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On the Strength Design of Ship Plates Subjected to In-Plane and Transverse Loading

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SUMMARY: A survey is made of the methods available for ultimate strength analysis and design of isotropic steel plates under in-plane loading and uniform transverse pressure. It is concluded that sufficient work has been carried out in formulating the elasto-plastic behaviour of plates, but that no solutions have been presented for plates under combined loading of interest to ship designers.

For a square panel with breadth to thickness ratio of 50, a total of 84 plate panels were tested under varying ratios of transverse pressure to edge-compression. The tests were based on a box model which consisted of four plates representing closely the conditions of ship panels. Interaction curves between the in-plane loading and transverse pressure are obtained which show the ultimate strength of the panels. It is found that for certain combinations of loading ($Q/N < 25$) the plates undergo a sudden collapse due to plastic instability while for other load combinations the plates exhibit excessive plastic deformation with load increments. The effects of residual stresses and initial deformations are investigated.

Proposals are made for the further development of the theoretical work and the extension of the present experimental programme which would provide a fuller understanding of the panel strength and would therefore lead to more rational design procedures.

NOTATION

l = Length of panels

b = Width of panels

h = Thickness of panels

q = Intensity of uniformly distributed transverse pressure

w = Out-of-plane deflexion of panels at centre

v = Axial shortening of panels in direction of applied end-thrust

σ_y = Yield stress

σ_m = Direct membrane stress

E = Young's modulus of elasticity

$Q = \frac{b^4 q}{h^4 E}$, non-dimensional transverse pressure ratio

$N = \frac{b^2 \sigma_m}{h^2 E}$, non-dimensional in-plane stress ratio

1. INTRODUCTION

Plate panels covering the sides and bottom of ships are subjected to a complex system of loading, the major components of which are: (i) transverse (normal) pressure due to still water pressure and the dynamic effects of waves and ship's motion, and, (ii) in-plane tension or compression as a result of longitudinal bending of the ship. The behaviour and strength of flat panels under such combined loading has for long been a problem of great interest to the ship designers, but research carried out in the field has not so far lead to a clear understanding of the behaviour and conclusive proposals for the design.

The following is a summary of the work carried out and the present standing of the available methods in dealing with the design of isotropic flat steel plates under in-plane and transverse loading.

The simplest approximate method which has been used for the design of plates under combined loading has been the superimposition of the available elastic solutions$^{(1)}$ for plates under transverse pressure alone, and the solutions of plates under in-plane loading. A marked improvement to the above approach, due to Bleich$^{(2)}$, is to allow for the presence of edge-compression in the flexural equation and to obtain linearised solutions in which, due to the reduced flexural stiffness, the deflection and bending stresses are magnified. A solution so obtained still needs to be superimposed on the in-plane stresses estimated independently to give the final stress.

A study of the elastic large-deflexion of plates$^{(3)}$ showed that the membrane boundary conditions, which were neglected in the previous studies, are at least as important as the flexural boundary conditions on the behaviour of plates even within the in-service deflexion range of ship panels. It was concluded that Bleich's solutions must be used with caution for plates which form part of a structure, as they refer to simple isolated boundaries and in certain cases may underestimate the actual stresses. A comprehensive analysis is made in Ref. 4 for plates of general interest, which allows for the complete interaction of the transverse pressure and in-plane loading, as well as the flexural and membrane boundary conditions together with the initial unfairness of plating. In Ref. 5 plates of specific interest to ship designers are similarly treated. The analysis is based on von Karman's$^{(1)}$ elastic large-deflexion plate equations and represents the true behaviour of the plate panels up to the onset of plasticity.

Experience has shown that due to the high reserve of strength in the plates undergoing plastic flow, ship panels can sustain loadings far beyond the elastic limits determined from small-deflexion or large-deflexion solutions. Plastic deformations and permanent sets may be allowed without violating the accepted safety limits of the structure. Although the elastic large-deflexions give an insight into the behaviour of plating under working conditions, they are not a measure of the plate strength and need to be accompanied by elasto-plastic analyses

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for an optimum design. No theoretical solutions are yet available beyond the elastic limit giving the elasto-plastic large-deflexion behaviour of rectangular plates under combined loading, upon which a limit design approach could be based. There have, however, been a number of attempts made to put forward approximate solutions to simplified cases or to assess the ultimate loading through experimental methods. An account of the early developments in the plastic design of steel plates is given in Ref. 6.

A theory of plasticity with applications to steel plates is developed by Prager (7) and Hill (8) who proposed several criteria for the conditions of plastic flow and the interaction of bending and membrane effects. As a result of the complexity of the theory and the difficulty in obtaining elasto-plastic solutions, idealisations of the process of yielding are made to enable the plate behaviour to be expressed in a way suitable for obtaining solutions. Several research workers have neglected the effect of membrane stresses and on the basis of Tresca (9) or von Mises' yield criterion (10) for bi-axial action, have obtained small-deflexion elasto-plastic solutions for circular plates under axisymmetrical loading. Based on the same simplifying assumption Prager (11) calculated the collapse load of a square clamped steel plate under transverse pressure. The solution offered is a lower bound approximation to the strength of the plate and grossly underestimates the plate strength. Jaeger (12) on the other hand made the simplifying assumption of neglecting the effect of bending and obtained solutions by accounting only for the membrane action. In respect of the importance of bending and membrane actions on the ultimate strength of steel plates, Wood (6) suggests that clamped plates with b/h > 40 may be considered to act wholly as a membrane in resisting the load to failure.

Hopkins (13) and Massonnet (14) have overcome the restrictions of axisymmetrical circular plates and have set out general equations of equilibrium and plastic flow for rectangular plates. Massonnet develops a large-deflexion general theory for the elastic and elasto-plastic behaviour of the rectangular plates taking into account both the membrane and bending actions of the plates as well as the initial imperfections. The method developed is directed towards modifying the elastic von Karman equations in the regions of plastic formation. No solutions are however obtained due to the complexity of the equations and limitations on computer space.

