0.9
At midspan, with 12 strands
$\Phi * M_n$ = 0.9* 12*99 *1860*0.9 *(760 – 70)/10^6
= 1234.99 kNm

Design moment at other locations are calculated in a similar manner.

B. Based on EC2

❖ Unbonded tendons

EC2 [41] requires that for beams reinforced with unbonded tendons the total amount of prestressed and nonprestressed shall be adequate to develop a factored load at least 1.15 times the cracking load computed on the basis of the modulus of rupture of the section. In practice, this is taken as cracking moment of the section M_{cr}.

The necessity and amount of rebar is defined as a function of cracking moment of a section (M_{cr}). For prestressed members
$M_{cr} = (f_r + P/A)*S$
Where, fr is the modulus of rupture[42]
$f_r = f_{ctm} = 0.3f_{ck}^{(2/3)} = 2.77$ MPa
P/A is the average precompression, and S the section modulus. The Table 8.4-3 summarizes the leading values and the outcome.

Since at the selected sections, the design capacity of the section with prestressing alone exceeds 1.15*M_{cr}, no additional rebar is required from this provision.

In design situations like above, where the design is initiated by determination of whether a value is less or more than a target, it is advisable to start the check using a simplified, but conservative procedure. If the computed value is close to the target, design check can be followed with a more rigorous computation.

TABLE 8.4-3 Cracking Moment Values and the Respective Data for EC2

Basic parameters and analysis		Section A	Section B&C	Section D
	S_{top} (mm³)		1.28e+8	
	S_{bot} (mm³)	6.22e+7		6.22e+7
	P (kN)	1428	1428	1071
	P / A (MPa)	-1.56	-1.56	-1.17
	fr + (P/A)	4.33	4.33	3.94
	Mcr (KNm)	269.33	554.24	245.07
	1.15 Mcr (kNm)	309.73	637.38	281.83
	Φ Mn (kNm)	1193.23	1193.23	616.07
	Status	OK	OK	OK

Assume the following:
Cover to strand CGS = 70 mm;
hence $d = h$ (thickness) - 70
Moment arm = 0.9d
Design force in strand = A_{ps}*1860 MPa/1.15;
At midspan, with 12 strands, 1860 MPa strength
$\Phi * M_n$ = 12*99 *(1860/1.15)*0.9 *(760 – 70)/10^6 = 1193.23 kNm

Design moment at other locations are calculated in a similar manner.

Strength computations performed herein were limited to points considered critical by inspection. When spans and loading are not regular, the selection of critical points by inspection becomes difficult. In such cases, stress and strength checks must be performed at a greater number of locations. Also, note that due to the contribution of tendon to ultimate strength, and change in drape of tendon along the length of a member, the most critical location for design is not necessarily the location of maximum moment.

The envelope of total reinforcement is given in Table 8.4-4.

8.5 One-Way Shear Design

The shear design for the right support of span 1 will be followed in detail, since this is the most critical location. The procedure for the shear design of other locations is identical.

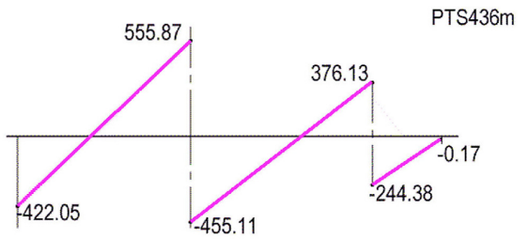
A. Based on ACI 318-11/IBC 2012

Distribution of design shear is shown in Fig. 8.5-1. The

[41] EN 1992-1-1:2004 (E), Section 9.2.1.1(4)
[42] EN 1992-1-1:2004 (E), Section 7.1(3). Here tensile stress limit for uncracked section is used.

