

## VIBRATION DESIGN OF CONCRETE FLOORS<sup>1</sup>

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**X.1 – INTRODUCTION**

This Technical Note outlines a practical procedure for the design and evaluation of concrete floors for vibration. The objective is to present a method of minimizing the effects of a floor's vibrations on its occupants and vibration-sensitive equipment.

The analytical background of vibrations is well understood. Detailed and rigorous computational tools can analyze the vibration characteristics and response of both simple and complex structures once the parameters of the structure are defined. The challenge in vibration design is the generally poor correlation between the analytical computations during design and the observed response of the constructed floor. There are inherent uncertainties in the material properties, damping characteristics, boundary conditions, the level of vibrations perceived by occupants, the vibration that the occupants consider objectionable, and the force and frequency of the exciting source. The selection of these parameters is subject to engineering judgment and prone to large variations.

Once a floor is constructed, its response can be measured with a high degree of accuracy by commercially available instruments. Nevertheless, it is often necessary to have a reasonable estimate of a floor's vibration response prior to construction. The information that follows provides a practical and expeditious procedure for estimating a floor's vibration and minimizing its undesirable effects.

The vibration response of a concrete floor can be of interest in a variety of situations including:

- ❖ The consequence of foot fall on residential and commercial floors
- ❖ Rhythmic vibrations, such as on dance floors and at sport events
- ❖ Vibration in laboratories and sensitive manufacturing facilities
- ❖ Vibrations due to vehicular traffic outside a facility
- ❖ Vibrations due to machinery
- ❖ Transient impulse due to earthquake, wind or impact loads

This Technical Note covers three sources of vibration and their evaluation:

- ❖ Vibrations due to footfall of individuals in residential and commercial facilities
- ❖ Vibrations resulting from rhythmic group activities, such as dancing and aerobics
- ❖ The impact of vibrations arising from footfall and rhythmic group activities on the operation of sensitive laboratory equipment

**X.2 – THE SIX-STEPS OF DESIGN FOR VIBRATION**

Regardless of the cause of vibration and the objective of the analysis, vibration analysis can be broken down into the following six steps.

- 1 : Determine the governing natural frequency of the floor system;
- 2 : Identify the exciting force and estimate its magnitude;
- 3: Define the floor's dynamic parameters;
- 4: Select and calculate the applicable weight of the vibrating floor;
- 5: Calculate the floor's peak acceleration;
  - If the objective is to evaluate the impact on sensitive equipment, the maximum induced velocity must also be calculated; and
- 6: Compare the floor's vibration response to acceptable thresholds.

Several of these steps, such as determining the natural frequencies and defining the relevant parameters, are common for all applications. In the following discussion, these common aspects are addressed first, followed by specifics for the different sources of vibration. Numerical examples are provided to clarify the calculations.

### **X.3 STEP 1 – NATURAL FREQUENCIES**

The key step in the vibration design of a concrete floor is determining the floor's natural frequencies and mode shapes, particularly the frequencies that excite the region of interest. Concrete floors are distinctly different from steel-framed floors with respect to vibration design. In contrast to steel-framed floors, which are made up of discrete skeletal members such as beams and girders, concrete floors are a continuum. Although there is a great deal of literature on techniques for estimating the vibration of simply-supported discrete skeletal members, these techniques are not applicable to concrete floors.

Most of the techniques for design and analysis of steel-framed floors are based on the premise that, as shown in part (a) of Fig. X.3-1, a simply-supported member's first mode of vibration closely matches its deflected shape under self weight. This premise generally does not apply to concrete floors that are continuous over their supports. As shown in part (d) of Fig. X.3-1 the displacement shape for an interior span of a concrete floor under self weight is analogous to that of a single panel fixed at its supports (part b). However, the first mode of vibration, shown in part (c), suggests that the vibration response of the floor is closer to that of a single panel simply supported along its four sides (part a). Thus for concrete floors, the displacement under self weight does not necessarily reflect the shape of the floor's first mode of vibration. Formulas that express the vibration of a member in terms of its deflection under self-weight, while appropriate for steel-framed floors, are not applicable to concrete floors.

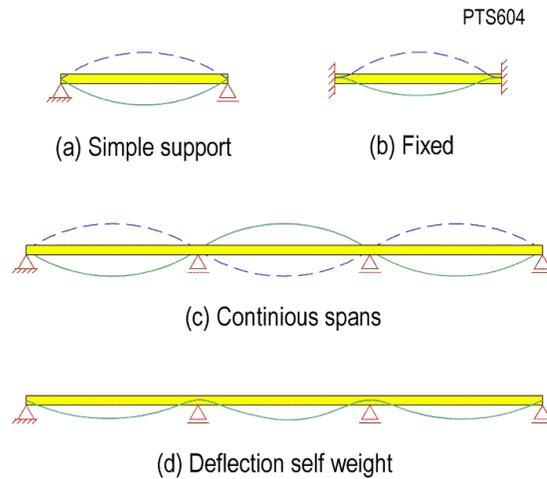


FIGURE X.3-1 First Mode Shapes and Deflection of simple and Continuous Spans

Where columns are arranged on a regular orthogonal grid, the first natural mode shape of a two-way concrete slab is likely to be in the form of a one-way slab deflecting in a cylindrical form. (Fig. X.3-2). The displacement will be along the longer span for rectangular grid column layout.

