The Challenge of the End Span

Selection of the post-tensioning tendon profile

by Bijan O. Aalami

Although post-tensioning was made a viable construction option by Eugene Freyssinet in the 1930s, its application remained limited to special structures for almost 30 years. It was not until the early 1960s, when T.Y. Lin introduced the "load balancing" approach to the greater design community,[1] that post-tensioning came into widespread use in building construction.

Load balancing overcomes the complexity of post-tensioning design. The method relies on concepts that are familiar to structural engineers, and it is noted for its computational simplicity. Unlike the alternative design approach, which considers post-tensioning tendons as reinforcement with initial stress, load balancing considers the post-tensioning as a load on the structure. When checking the structure under service conditions, load from the post-tensioning tendon counteracts dead and live loads on the structure, thus improving its performance.

The ease of implementation of the load balancing approach is largely due to the properties of a parabola. Figure 1 illustrates that a tendon in the shape of parabola tensioned with force P provides a uniform uplift w_p on the concrete. The uplift force is a function of the tension P and the geometry of the parabola, and it is given by

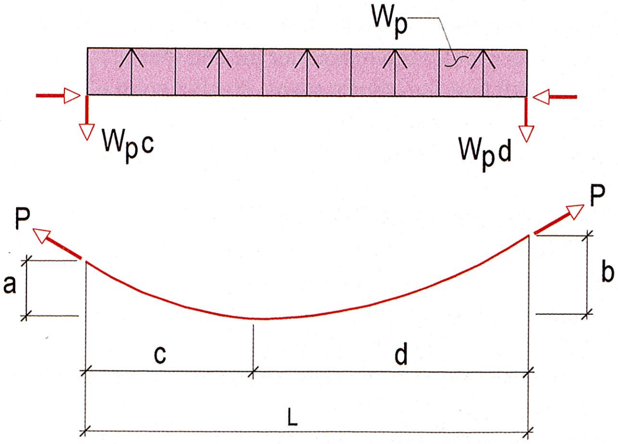

Fig. 1: Geometry and forces imposed on the concrete by an asymmetrical parabolic tendon

$$w_p = 2Pa/c^2 \tag{1}$$

where a and c locate the low point of the parabola relative to the left end of the parabola. The general shape of a parabola is given by a quadratic equation, and the distance c is a function of the height of both ends of the parabola. When these heights are the same, the low point will be at midspan. When these heights are not the same, as is generally the case of end-span tendons, the location of the low point will be given by

$$c/L = (\sqrt{a/b})/(1+\sqrt{a/b}) \tag{2}$$

Figure 2 illustrates the design of a post-tensioned member for service conditions using load balancing. As profiled in Fig. 2(a), the tendon creates a uniform uplift on the member. The tendon can thus be "removed" from the member and replaced with the uniform uplift (Fig. 2(b)). The uplift reduces the effect of the downward load on the member to DL-PT (Fig. 2(c)). The reduced downward force in turn reduces the stresses and deflections, as well as the probability of cracking.

When checking the safety of the structure, the common practice is to consider the tendon as reinforcement with an initial stress. The post-tensioning tendons, along with nonprestressed reinforcement, if any, provide strength for the safety of the structure.

End Span Challenge

A central premise of the load balancing approach is the separation of bending and axial effects. To avoid creating a moment at the anchorage, tendons are typically anchored at the centroid of the member. This results in a uniform