If it is assumed that the maximum deflexion and stresses as given by the large-deflexion theory, for plates under uniform transverse pressure and with length to width ratio greater than 1·5, differ very little from the corresponding quantities for plates with infinite aspect ratio, for a plastic analysis of long plates the plate may be assumed to be infinitely long and to act as a plate strip. The two-dimensional plate problem thus transforms into a one-dimensional strip problem being less complicated to analyse. On this assumption Clarkson (15) and Wah (16) developed an elasto-plastic analysis of transversely loaded long rectangular plates. The above assumption, however, needs to be employed with caution as its accuracy depends largely on the plate boundary conditions. In Ref. 5 (Figs. 6 and 7) a study of the distributions of membrane stresses obtained for a 3/1 rectangular plate under uniform transverse pressure and with four different boundary conditions indicates that (i) the distribution is primarily influenced by the membrane boundary conditions, and (ii) in the large-deflexion range a 3/1 plate is not long enough to ensure cylindrical bending of the middle region so that the plate's end-effects may be neglected. Clarkson derived relationships for deflexions and permanent sets of long rectangular plates under transverse pressure for the various phases of plastic formation in the plate. Wah extended Clarkson's work and included in his analysis the effects of initial unfairness of plating, residual stresses as well as edge-displacement.

On the experimental side a series of tests on clamped square and rectangular plates under transverse pressure and for a number of breadth to thickness values and different aspect ratios were carried out by Clarkson (17). The test results, though of value are too few in number with too many variables to enable any clear conclusions to be drawn. Hook and Rawlings (18) conducted an experimental investigation into the behaviour of clamped, rectangular, mild steel plates subjected to uniform transverse pressure with breadth to thickness ratios ranging from 53 to 168. In a later paper they (19) reported a survey of available methods of predicting the post-elastic load carrying properties of initially flat plates under transverse pressure, where for the design of plates under transverse pressure alone they proposed that for (i) plates having aspect ratios greater than 2, Clarkson's proposals (15) may be used, (ii) for square plates, the plate may be approximated by a circular plate for which solutions are available, and for (iii) plates with aspect ratios between 1 and 2 they proposed an empirical design formula.

For plates under end-thrust alone, Timoshenko in Ref. 20 proposed a method of evaluating the ultimate capacity of plates under edge-thrust which may be readily extended to apply to clamped plates. The method is based on an effective-breadth concept and on equating the critical buckling stress of the plate to yield stress. The formulae proposed by Timoshenko may be used for thin plates with critical buckling stress below the yield point (for simply supported square plates b/h between 55 to 60).

There are no test results known to the authors for the ultimate strength of isotropic unstiffened rectangular steel plates under the simultaneous action of edge-compression and transverse pressure. Young (21) has carried out a series of tests for the behaviour of rectangular plates under combined loading in the elastic range.

Due to the lack of analytical solutions and the very limited amount of experimental results available, and also that the available data in most cases does not reflect clearly the membrane boundary conditions, it was considered necessary to carry out a series of pilot tests on the ultimate strength of steel plates under combined loading. The object of the tests was to study the interaction relationship between the transverse pressure, edge-compression and the ultimate strength of the plates, with a view to applying the results to a limit stage analysis approach.

2. NATURE OF THE PROBLEM

The plate panels skinning the bottom of the ship are about 30 in wide and have width to length ratios varying between 1/1 to 1/4. They cover the grid structural system of the ship bottom which consists of floors, intercostals and stiffeners. Their breadth to thickness ratio varies between 20 to 60. This is higher than the ratio used in plates of steel bridge decks which are subjected to combined loading (about 25). In light gauged structures, however, such as aircraft this ratio reaches the order of 100 to 120.

Adjacent panels at the bottom shell may be assumed to be under the same transverse pressure and in-plane loading.

With regard to boundary conditions, the panels rest on either frames or intercostals which may be regarded as rigid, or may be resting on stiffeners which allow a certain amount of sinking of the boundary. The rotational condition of the adjacent panels can be assumed as fixed due to symmetry, provided there are no initial deformations. But initial deformations are often present in the actual structure, and, if large enough, depending on the shape of the unfairness, they may force the behaviour of the adjacent panels into skew symmetrical buckling. In such a case the rotational restraint of the stiffeners or the web supports will be mobilised. For the ship under pure bending, the membrane shear stresses along the boundaries perpendicular to the ship's longitudinal axis, and the boundaries parallel to the ship's longitudinal axis, are zero. There will however be a membrane shear restraint exerted by the web or stiffener support when the panel deflexions become large due to transverse pressure. The extensional membrane continuity is that due to symmetry the common boundaries remain straight, and that the membrane

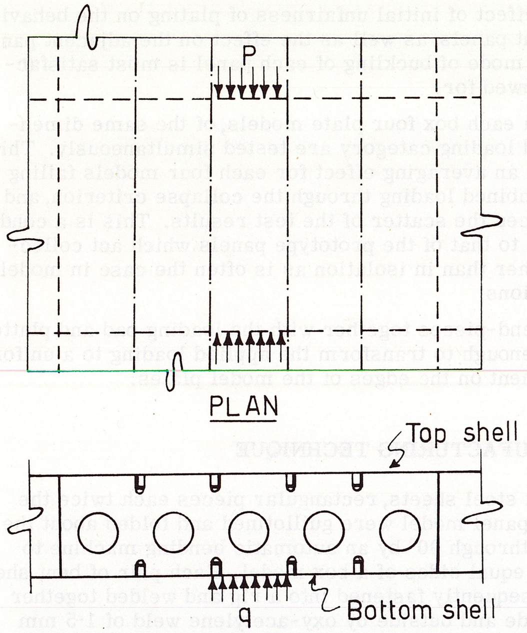

Fig. 1. Portion of a ship's double bottom showing transverse pressure (q) and axial loading of one of the bottom shell plate panels

direct stresses developed will be such that they integrate to the applied loading along the loaded edges, which may be assumed to have been uniformly displaced.

In Fig. 1 transverse and in-plane loadings on a double bottom panel of plating are shown.

3. FAILURE CRITERIA

The failure criteria of panels under the simultaneous action of in-plane and transverse loading may be based on several factors.

For certain ratios of transverse pressure to end-thrust, the plate undergoes a sudden collapse as the loads are increased proportionally. For such cases, in which the end-thrust may be considered as dominant, the failure load is clearly the maximum load the plate can sustain.