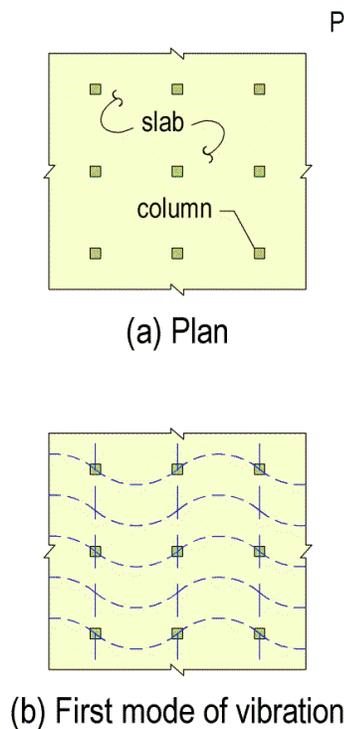


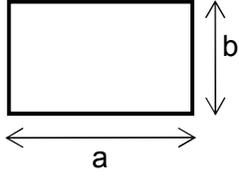
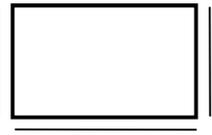
FIGURE X.3-2 First Mode of Vibration of a Concrete Floor on a Regular and Orthogonal Array of Supports

As discussed below, there are several ways to determine the natural frequencies and associated mode shapes of concrete floors.

### X.3.1 Floors with Regular Geometry

Tables available in the literature, such as the one reproduced in Table X.3.1-1 [Bares, 1971], can be used to calculate the first natural frequency of a panel in a floor of uniform thickness resting on regularly-spaced supports. Table X.3.1-1 includes the different boundary conditions and various aspect ratios for a single panel of a floor system.

TABLE X.3.1-1 First Natural Frequency Constant  $\varphi$  for Rectangular Slab Panels of Uniform Thickness (T183)

Case	Boundary conditions	Constant $\varphi$
1		$\varphi = 1.57(1 + \gamma^2)$
2		$\varphi = 1.57\sqrt{1 + 2.5\gamma^2 + 5.14\gamma^4}$
3		$\varphi = 1.57\sqrt{5.14 + 2.92\gamma^2 + 2.44\gamma^4}$
4		$\varphi = 1.57\sqrt{1 + 2.33\gamma^2 + 2.44\gamma^4}$
5		$\varphi = 1.57\sqrt{2.44 + 2.72\gamma^2 + 2.44\gamma^4}$
6		$\varphi = 1.57\sqrt{5.14 + 3.13\gamma^2 + 5.14\gamma^4}$

$$\gamma = a / b$$

Using Table X.3.1-1, the first natural frequency,  $f$ , of the panel in Hz is calculated as:

$$f = \frac{c}{a^2} \phi \quad (\text{Exp X.3.1-1})$$

Where,

$$c = \sqrt{\frac{Eh^3}{12(1-\nu^2)}} \times \frac{g}{q} \quad (\text{Exp X.3.1-2})$$

$a$  = longer panel dimension;

$b$  = shorter panel dimension;

$E$  = dynamic modulus of elasticity of concrete [1.2 static  $E_c$ ];

$f$  = first natural frequency [Hz];

$g$  = gravitational acceleration [32.2 ft/sec<sup>2</sup> ; 9.81 m/sec<sup>2</sup>]; and

$h$  = slab thickness [in; mm];

$q$  = weight per unit surface area of the slab [psi; N/mm<sup>2</sup>].

$\nu$  = Poisson's ratio [0.2];

Consistent units, either in lb,in or N,mm must be used for the calculations.