TABLE 8.4-4 Envelope of Reinforcement for Serviceability (SLS) and Strength Conditions (ULS) (T137SI)

Code	Unbonded				Bonded			
	Point A	Point B & C	Point D	Point E	Point A	Point B & C	Point D	Point E
ACI/IBC	2368	2631	1161	966	2463	2625	926	0
EC2	2915	3399(top) 1180(bot)	1134	0	2915	3399(top) 1180(bot)	1134	0
TR43	2703	3821(top) 1602(bot)	986	0	2836	3708(top) 1489(bot)	1134	0



Distribution of Shear (kN)

FIGURE 8.5-1

design shear (V_u) is computed from the results of the standard frame analysis performed for the loading conditions D, L and PT. The following combination was used:

The design starts with the calculation of vc, the code allowable shear stress contribution of concrete over the shear area of the section

$V_u = 1.2*V_D + 1.6*V_L + 1.0*V_{HYP}$
Span 1
b_w = 460 mm
d = 760 –40-22/2 =709 mm
point of zero shear
= 422.05*20/(422.05 + 555.87)= 8.63 m
Design at distance = column width/2 + d
For left support: 350/2 + 709 = 884 mm
For right support: 450/2 + 709 = 934 mm
For left support:
V_u = -422.05*(8.63-0.884)/8.63 = -378.82 kN
For right support:
V_u = 555.87*(20-8.63-0.934)/(20-8.63)
= 510.21 kN
Hence, the right support governs.
b_w = 460 mm
d = 0.8*h = 0.8*760 = 608 mm

Distance "d" can be calculated from the position of reinforcement in the section. However, ACI-318-11[43] stipulates that d need not be taken less than 0.8h. For hand calculation, this option is used conservatively.

d_p = 690mm > 0.8h = 608mm
Conservatively assumed 608mm
v_{cmin} = 0.166*√28 = 0.88 MPa
v_{cmax} = 0.420√28 = 2.22 MPa
v_c [44] = 0.05*√f'$_c$ + 4.8*V_u*d/ M_u
The term (V_u*d/ M_u) must be less than 1 or use 1.
V_u*d/ M_u = 510.21*608/(1412.22*1000)
= 0.22 < 1 OK
v_c = 0.05*√28 + 4.8*0.22
= 1.32 MPa > $v_{c\,min}$ = 0.88 MPa
< $v_{c\,max}$ = 2.22 MPa
Hence v_c = 1.32 MPa governs the design

For this example the ultimate moment was taken at the face-of-support for brevity, while the shear check is done at a distance h/2 away from the support. This assumption is conservative and does not have a significant impact on the outcome of the calculation. In the general case, the value of V_u*d/ M_u varies along the length of the member. But, it is assumed constant for this expeditious hand calculation.
v_u = (510.21*1000)/(460*608) = 1.82 MPa
> Φ v_c = 0.75*1.32 = 0.99 MPa
Hence shear reinforcement is required by calculation.
Assume 12 mm stirrups with two legs:
A_v = 2*129 = 258 mm^2
The spacing, s, between the stirrups is given by:
s = Φ*A_v*f_v/[b_w (v_u – Φ*v_c)]
= 0.75*258*460/[460*(1.82 – 0.99)]
= 233 mm
also s<= 0.75*h = 0.75*760 = 570 mm
and s <= 600mm[45]

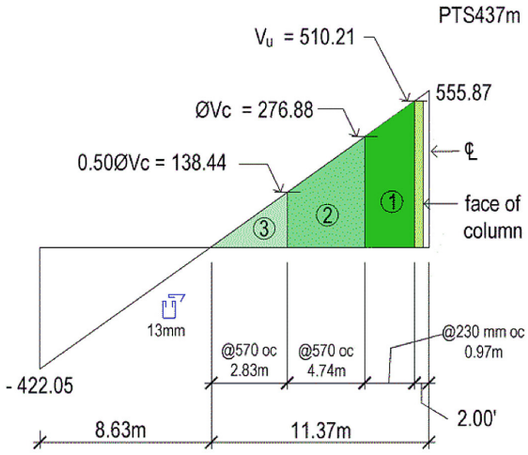
43 ACI 318-11, Section 11.3.1
44 ACI 318-11, Section 11.3.2
45 ACI 318-11,Section 11.4.5.1