Under other load combinations, when the transverse pressure is dominant, a plate panel continues to deform while the load is being increased. The plate passes through the fully elastic and the elasto-plastic stages to the plastic membrane condition with large deformations. For this group of plates the failure criterion may be set as a limit to the maximum central deflexion of the panel. Alternatively some investigators have proposed (15, 18) a limit to be placed on the permanent set of a plate after loading is removed. This entails the difficulty of measuring or computing the panel's permanent set corresponding to the 'shake down' condition of the given maximum values of transverse pressure and end-thrust considered for design. For in partially yielded plates under a number of loadings (such as the present in-plane and transverse loads), the maximum deflexion and permanent set depend upon the sequence of the application of loads, the number of times the loads are introduced and omitted and the initial stress condition of the plate. But for any assumed values of in-plane and transverse loading which a plate can sustain without collapsing, there is a limiting value for the permanent set reached after repeated random applications of the loads called the 'shake down' limit. It is possible to calculate the shake down permanent set of simple rigid frames (22), but the corresponding theoretical work in respect of plates is not yet developed. Experimentally also, the problem is complicated.

While for the case of plates under uniform transverse pressure alone, for which there is no shake down phenomenon, it is best to base the design on a permanent set criterion, for plates under combined loading the maximum central deflexion under proportional increments of both loadings may prove a more convenient means of measuring the plates' resistance to the applied loads. This is so because both the sequence history of loading and the initial state of stress are known which enable the experimental results to be readily repeated and checked.

For large plastic deformations, the central deflexion of the panel under loading is a measure of the plastic strength reserve of the panel consumed through plastic deformation to withstand the applied loading and hence its selection as a design criterion may be justified.

In the present work, as the bulk of the results relate to plates under combined loading, the latter approach of deformation under applied loading is adopted for plates where no clear collapse loading has been reached. Three maximum deflexion criteria of various degrees of plate resistance are set, namely maximum central deflexion of plates reaching once, twice or three times the plate thickness.

4. CHOICE AND DESIGN OF THE MODELS

The difficulty in the experimental analysis lies in the simultaneous application of transverse pressure and edge-compression, and the simulation of the idealised boundary conditions outlined previously for the model. Whereas there are a few experiments reported in the literature for plates under in-plane loading or transverse pressure alone, the only experimental results for plates under combined loading known to the authors are those due to Young (21).

Young's testing rig was designed to test single panels under uniform transverse pressure, applied through an air bag, and uniaxial or biaxial edge-compression. A system of closely spaced hydraulic jacks acting through adaptors on the extensions of the test panel beyond its boundaries and in the plane of the panel served to apply the edge-compression. The model panel rested on rigid discrete roller supports. Zero slope was maintained by applying fixing moments through a system of screwing studs on the extended portion of the plate. The membrane shear and extensional restraints are not very clear. They depend on the extensions of the model, and the attachments on it.

A test arrangement similar to that used by Young though suitable for small deflexion behaviour in the elastic range was not considered satisfactory for the present work. Apart from its complexity, (i) the membrane boundary conditions do not represent the symmetry of the prototype panels, (ii) the corrections of slope affect the loading history of the plate and hence its behaviour in the elasto-plastic range, and (iii) the distribution of the applied in-plane loading is not well defined in the large-deflexion range.

The experimental part of the present work is based on a box type model as shown in Fig. 2, the walls of which represent the prototype plate panels under combined loading. Two rectangular steel sheets are bent and welded together to form the four sides of the box model. The ends of the box are covered by two stiff end-pieces. Each wall of the box then represents a plate panel which can be tested under combined loading.

The end-thrust is applied through a hydraulic ram to the end-pieces of the box model. Transverse pressure is exerted by oil under pressure inside the box (see Fig. 3).

The model presented has the following features:

(a) It is simple to construct.

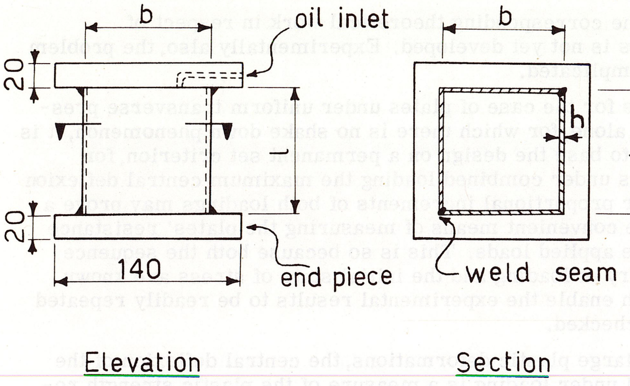

Fig. 2. General arrangement of model (dimensions in millimetres)

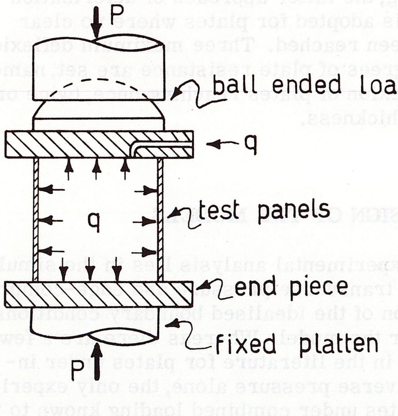

Fig. 3. Loading arrangement of model

(b) A boundary continuity close to the prototype conditions is achieved at the common boundaries of the plate panels. The rotational and membrane shear boundary conditions are truly represented. The stiffener or the web support of the prototype plate is omitted. Each model plate has on its sides two of the adjacent walls of the box acting as supports.

The membrane direct forces at the boundaries equal the shear forces of the adjacent panel because the walls are at right angles to one another. Under large-deflexions, high membrane tensile forces develop across the boundaries of the prototype plates having a stabilising influence on the behaviour. The model, however, exhibits a more flexible behaviour due to the lack of sufficient restraint to develop the mentioned tensile forces across its boundaries. The average tensile stress developed in the walls of the model is equal to qb/4h, being the reaction of the adjacent walls to the transverse pressure on the panel under consideration. For the same box dimensions and plate aspect ratio, the magnitude of this membrane tensile stress and hence its stiffening effect depends on the b/h ratio becoming more significant for thinner plates. For a b/h = 50, a large-deflexion analysis of the model plates under transverse pressure alone, based on the work of Ref. 4, showed that the plates become about 4% stiffer flexurally due to the presence of the mentioned tensile forces acting on their boundaries. This is lower than the stiffening effect of the 'edges remaining straight' condition of the prototype. The results obtained will, therefore, be on the safe side in respect of the influence of the extensional membrane boundary conditions. Due to the design of the model, the oil pressure on the end-pieces results in an axial tension in the plates in the direction of the applied end-thrust. This pressure however may be readily subtracted from the applied axial force to give the effective end-thrust on the wall panels.

(c) The effect of initial unfairness of plating on the behaviour of adjacent panels, as well as the effect on the adjacent panels of the mode of buckling of each panel is most satisfactorily allowed for.