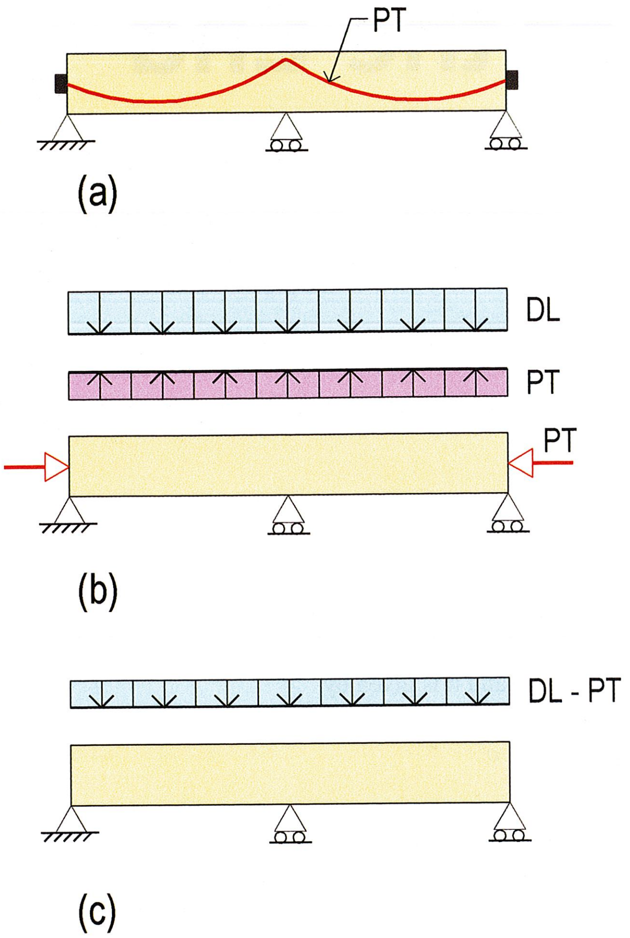

Fig. 2: Contribution of post-tensioning in reducing the effect of dead load on member: (a) post-tensioned member with post-tensioning tendon (PT); (b) member with tendon removed and replaced with uplift imposed by PT (DL is the dead load on the member); and (c) net effect of PT and DL

compression in the member, which is added to the stresses from the bending of the member to complete the design. Anchoring the tendon at the member centroid leads to a challenge in the selection of the tendon profile for the end span, however, as the tendon is typically positioned high over the first interior support to maximize the uplift in the first interior span. The tendon profile must thus be nonsymmetrical as shown in Fig. 3(a) to create uniform uplift in the end span.

Specifying the tendon low point at midspan in the end span—as may be desired to simplify construction—breaks the continuity of the tendon curve. It creates two separate parabolas, one on each side of the low point, each with its own value of uplift (Fig. 3(b)). Designing for the two parabolas, along with the different values of uplift, is a valid approach, but it deviates from the simplicity of uniform uplift concept that was the cornerstone in the adoption of load balancing by structural engineers.

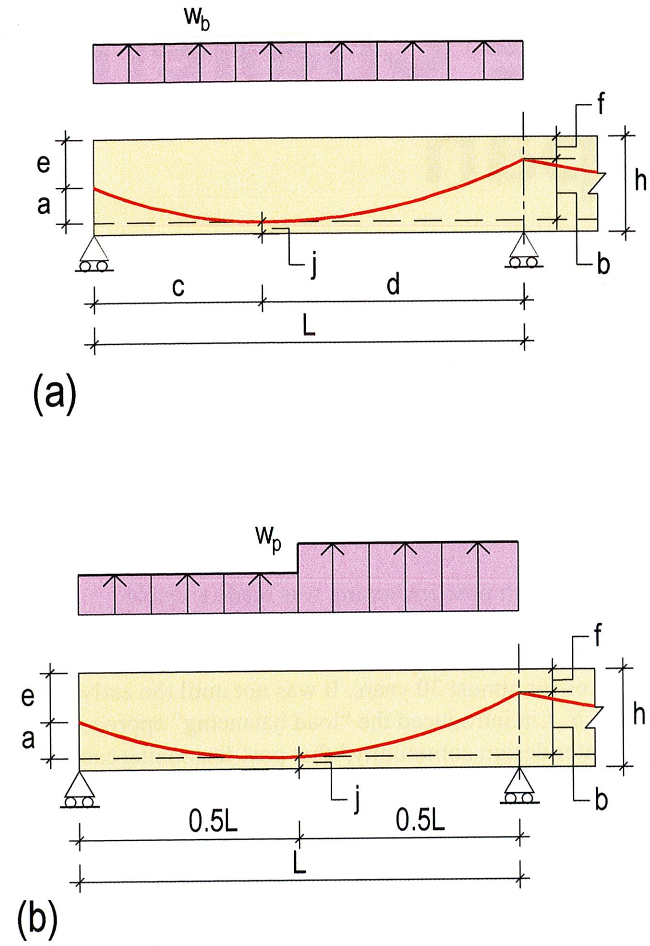

Fig. 3: End span tendon profiles and effects: (a) asymmetrical parabolic tendon; and (b) two parabolic profiles with common low point at midspan

There are approximations used by the engineering community for the end span profile.[2,3] The more rigorous approach is based on a relationship where the location of tendon's low point used for calculation and construction differs from the geometry of the parabola for uniform uplift. The approximation depends on the specifics of the project.

A second profile requirement for the end span is to provide a smaller uplift than the interior spans. Smaller uplift of the end span results in an effective larger net downward force, which is generally beneficial to the performance of the penultimate span.

Preferred Tendon Profile

The preferred end span tendon profile (Fig. 3(a)) is one that satisfies the following requirements:

- Tendon at high point over the penultimate support to meet the demand of the interior span;
- Tendon anchored at the slab centroid to avoid introducing moment at the slab edge;
- Uniform uplift along the length of the end span; and
- Uplift from post-tensioning to be equal to the designer-defined fraction of dead load.