Select s = 230mm for the entire region where stirrups by calculation governs.
Using similar triangles, the three regions for the calculation of shear reinforcement are worked out and shown graphically in Fig. 8.5-2.
For the first region V_u > = Φ*V_c = 460*608*0.99/1000 = 276.88 kN
Use stirrups at 230mm spacing.
For the second region V_u > = 0.5*Φ*V_c = 0.5*460*608*0.99/1000 = 138.44 kN
Use the minimum value specified by code.
For the third region V_u < 0.5*Φ*V_c = 138.44 kN
No web shear reinforcement required by code. Conservatively, use the same stirrups at 570 mm spacing (s <= 0.75 h = 570 mm).
For the region governed by the minimum rebar, the spacing shall be the smallest of the following:

In the following the three applicable code relationships [46] are rearranged to express them in terms of "s" spacing. The format of the relationships in the code is in terms of "A_{min}." In this case, since we have already selected a two-legged 12mm bar, we work out the spacing that is appropriate for our selection. Hence, $A_{min} = A_v$ = 2*129 = 258 mm^2.

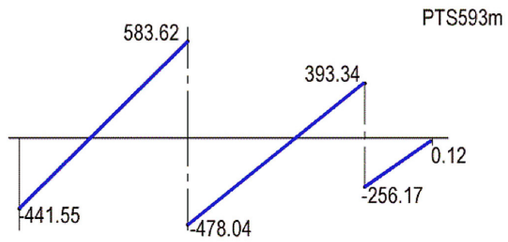
(i) s = A_v f_v/(0.35 b_w)
= 258*460/(0.33*460) = 782 mm
(ii) s = 80 *A_v*(f_v/f_{pu})*d*(b_w/d)$^{0.5}$ /A_{ps}
= 80*258 *(460/1860)*608*(460/608)$^{0.5}$/1188
= 2272 mm
(iii) s = 16*A_v*f_v/(b_w*$f'_c{}^{0.5}$)
= 16*258*460/(460*280.5) = 780 mm
At the same time, spacing "s" shall not be more than 600 mm, nor 0.75*h=570 mm.
Use 12 mm two-legged stirrups at 570 mm on spacing for this region.

B. Based on EC2
V_{ED} =1.35*V_D + 1.5*V_L + 1.0*V_{HYP}
Span 1
b_w = 460 mm
d = 760 –40-22/2 =709 mm
Point of zero shear = 441.55*20/(441.55 + 583.62) = 8.61 m
Design at distance = column width/2 + d
For left support: 350/2 + 709 = 884 mm
For right support: 450/2 + 709 =934 mm
For the left support:
V_{ED} = -441.55*(8.61-0.884)/8.61= -396.22 kN
For the right support:



Shear Force and Reinforcement for Right Side of Span 1 (kN; mm UNO)

FIGURE 8.5-2



Distribution of Shear (kN)

FIGURE 8.5-3

V_{ED} = 583.62*(20-8.61-0.934)/(20-8.61)
= 535.76 kN
Hence, the right support governs.
$V_{Rd,c}$ [47] = [$C_{Rd,c}$* k*(100*ρ1* f_{ck} *)$^{1/3}$+ k 1* σ_{cp}]*b_w *d
but not less than (v_{min} + k $_1$* σ_{cp}) b_w *d

Where,
f_{ck} = 28 MPa
k = 1+ (200/d)$^{1/2}$ = 1+(200/709)$^{1/2}$ = 1.53 < 2.0
ρ1 = Asl/ (b_w d) = 9* 387/(460* 709) = 0.01068
σ_{cp} = N_{ED}/A_C = 1428*10^3/917100 = 1.56 MPa
< 0.2*19 = 3.8 MPa
$C_{Rd,c}$ = 0.18/γ_c = 0.18/1.50 = 0.12
k $_1$ = 0.15
v_{min} = 0.035* k$^{3/2}$* f_{ck} $^{1/2}$ = 0.035* 1.53$^{3/2}$* 28$^{1/2}$
= 0.35 MPa
$V_{Rd,c}$ = [0.12* 1.53*(100*0.01068* 28*)$^{1/3}$+ 0.15* 1.56]*460 *709/1000 = 262.18 kN

