TN503_hyperstatic_computation

HYPERSTATIC MOMENTS FROM POST-TENSIONING
COMPUTATION AND VERIFICATION FOR DESIGN STRIPS

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This technical note illustrates the calculation of hyperstatic moments for a floor slab design strip. The tendon profile and non-essential aspects of the design are kept simple, in order to focus on the items critical to the understanding and computation of the hyperstatic values.

In practice, the computations are done using specialized software. The following is intended to illustrate the computational steps and to provide verification for a design example.

The computation steps are:

1 – Determine the balanced load from prestressing
2 – Verify the validity of the balanced loads
3 – Apply the balanced load to the structure; determine the structure's reactions from the balanced loads
4 – Use the reactions to calculate the hyperstatic moments

The example given below is extracted from the reference [Aalami, 2014]

1 – DETEREMINATION OF BALANCED LOAD

Balanced loads are the forces from prestressing tendons that result in "bending" of the member. The precompression from the axial forces of prestressing are accounted for separately. Only the bending aspects of the prestressing are covered herein.

To illustrate the computational steps, the slab strip shown in Fig. 1-1 is used. The figure represents a design strip extracted from a post-tensioned floor. It is important that the extracted model retains all the features of the slab geometry. Changes in geometry, such column drop, drop panel, steps and slab thickness influence the outcome.

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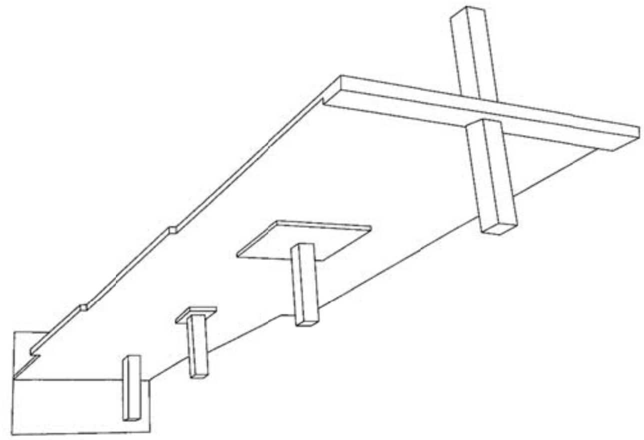


FIGURE 1-1 View of the design strip.
The design strip features column drop, drop panel, wall and column supports (P473)

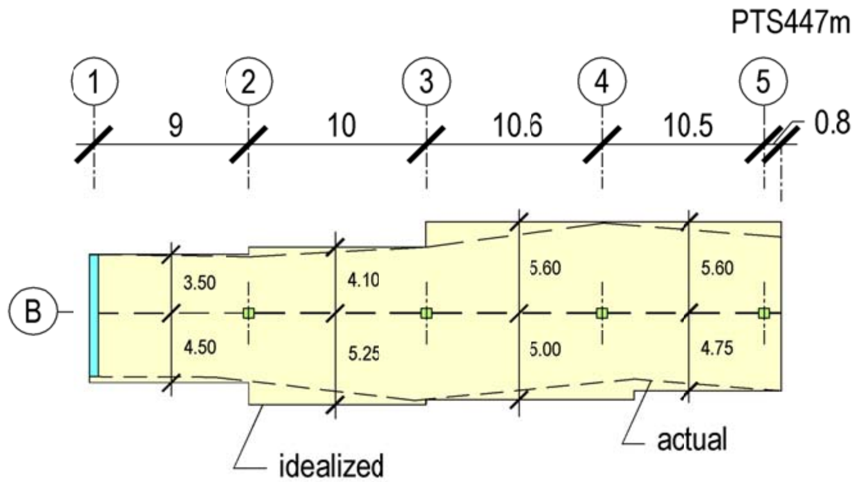
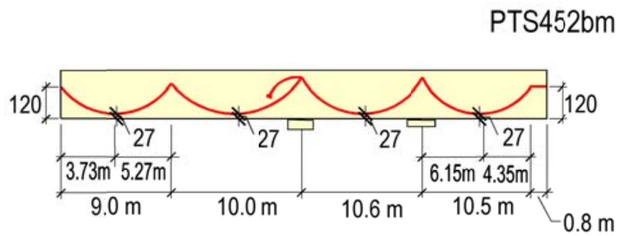
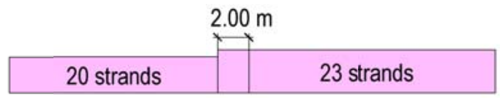