(d) From each box four plate models, of the same dimensional and loading category are tested simultaneously. This results in an averaging effect for each four models failing under combined loading through the collapse criterion, and thus reduces the scatter of the test results. This is a condition close to that of the prototype panels which act collectively rather than in isolation as is often the case in model presentations.

(e) The end-pieces together with the loading pad and platten are stiff enough to transform the applied loading to a uniform displacement on the edges of the model plates.

5. MANUFACTURING TECHNIQUE

From flat steel sheets, rectangular pieces each twice the size of a panel model were guillotined and folded about the mid-line through 90° by an automatic bending machine to form two equal sides of a box model. Each pair of bent sheets were subsequently fastened into a rig and welded together from inside and outside by oxy-acetylene weld of 1·5 mm throat thickness. The above made walls of the box model were then positioned on the premarked end-pieces and welded to it using an electric arc while in a second rig. The welding was carried out from the outside, and the material deposited had an average throat thickness of 2·5 mm. During welding, precautions were taken to avoid overheating as well as rapid cooling of the welded joint.

The models constructed had an initial unfairness due to handling and welding scattered irregularly over the walls of the box. The average of the absolute maximum initial deformations of plate models (walls) was about $w_{o\,max}/h \approx 0{\cdot}08$. However the maximum value for individual plate models in certain cases reached a ratio of 0·3.

6. MODEL DIMENSIONS AND MATERIAL PROPERTIES

In the present work a series of tests was carried out under in-plane and transverse pressure on square plates having the following dimensions:

$b = l = 100$ mm

$h = 2$ mm (±1%), hence $b/h = 50$

$E/\sigma_y = 825$

End pieces 150 × 150 × 20 mm

The material used was from a single batch of mild steel sheets having a clearly defined yield point of 25·5 kg/sq. mm and a maximum stress of 33 kg/sq. mm.

7. INSTRUMENTATION AND TEST PROCEDURE

On the basis of the failure criteria adopted, it was necessary to measure the axial load/deformation of the box model and the transverse (out-of-plane) central deflexions of the walls. As shown in Fig. 4, dial gauges 1-4 are positioned at centre of the panels for out of panel deflexions. Dial gauges 9-12 were fixed at the corners of the end-pieces in such a way as to show the axial displacements of the corners. The average axial shortening was also measured independently through a displacement transducer coupled to the axial loading pad and a recorder. Gauges 5-8 were placed mid-way between the end-pieces on the edges of the panels in the directions shown to monitor the out-of-plane edge displacement of the walls.

The instrumented models were placed in turn between the compression jaws of a 60 tons Amsler Universal testing machine geared to a hydro-pacer which acted as a load main-

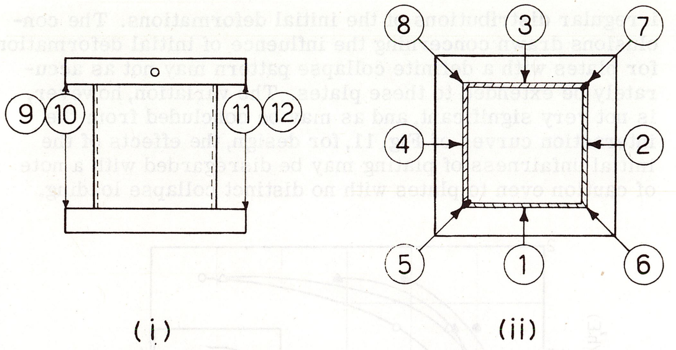

Fig. 4 Position of dial gauges
(i) Elevation showing dial gauges 9-12 measuring shortening of distance between tips of end-pieces
(ii) Section showing dial gauges 1-4 at centre of the panels and dial gauges 5-8 at mid-point of the edges

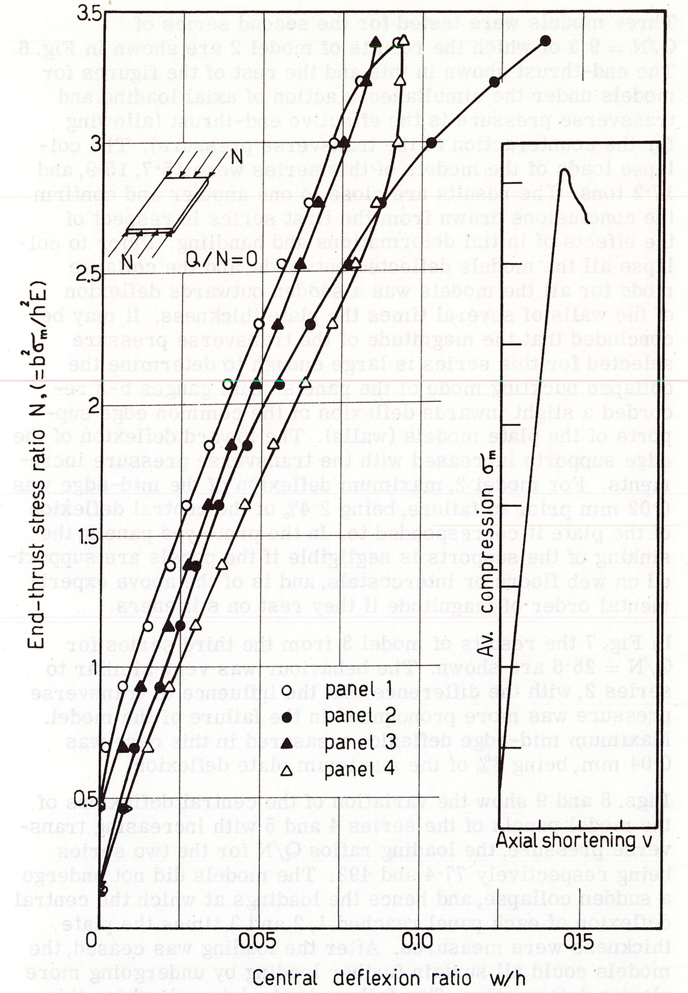

Fig. 5. Model 1 under end-thrust alone. Variations of panels' central deflexions with end-thrust, and of axial shortening with end-thrust

tainer. The internal pressure was applied by a separate Amsler dynamometer which consisted of an oil pump and a pressure measuring device.

For models under combined loading, the transverse pressure and the end-thrust were increased simultaneously keeping the predetermined ratio constant throughout the experiment.