46 ACI 318-11,Sections 11.5.5.3 and 11.5.5.4

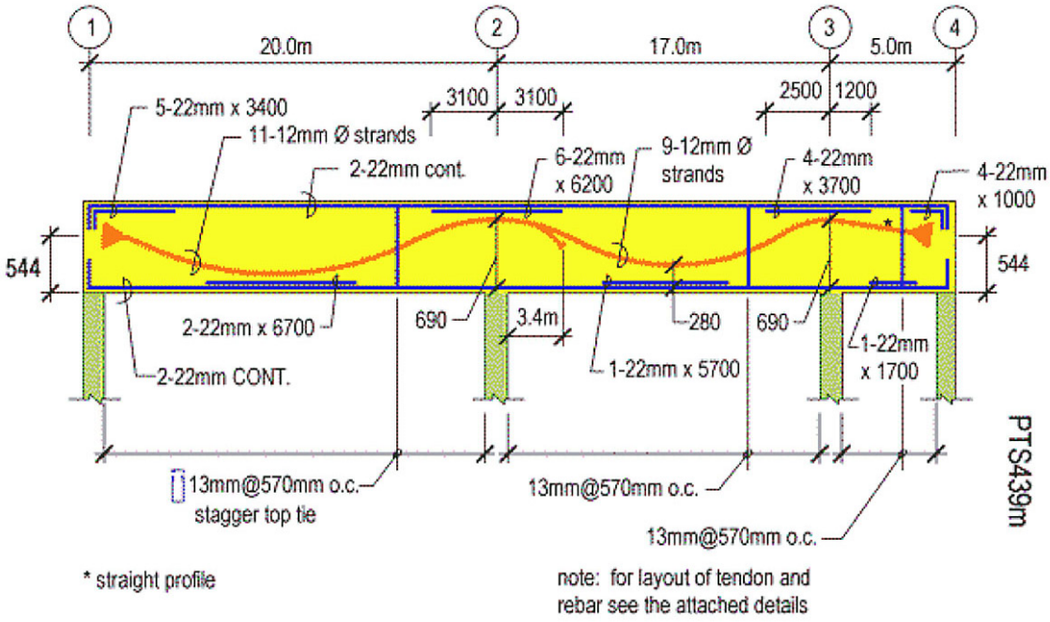
47 EN 1992-1-1:2004 (E) Section 6.2.2

$V_{Rd,cmin}$ = (0.35 + 0.15* 1.56) 460 *709/1000
= 190.47 kN
$V_{Rd,c}$ = 262.18 kN
$V_{ED} > V_{Rd,c}$, Shear reinforcement is required by calculation.
Assume 12 mm stirrups with two legs
A_{sw} = 2*129 mm^2 = 258 mm^2
The spacing [48], s, between the stirrups is given by:
$s = (A_{sw} / V_{Rd,s})^* z^* f_{ywd} \cot\theta$
Where,
Assume $\theta = 40^0$, $\cot\theta = 1.20$
$V_{Rd,s} = V_{ED} - V_{Rd,c}$ = 535.76– 262.18 = 273.58 kN
z = 0.9 d = 0.9*709 = 638 mm
s = [258 /(273.58*1000)]* 638*(460/1.15)*1.20
= 289 mm
$V_{Rd,max}$ [49] $= \alpha_{cw}^* b_w^* z^* \nu_1^* fcd / (\cot\theta + \tan\theta)$
Where,
ν_1 [50] $= 0.6[1-(f_{ck}/250)] = 0.53$ since $f_{ywd} > 0.8 f_{yk}$
f_{cd} = 19MPa
α_{cw} [51] = $(1+ \sigma_{cp}/f_{cd})$ for σ_{cp} = 1.56MPa < $0.25 f_{cd}$ = 0.25*19 =4.75 MPa
α_{cw} = (1+ 1.56/19) = 1.08 α_{cw} equation is as per σ_{cp} values,

$V_{Rd,max}$ [52] = 1.08 *460* 638* 0.53*19 /(1.20 +0.84)
= 1564.59 kN > 273.58 kN OK
Select s = 280 mm
s [53] <= 0.75 d (1+cotα)= 0.75* 709 = 532 mm for the entire region where stirrups by calculation governs.
If $V_{ED} < V_{Rd,c}$, Use the minimum rebar specified by code [54].
For the region governed by the minimum rebar, the spacing shall be the following:

In the following the applicable code relationship is re-arranged to express it in terms of "s" spacing. The format of the relationships in the code is in terms of "A_{min} ." In this case, since we have already selected a two-legged 12mm bar stirrups, we work out the spacing that is appropriate for our selection. Hence, $A_{min} = A_v$ = 2*129 = 258 mm^2.