FIGURE 1-2 Top plan view of the design strip, showing span dimensions and the tributaries (m)
Wall thickness = 203mm; supporting columns 610 mm square;
slab thickness 241 mm; there is a column drop at grid line 3 and drop panel at gridline 4 not visible on the top view.



(a) Assumed tendon profile (mm, UNO)



(b) Force diagram

FIGURE 1-3 Tendon profile and number of strands
There are 20 strands along the entire length of the member; 3 added strands extend over spans 2 and 3.

The balanced loads from the prestressing tendon consist of:

(I) up and down forces from tendons that extend over the entire span;
(ii) up and down forces of the curtailed tendons extending from span three into span two; and
(iii) moments generated by tendons at change in centroidal axis of the member.

The tendon profiles are assumed simple parabola between their control points.

The up and down forces from the tendons that extend over the entire span are obtained from the equation of parabola applied to the geometry of the respective span and the tendon force [Aalami, 2014].

Three tendons from span 3 extend over the third support and anchored in the second span. The Balanced loads from tendons that are terminated over interior supports are shown in Fig. 1-4

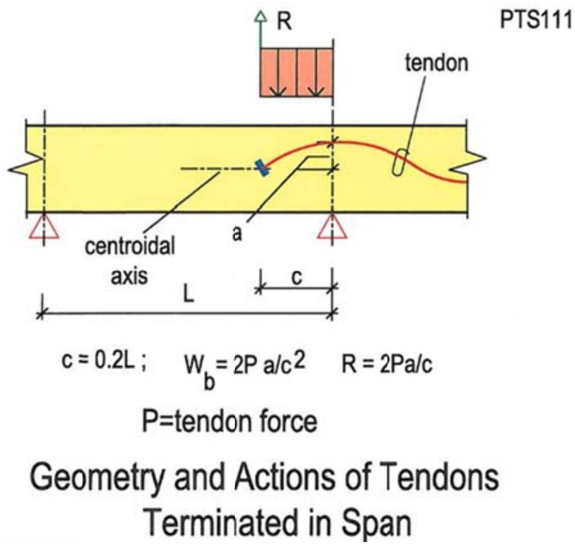


FIGURE 1-4 Forces from tendons terminating over interior supports
Tendons follow parabolic path and are anchored at one-fifth of the span.

The presence of column drops and drop panel over the supports results in added moments to the balanced loads.

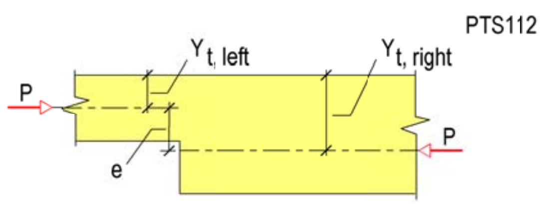


FIGURE 1-5 Change of centroidal axis along a post-tensioned member
The change results in an applied moment equal to force times change in centroid;
$M = P(Y_{t,right} – Y_{t, left})$

The up and down forces and the bending moments from the change in centroidal axis form the balanced load from post-tensioning together form the balanced loads.

For the current example, the balanced loads are shown in Fig. 1-6. The up-and-down forces are shown in part (a) of the figure. The moments are shown in part (b). The two act concurrently on the member. They are shown separately for ease of visualization.