Rate of loading was about two minutes for each load increment and between 10 to 17 increments were made for each model to failure.

8. RESULTS

To obtain an ultimate strength load interaction curve between transverse pressure and end-thrust for $b/h = 50$ plates, six ratios of transverse pressure to end-thrust, Q/N, were selected, namely 0, 9·5, 25·6, 77·4, 492 and −201. The first refers to axial loading only and the last to transverse pressure, which is coupled with an axial tension due to pressurised oil.

For each ratio a series of at least three box models were tested giving results for a minimum of 12 plate models for any load combination. The behaviour of one model from each series is shown separately in Figs. 5-10, and the interaction relationship obtained which incorporates the results of the 21 models tested (consisting of 84 plates) is given in Fig. 11.

The experimental results obtained are all presented in a non-dimensional form in order to increase the generality of the values given and to render reference and their comparison with other solutions readily possible. The actual measurements taken in each case can be evaluated from the definition of non-dimensional parameters given in the notation.

Figs. 5-7 relate to models 1-3 with Q/N ratios of 0, 9·5, and 25·6 and show the variation of panel's central deflexion with increasing end-thrust. At the right hand side of each figure, the variation of the average axial shortening of the four walls with the average axial load monitored by the displacement transducer and the recorder is reproduced. The end-thrust versus axial shortening of the models 1-3 shows the sudden collapse type of failure of the models. The central deflexions shown in the figures reflect the behaviour to the point of collapse. The deflexions after collapse were several times the plate thickness.

Five models were tested in the first series. Prior to collapse most of the models deflected outwards, despite the irregular distribution of initial deformations both inwards and outwards. The models all collapsed suddenly into a mode which in three cases was a regular pattern of two opposite walls bulging outwards and the other two dishing inwards. In the remaining two models the failure modes were complex and irregular with sides partly deflecting inwards and partly outwards. The collapse loads of the five models measured were 21·0, 21·5, 24·3, 23·4, and 21·1 tons each. The first three failure loads refer to the models with the regular collapse pattern. No consistent and obvious relationship was observed between the final mode at failure and initial out-of-plane deformations of the walls of the models. The initial deformations were considered to be too small to affect the ultimate behaviour, hence their measured distribution is not reported.

The crushing load of the plate panels calculated on the basis of total resisting cross sectional area times the yield stress equals 20·2 tons, showing that for square clamped plates with the experimental b/h ratio an ultimate strength close to the yield strength of the plate may develop. Other factors which may be considered to affect the strength are the residual stresses set up together with metallurgical changes due to welding. An investigation into the above effects (see the Appendix) revealed that welding and locked-in stresses did not reduce the ultimate strength of the plates under investigation, although they may affect the elastic behaviour and may cause premature yielding. The results of the investigstion imply that the slightly higher experimental values may be attributed to the outcome of welding and strain hardening.

The agreement between the ultimate strengths of the five models, as well as the calculated failure, is so close that for design of plates under the above conditions it may be assumed that the prevalent initial unfairness of plating, if within the range encountered in the experiment, and variations in the normal procedures of handling and workmanship do not affect the ultimate strength of the plates significantly.

Three models were tested for the second series of Q/N = 9·5 of which the results of model 2 are shown in Fig. 6. The end-thrust shown in this and the rest of the figures for models under the simultaneous action of axial loading and transverse pressure is the effective end-thrust (allowing for the counteraction of the transverse pressure). The collapse loads of the models of this series were 15·7, 15·9, and 17·2 tons. The results are close to one another and confirm the conclusions drawn from the first series in respect of the effects of initial deformations and handling. Prior to collapse all the models deflected outwards, and the collapse mode for all the models was a sudden outwards deflexion of the walls of several times the plate thickness. It may be concluded that the magnitude of the transverse pressure selected for this series is large enough to determine the collapse buckling mode of the panels. Dial gauges 5-8 recorded a slight inwards deflexion of the common edge supports of the plate models (walls). The inward deflexion of the edge supports increased with the transverse pressure increments. For model 2, maximum deflexion of the mid-edge was 0·02 mm prior to failure, being 2·4% of the central deflexion of the plate it corresponded to. In the prototype panels, the sinking of the supports is negligible if the panels are supported on web floors or intercostals, and is of the above experimental order of magnitude if they rest on stiffeners.

In Fig. 7 the results of model 3 from the third series for Q/N = 25·6 are shown. The behaviour was very similar to series 2, with the difference that the influence of transverse pressure was more pronounced in the failure of the model. Maximum mid-edge deflexion measured in this case was 0·04 mm, being 3% of the maximum plate deflexion.

Figs. 8 and 9 show the variation of the central deflexions of the model panels of the series 4 and 5 with increasing transverse pressure, the loading ratios Q/N for the two series being respectively 77·4 and 492. The models did not undergo a sudden collapse, and hence the loadings at which the central deflexion of each panel reached 1, 2 and 3 times the plate thickness were measured. After the loading was ceased, the models could all sustain further loading by undergoing more plastic deformation. The failure loads determined for this series, based on the deflexion criteria adopted, agreed less closely to one another compared with the models with a distinct collapse load. The variations may be attributed to the irregular distributions of the initial deformations. The conclusions drawn concerning the influence of initial deformation for plates with a definite collapse pattern may not as accurately be extended to these plates. The variation, however, is not very significant, and as may be concluded from the interaction curves of Fig. 11, for design, the effects of the initial unfairness of plating may be disregarded with a note of caution even to plates with no distinct collapse loading.