$s = A_{sw}^* f_{yk} / (0.08^* \sqrt{f_{ck}}^* bw)$
= 258* 460 /(0.08*√28 *460) = 609 mm
At the same time, spacing "s" shall not be more than 532 mm.



Beam Elevation

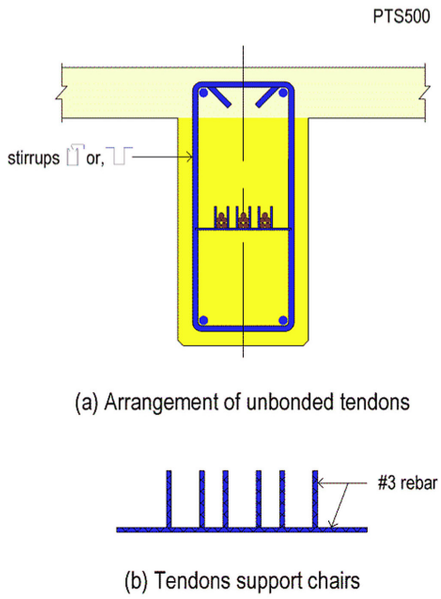
FIGURE 10.1

[48] EN 1992-1-1:2004 (E) Exp: 6.8
[49] EN 1992-1-1:2004 (E) Exp: 6.9
[50] EN 1992-1-1:2004 (E) Exp: 6.6N
[51] EN 1992-1-1:2004 (E) Exp: 6.11aN
[52] EN 1992-1-1:2004 (E) Exp: 6.9
[53] EN 1992-1-1:2004 (E) Exp: 9.6(N)
[54] EN 1992-1-1:2004 (E) Exp: 9.4 &9.5(N)

TABLE 9-1 Stresses at Transfer of Post-Tensioning (T139SI)

	Point A	Point B	Point C	Point D	Point E
M_D (kN-m)	636.00	-926.00	-802.80	262.40	-68.88
M_{PT}(kN-m)	-434.80	592.70	495.30	-118.30	54.75
P (kN)	1428	1428	1428	1071	1071
1.15*P/A (MPa)	-1.79	-1.79	-1.79	-1.34	-1.34
f_t (MPa)	-2.85	0.12	0.03	-2.33	-1.27
f_b (MPa)	0.40	-5.72	-5.54	0.69	-1.45
ACI-08/IBC 2012					
F_t (MPa)	-12.00	1.12	1.12	-12.00	-12.00
F_b (MPa)	1.12	-12.00	-12.00	1.12	-12.00
	OK	OK	OK	OK	OK
EC2					
F_t (MPa)	-12.00	2.21	2.21	-12.00	-12.00
F_b (MPa)	2.21	-12.00	-12.00	2.21	-12.00
	OK	OK	OK	OK	OK
TR-43					
F_t (MPa)	-10.00	1.59	1.59	-10.00	-10.00
F_b (MPa)	1.59	-10.00	-10.00	1.59	-10.00
	OK	OK	OK	OK	OK

Note: Section properties I, A, S_{top}, S_{bot} are the same as used for service condition stress check
F_t and F_b are allowable stresses at top and bottom respectively.



Placement of Tendons in Beam

FIGURE 10-2

Use 12 mm two-legged stirrups at 530 mm on spacing for this region.

C. Based on TR-43 [55]

TR-43 refers to EC2 for one-way shear design. But TR-43 includes the safety factor, γ_p, in the calculation of σ_{cp}. Where γ_p equals 0.9 if the prestress effect is favorable and 1.1 when it is unfavorable.

Precompresssion (σ_{cp}) enhances the shear capacity. Hence, in using the EC2 as outlined in the preceding, reduce the value of precompression by factor 0.9

9. CODE CHECK FOR INITIAL CONDITION

At stressing (i) concrete generally has not reached its design strength; (ii) prestressing force is at its highest value; and (iii) live load generally envisaged to be counteracted by prestressing is absent. As a result, the stresses experienced by a member can fall outside the envelope of the limits envisaged for the in-service condition. Hence, post-tensioned members are checked for both tension and compression stresses at transfer of prestressing. Where computed compression stresses exceed the allowable values, stressing is delayed until either concrete gains adequate strength or the member is loaded. Where computed tension stresses are excessive, ACI/IBC[56] suggest adding non-prestressed reinforcement to control cracking.