Mathematically, the requirement of uniform uplift and low point at center cannot be met for the defined requirements. Low point at center leads to an uplift distribution shown in Fig. 3(b). The alternative is described next.

Tendon Profile Values

The listed requirements can be met by applying the general equation for a parabola: $y = Ax^2 + Bx + C$, where y and x are the vertical and horizontal coordinates of the tendon and A, B, and C are constants determined by the boundary conditions. For the slab tendon profile shown in Fig. 3(a), the application simplifies to

$$d = [(2P / w_p)(e - f) + L^2] / 2L \qquad (3)$$

and

$$j = h - f - d^2 w_p / 2P \qquad (4)$$

where L is the length of the end span; e is the vertical distance to the tendon anchor from a datum at the top of the slab; f is the vertical distance from the datum to the center of gravity (CGS) of the tendon at the interior support; d is the horizontal distance from the penultimate support to the low point of the tendon profile; and j is the distance from the bottom of the slab to the CGS of the tendon at the low point. In summary, the parameters d and j define the low point of the tendon, P is the force in the tendon, and w_p is the uplift created by the tendon.

Example

The following example, along with Fig. 4 and 5, illustrate the application of these relationships.

Dead load: $W_d = 152$ lb/ft^2 (7.28 kN/m^2)

Post-tensioning: $P = 21$ k/ft (306 kN/m)

Minimum distance to CGS: $f = 1$ in. (25 mm)

Slab thickness is 8.5 in. (216 mm); hence, $e = 4.25$ in. (108 mm).

Other parameters are as shown in Fig. 4 and 5.

Determine the tendon profile to balance 60% of the end span's dead load:

$w_p = 0.6 \times 152 = 91.2$ lb/ft^2 (4.37 kN/m^2)

Substituting in Eq. (3):

$d = [(2 \times 21{,}000 / 91.2)(4.25 - 1) / 12 + 23^2] / (2 \times 23) =$ 14.34 ft (4371 mm)

From Eq. (4):

$j = 8.5 - 1 - [(14.34 \times 12)^2 \times (91.2 / 144)] / (2 \times 21{,}000/12) =$ 2.14 in. (54 mm)

Tendon low point at 14.34 ft (4371 mm) from the interior support. Height to CGS of tendon at low point 2.14 in. (54 mm).

Check the validity of design:

$b = 8.5 - 1 - 2.14 = 5.36$ in. (136 mm)

$w_p = 2pb/d^2 = 2 \times 21{,}000\,(5.36 / 12) / 14.34^2 = 91.22$ lb/ft for a 1-ft strip (4.37 kN/m for a 1-m strip) equal to 60% DL → OK

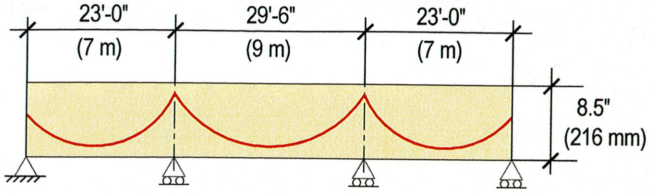

Fig. 4: Slab elevation

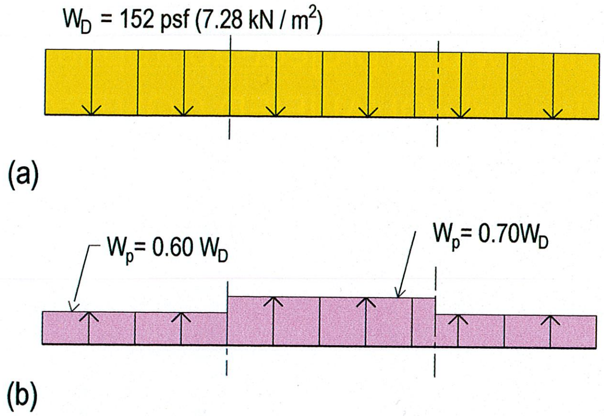

Fig. 5: Slab load distributions resulting from: (a) dead load; and (b) post-tensioning tendon

References

1. Lin, T.Y., "Load-Balancing Method for Design and Analysis of Prestressed Concrete Structures," *ACI Journal Proceedings*, V. 60, No. 6, June 1963, pp. 719-742.

2. "PTI TAB,1-06: Post-Tensioning Manual," sixth edition, Post-Tensioning Institute, Farmington Hills, MI, 2006, 354 pp.

3. Nawy, E.G., *Prestressed Concrete, a Fundamental Approach*, third edition, Prentice Hall, Upper Saddle River, NJ, 1999, 938 pp.

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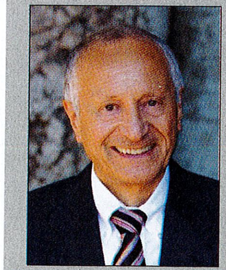

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