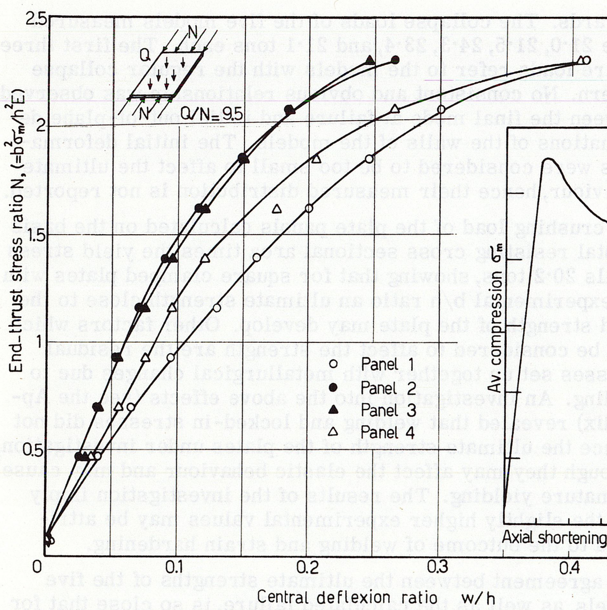

Fig. 6. Model 2 under combined end-thrust and transverse pressure (Q/N = 9·5). Variations of panels' central deflexions with end-thrust, and of axial shortening with end-thrust

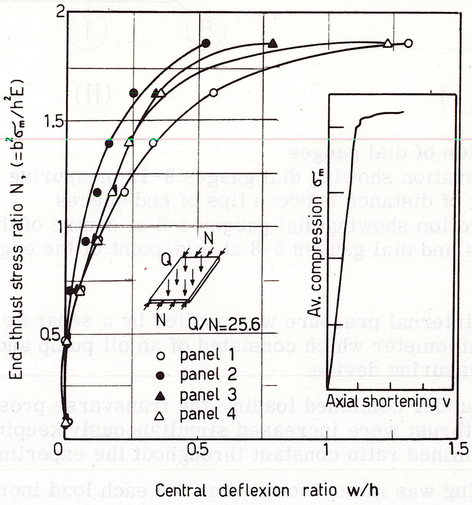

Fig. 7. Model 3 under combined end-thrust and transverse pressure (Q/N = 25·6). Variations of panels' central deflexions with end-thrust, and of axial shortening with end-thrust

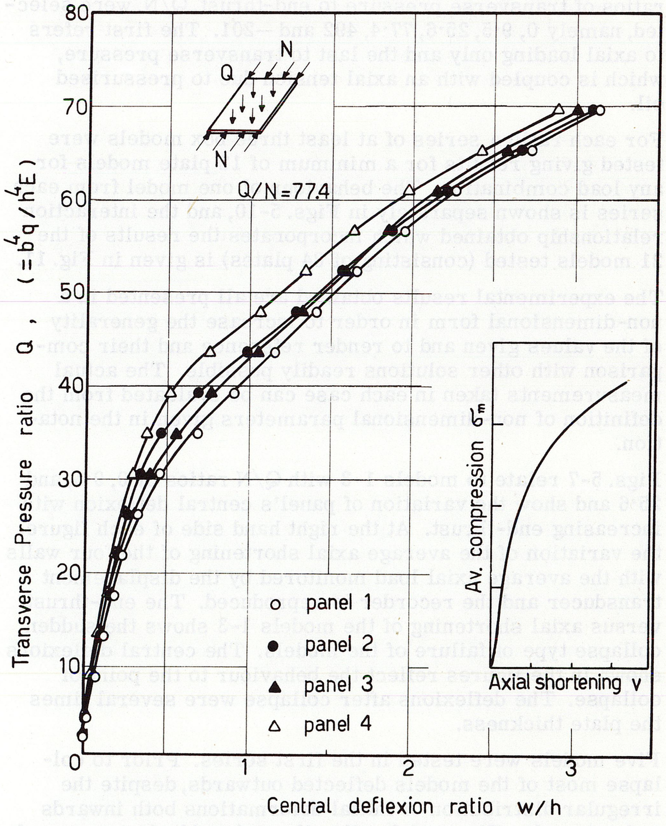

Fig. 8. Model 4 under combined end-thrust and transverse pressure (Q/N = 77·4). Variations of panels' central deflexions with end-thrust, and of axial shortening with end-thrust

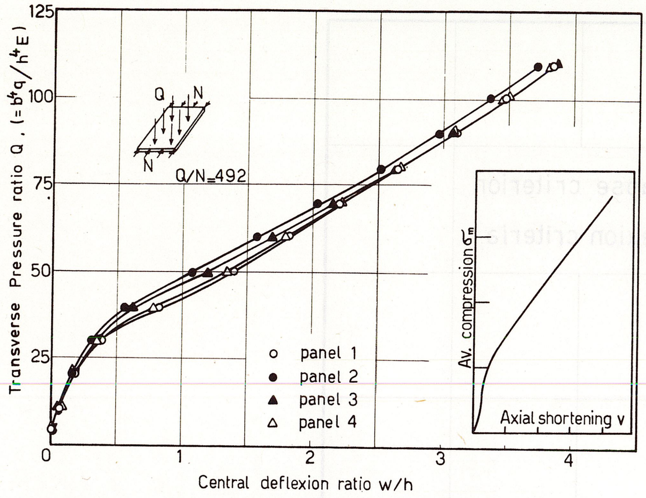

Fig. 9 Model 5 under combined end-thrust and transverse pressure (Q/N = 492). Variations of panels' central deflexions with end-thrust, and of axial shortening with end-thrust

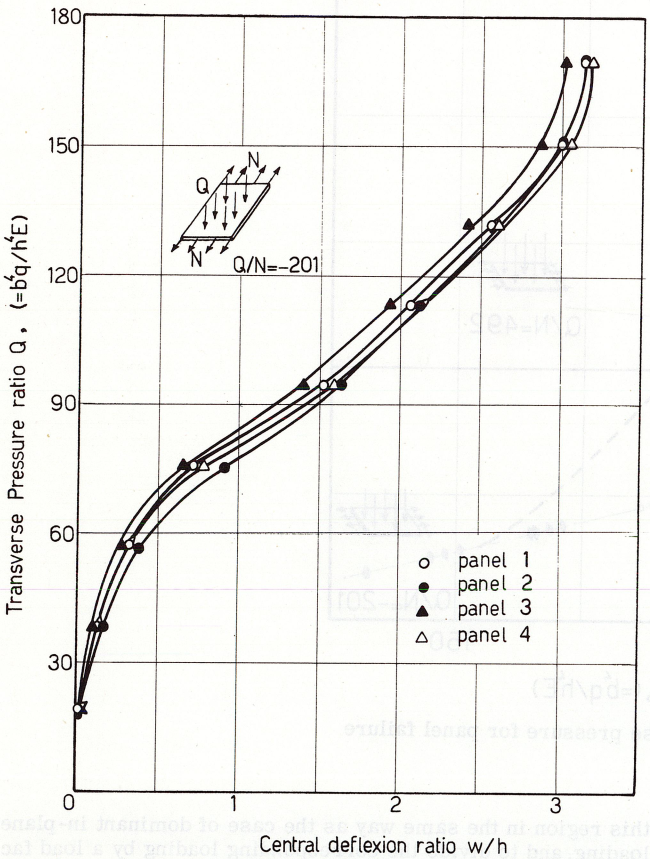

Fig. 10 Model 6 under transverse pressure and edge-tension (Q/N = −201). Variations of panels' central deflexions with transverse pressure