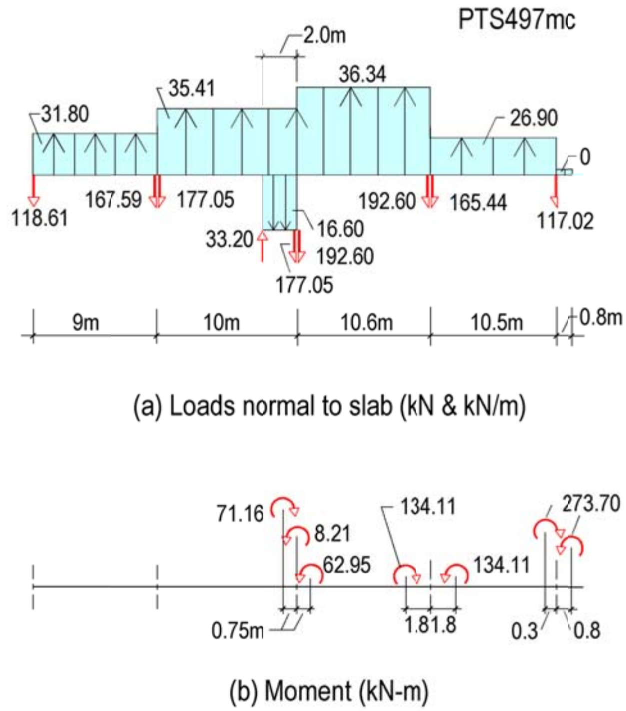


FIGURE 1-6 Balanced loads from prestressing
The up-and-down forces shown in part (a) and the moments in part (b) together form the balanced loads of the member. One is incomplete without the other

2 – VERIFY THE VALIDITY OF BALANCED LOAD

Balanced loads from post-tensioning tendons must be in equilibrium. Equilibrium of forces validates that the forces calculated are complete and correct.

Lack of equilibrium of resisting forces in a member requires the verification that the error is on the "safe" side.

Equilibrium check:
Sum of forces in the vertical direction:

$\sum$Forces $\uparrow$= -118.61 +(9*31.8)-167.59-177.05+(10*35.41)-(2*16.60)+33.20-177.05- 192.60+(10.6*36.34)-192.60-165.44+ (10.5*26.90)-117.02 = 0.006 kN $\approx$ 0 OK

Sum of moments about the third support:

$\sum M_{3rd\ Support}$ = 118.61*19.0 + (167.59+177.05)*10.0-33.20*2.0- 31.8*9.0*14.5-35.41*10^2 / 2+16.60*2^2 / 2+36.34*10.6^2 / 2 + 26.90*10.5*15.85-(192.60+ 165.44)*10.6-117.02*21.1-71.16+8.21+134.11+62.95-134.11-273.70+273.70 = 0.46 $\approx$ 0 kN-m OK

3 – APPLY THE BALANCED LOAD TO THE STRUCTURE AND DETERMINE THE REACTIONS

The balanced loads shown in Fig. 1-6 are applied as load to the structural frame of the design strip shown in Fig. 1-2. The resulting bending moments and reactions are shown in Fig. 3-1.

The reactions displayed in part (b) of the figure are the hyperstatic reactions from prestressing. These reactions result in forces, such as moment and shear in the member.

It is noteworthy that the hyperstatic reactions are in self-equilibrium, since the balanced load that generates them is also in equilibrium.

The moments and reactions shown in Fig. 3-1b are at center of support – centerline moments.

The values shown in part (b) will be used to construct the hyperstatic bending moment that will be included in the load combination for strength check of the member.