9.1 Load Combinations

The codes covered are not specific on the applicable load combination at transfer of prestressing. The following is the combination generally assumed among practicing engineers.

[55] TR-43 Second Edition, Section 5.9.1 and 5.9.2.

[56] ACI 318-11; Section 18.4

Load Case: 1.0*DL + 1.15*PT
Specification of this design example calls for tendons to be stressed with concrete cylinder strength is not less than 20 MPa.
f'_{ci} = 20 MPa

9.2 Stress Check

$\sigma = \pm(M_D + 1.15*M_{PT})/S + 1.15*P/A$
$S = I/Y_c$

9.3 Allowable Stresses

A. Based on ACI 318-11; IBC 2012
Tension = 0.25*√20 = 1.12 MPa
Compression = 0.60*20 = -12 Mpa

B. Based on EC2
Tension = f_{cteff} = 2.21 MPa
Compression = 0.60*20 = -12 Mpa

C. Based on TR-43
Tension = $0.72f_{ctm}$ = 1.59 MPa
Compression = 0.50*20 = -10 Mpa

Farthest fiber stresses are calculated in a similar manner to service condition as outlined earlier. The outcome is summarized in Table 9-1.

If in any of the above locations the stresses exceeded the allowable values the following would have been done.

If compression stresses exceed the allowable value, the design parameters must be modified to bring the stresses within the code limits. If tensile stresses exceed the allowable value, bonded additional reinforcement (nonprestressed, or prestressed) shall be provided in the tensile zone to resist the total tension force in concrete computed with the assumption of an uncracked section.

10. DETAILING

The final tendon and reinforcement layout for the designed beam frame are shown in Fig. 10-1 through 10-3 for unbonded tendons.

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FIGURE 7.SI.1 Beam and Slab Construction; Beam Released from Column;
Tendons crossed and stressed on stem sides
(California; Courtesy DES and ADAPT; P731)

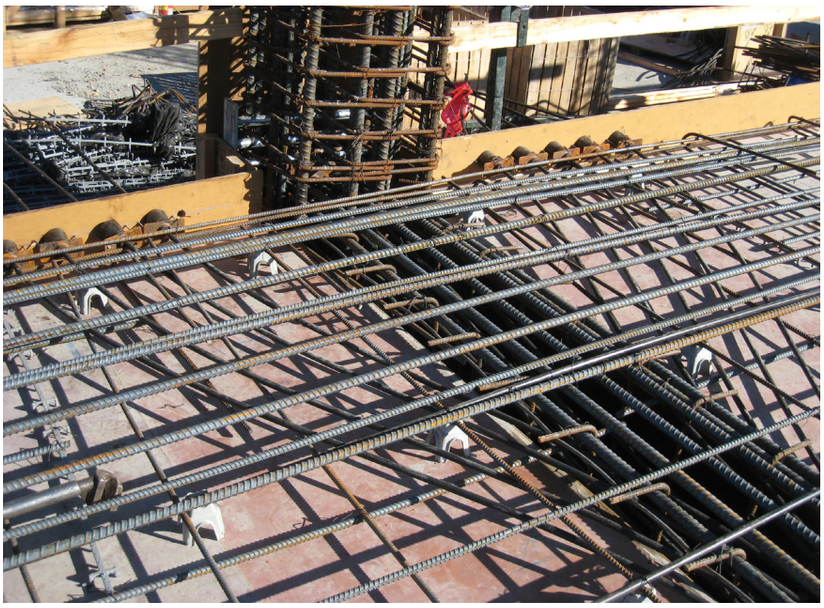

FIGURE 7.SI.2 Beam and Slab Construction;
Monolithic Beam-Column Connection;
A number of tendons are crossed over within the beam stem and raised to anchor at slab edge (California; Courtesy DES and ADAPT; P732)