Fig. 10 refers to model 6 of the three models tested under transverse pressure alone. Due to the model geometry, the panels are under an average biaxial tension equal to Q/201. The variation of transverse pressure with the panels' central deflexion shows that for large-deflexions (w/h = 3) the panels became stiffer due to their behaviour in resisting the loading as a membrane skin. The collapse load of the model panels calculated on the basis of Prager's (11) formula, in which the membrane action is not accounted for is—

$$Q = \frac{b^4}{h^4E} \cdot \frac{12h^2}{b^2} \cdot \sigma_y = 36{\cdot}5$$

The comparison of the above value with the transverse pressure reached and shown in Fig. 10 indicates that Prager's collapse load for steel plates grossly underestimates the load capacity.

Fig. 11 shows the experimental failure interaction curves obtained for plates under in-plane loading and transverse pressure. The experimental results obtained form a consistent interaction curve, indicating that the major factors affecting the ultimate behaviour of plates are duly accounted for. For plates with a sudden collapse loading, the interaction curve may be closely approximated by a straight line as shown in Fig. 11. For the loadings which do not produce a clear collapse, the broken lines show the adopted failure criteria on the basis of the maximum plate deflexion. Depending on the load factor required for the design of the plating in mind, design curves may be drawn on the basis of similar interaction curves for plates under combined loading.

9. CONCLUSIONS

From a survey made of the methods available for the analysis and design of rectangular plates under the combined action of in-plane loading and uniform transverse it is concluded that:

(i) For the elastic range, the large-deflexion theory is fully developed and is being employed to investiagte the behaviour of plate panels under various loading and boundary conditions.

(ii) For the elasto-plastic behaviour of plates, there is a good background of theoretical work available accounting both for bending and membrane actions. But so far, due to the complexity of the formulations, there are no solutions offered. The analytical methods presented relate to simple cases with simplifying assumptions and do not represent the panel behaviour satisfactorily.

(iii) For the ultimate strength of plates there is no rigorous theoretical work developed in line with the yield line collapse theories of reinforced concrete slabs. The available methods are for transverse pressure alone which take either the membrane or the bending action into account, or for thin plates under edge-compression alone.

The strength of a plate is a function of its geometry (aspect ratio and thickness), its boundary conditions and loading, and its material properties (E, σ_y, and Poisson's ratio). Keeping all the above variables constant, for square panels with b/h = 50, a box model and test arrangement was successfully developed to represent closely the bottom and side plate panels of a ship. Twenty one identical box models each consisting of four panels were tested under a number of combinations of transverse pressure and edge-compression in order to investigate the interaction of the above loadings on the panels' ultimate strength. For plates under the combined actions of in-plane loading and transverse pressure, there is a range of combinations for which the in-plane loading is dominant, when with a proportional increase of loading the plate undergoes a sudden collapse. For this range of loading, the interaction relationship between the transverse pressure and in-plane loading may be closely approximated by a straight line (Fig. 11), in which an increase in transverse pressure corresponds to a decrease in the end-thrust carrying capacity of the plate. Plates falling in this region may be directly designed by dividing the failure loads obtained by a suitable load factor.

There is some evidence that contrary to the general opinion, the effect of residual stresses, variations in the normal procedures of workmanship, and initial deformations—if within the range encountered in the experiments—are not as crucial to the ultimate strength of clamped square plates. The results suggest that in certain cases the thermal effects of welding enhance the ultimate strength.