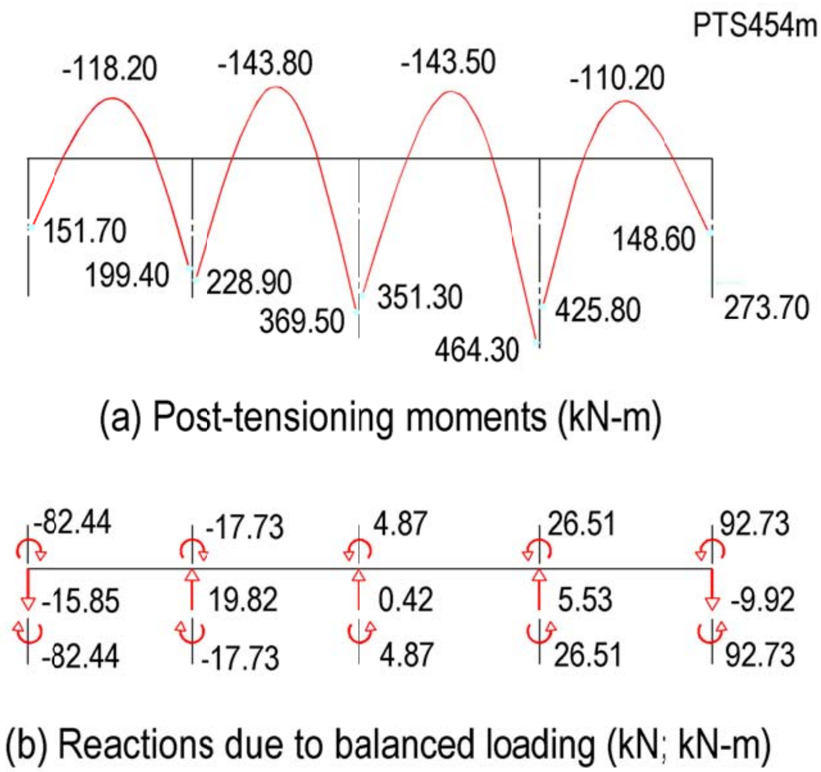


FIGURE 3-1 Post-tensioning moments (part a), and hyperstatic reactions.
The hyperstatic reactions shown in part (b) are centerline values. The moments shown in part (a) are values at the face of support.

4 – USE THE HYPERSTATIC REACTIONS TO DETERMINE THE DISTRIBUTION OF HYPERSTATIC MOMENTS

Values of hyperstatic actions shown in Fig. 3-1 will be used to determine the distribution of hyperstatic moments in the design strip.

4.1 Computation of Centerline Moments

Refer to Fig. 1-2 for span lengths and support widths

Left of Span 1 = 82.44 + 82.44 = 164.88 kN-m

Right of Span 1 = 164.88 -15.85 * 9.00 =22.23 kN-m

Left of Span 2 = 22.23 + 17.73* 2 = 57.69 kN-m

Right of Span 2 = 82.44*2+ 17.73* 2- 15.85*(10 + 9)+19.82*10.00 = 97.39 kN-m

The remainder of the moments at left and right of each span can be determined in a similar way

4.2 Face of Support (FOS) moments

These are shown in Fig. 4.2-1. The computation is as follows

Hyperstatic moments vary linearly between the supports. For face of support (FOS) hyperstatic moments linear interpolation between the centerline moments is used.

Span 1 = 9.00 m
First support with = 0.203 m
Second support width = 0.610 m

Span 1, Left(FOS) = 82.44*2 -15.85*(0.203/2) =163.3 kNm (163.30 kNm in Fig. 6.5-1c, OK)

Span 1, right(FOS) = 82.44*2 -15.85*(9.00 - 0.305) =27.06 kNm (26.99 kNm in Fig. 6.5-1c, OK)

Span 2= 10.00 m

Left of Span 2 (FOS) = 82.44*2 + 17.73*2 - 15.85* (9.00 + 0.305) +19.82*0.305 = 58.90 kNm
(58.88 kNm in Fig. 6.5-1c, OK)
Right of span 2(FOS) = 57.69 + (97.39 – 57.69)*9.695/10 = 96.18 kNm (96.24 kNm in Fig. 6.5-1c, OK)

The remainder of the hyperstatic moments can be calculated in a similar manner.

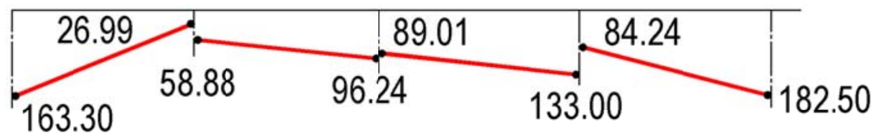


FIGURE 4.2-1 Distribution of hyperstatic moments in the member (kNm)
The values shown are at the face of support (FOS)

REFERENCES

Aalami, B. O.," (2014),Post-Tensioned Buildings; Design and Construction," www.adaptsoft.com, PT-Structures.com, Rewood City, CA, Mar 2014, 450 pp.