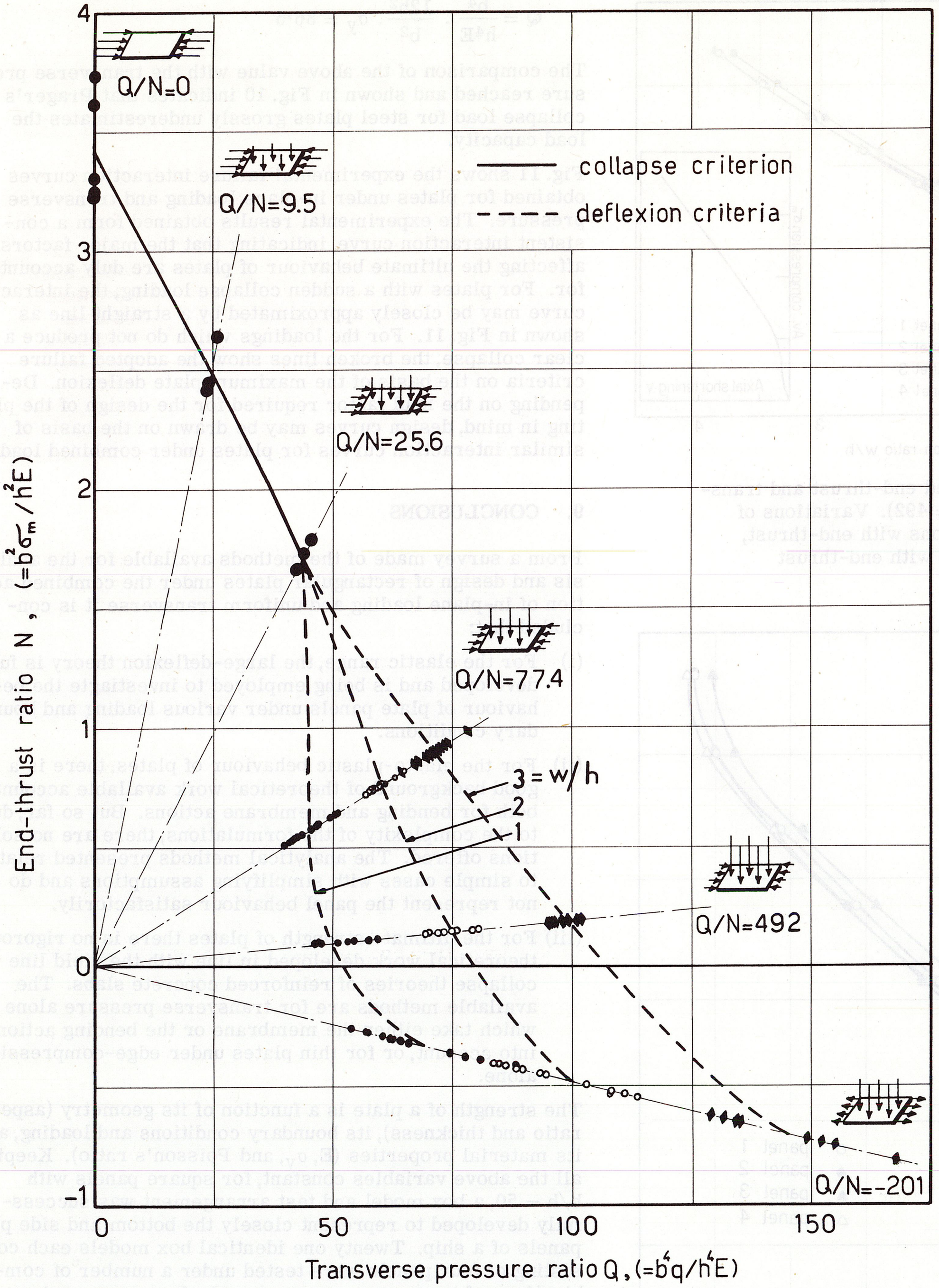

Fig. 11. Interaction curves between end-thrust and transverse pressure for panel failure

For combinations of loading when transverse pressure is dominant (for b/h = 50 square panels Q/N > 25), the panels do not exhibit a distinct collapse loading. The panels dish away in resisting the applied loading through continued plastic deformation, and a limit to central deflexion under the applied loading is proposed for consideration as a design criterion. Under very large deflexions (deflexions about three times the plate thickness), the panels behave as a membrane and the deflexions reached are a measure of the plates' reserve of strength mobilised through plastic flow in resisting the applied loading. The central deflexions are on this ground considered to be a measure of plate's strength. The results do not reflect the ultimate strength, which is thought to be significantly larger than the loads reached, especially in the regions of low edge-compression. For design, depending on the amount of deflexion permissible for the panel in mind, it is proposed to treat the interaction curves obtained for this region in the same way as the case of dominant in-plane loading, and to divide the corresponding loading by a load factor to obtain the permissible design load.

10. FUTURE WORK

For future work, on the theoretical side, it would be a major improvement to the present standing of the analytical data on the subject to (i) start from the elasto-plastic large-deflexion plate theories, which account for the geometrical non-linearity as well as material non-linearity of the plate and obtain solutions of interest to ship designers, and (ii) to develop an independent ultimate strength analysis based on the assumption of appropriate failure mechanisms allowing for the sheets biaxial bending and membrane actions.

On the experimental side, the present line of experiments may be extended to cover the breadth to thickness ratios of 20 to 120 for aspect ratios 1/1 to 3/1, and also for several values of E/σ_y, in order to provide the designer with a complete range of interaction curves for the strength design of plates.

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APPENDIX—RESIDUAL STRESSES

Residual stresses present in the prototype panels are those due to handling of the sheets and those due to welding. Thermal metallurgical effects due to welding and thermal residual stresses are concentrated around the weld seams. Their influence depends on the welding technique and the material deposited.

The presence of residual stresses may result in a premature local yielding of the plate and may affect its behaviour within the elastic range. But for ultimate strength of the panels, where the sheets undergo large plastic deformations the initial residual stresses within the regions of large plastic deformations will be ironed out by compressive or tensile biaxial stresses. Provided the plate remains stable, its full strength will be developed.

For a quantitive assessment of the effects of the thermal residual stresses on the ultimate strength of the models the following tests were carried out on two groups of models.

GROUP 1. Two completely made and welded models were annealed in an electric oven at 650°C for 12 hours. The models were then cooled down in the switched off oven to room temperature in 24 hours.

GROUP 2. Two models with the four walls prepared ready to be welded to the end-pieces were heat treated in the same oven. After removal of the walls from the oven they were

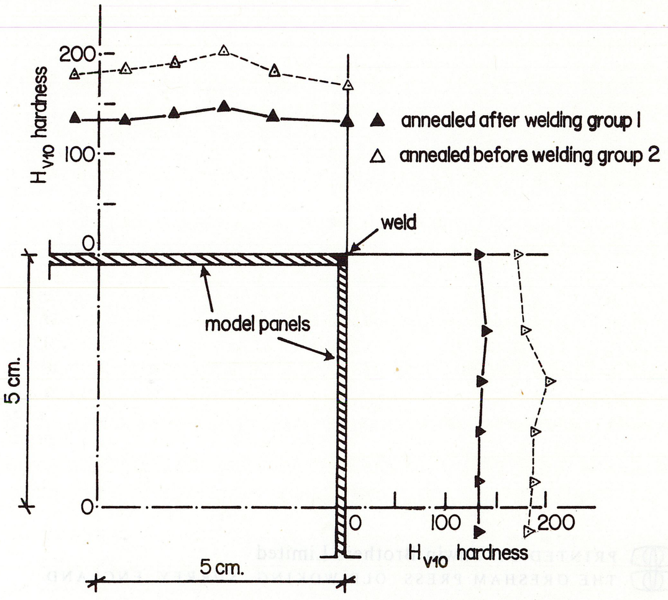

Fig. 12. Box model cross section, showing the H_V hardness measured on first quadrant of the model

welded to the end-pieces in the same manner as group 1 models were welded prior to the heat treatment.

All the models were tested under axial loading. The average failure loads of the models of group 1 were 93% of group 2 models. The difference between group 1 and group 2 models is the application of weld in group 1 before, and in group 2 after the heat treatment. Group 2 models exhibit the combined effects of thermal residual stresses and the metallurgical changes due to welding. The materials of both groups are softened due to annealing. The welding of group 2 models after heat treatment and its cooling in the open air has resulted in a hardening effect of the weld seam and an increase in the yield stress. As a result the models have collapsed under a slightly higher load despite the existence of thermal residual stresses.

The above conclusion is in agreement with Clarkson's observation(15) that when welding 'locked-in' stresses are set up, and often the sign is such that they give a larger elastic range.

In Fig. 12 the H_V (10 kg) hardness of a model of each group was measured, after testing, across a cross section of the models perpendicular to the applied end-thrust. The measurements were taken on cut out and polished pieces of the models using a Wolpert Standard Hardness Testing Machine. The effect of weld in raising the H_V hardness is clearly shown for group 2 models. It should be noted that H_V is based on plastic deformation of the material and may be a measure of rupture stress, and not the yield point which is the cause of failure of the present models.

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