Alternatively, the following relationship can be used to calculate the first natural frequency of a simply supported panel – or a central panel of a floor on regular supports [Szilard, 1974].

$$f = \frac{\omega}{2\pi} \quad (\text{Exp X.3.1-3})$$

Where,

$$\omega = \pi^2 \left( \frac{1}{a^2} + \frac{1}{b^2} \right) \sqrt{c} \quad (\text{Exp X.3.1-4})$$

The above approximations are appropriate for floors on regularly arranged supports, such the floor shown in Fig. X.3-2a. The first natural frequency of a floor like this with long, narrow panels (“ $a$ ” much larger than “ $b$ ”) is likely to be as shown in Fig. X.3-2b, with displacements in the form of cylindrical waves. The panels can be modeled as simply supported along the long dimension, which means that Equation X.3.1-3 reduces to:

$$f = (1.57 / b^2) \sqrt{c} \quad (\text{Exp X.3.1-5})$$

### X.3.2 Floors with Irregular Geometry

Driven by higher-strength materials, greater analytical capabilities, and architectural aspirations, along with flexibility of concrete to assume any shape, floor plans are increasingly featuring thin slabs, long spans, and irregular support layouts. This places greater emphasis on the floor's vibrational response, and may make it necessary to consider higher modes of vibration, in order to capture the governing design values.

Consider a typical floor of the multistory building shown in Fig. X.3.2-1. Let the area of interest for vibrational response be the panels at the bottom left of the floor.

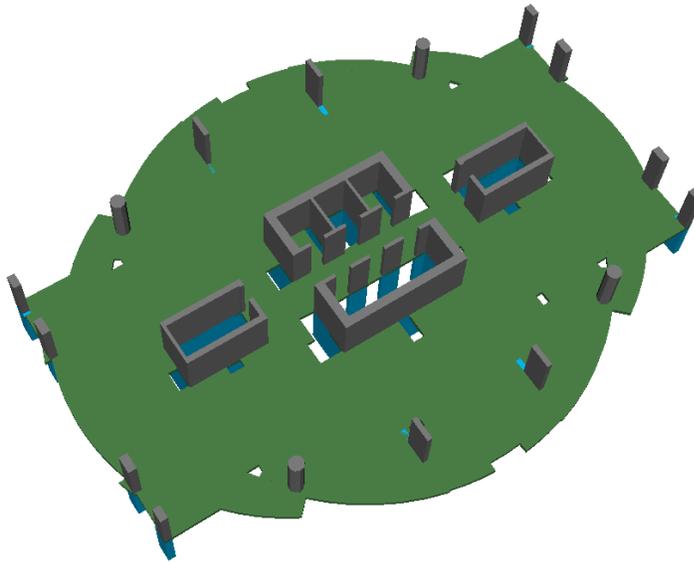


FIGURE X.3.2-1 View of Typical Floor (Jaleh – P736)

The vibration analysis of this floor results in the natural mode shapes for several of the lower frequencies as shown in Fig. X.3.2-2. Note that it is the fifth mode of the floor's natural vibration that excites the area of interest. For irregular floors it may be necessary to obtain several frequencies and their associated mode shapes, in order to determine the governing mode shape and frequency

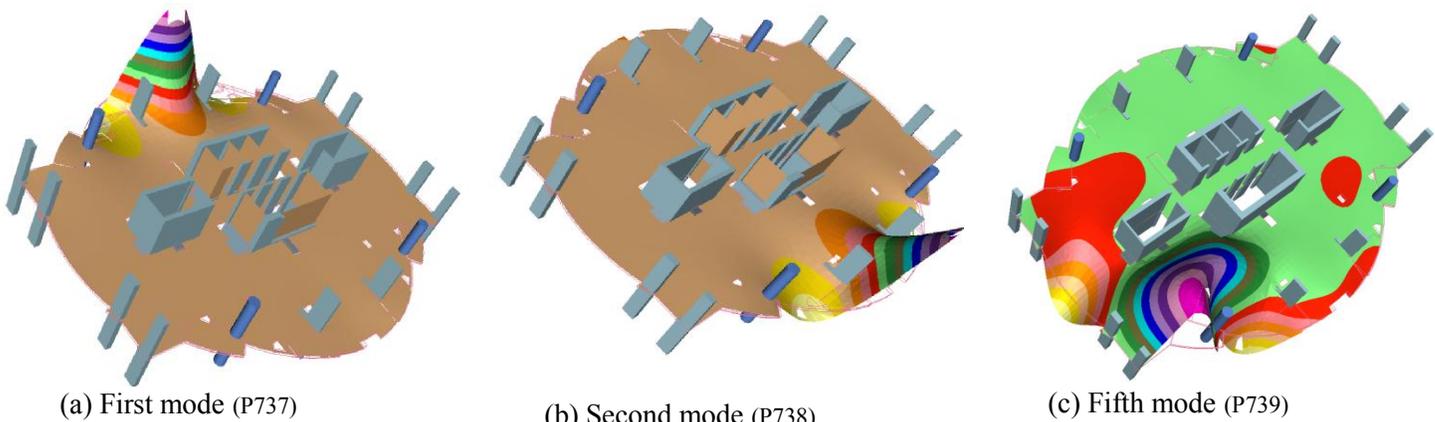


FIGURE X.3.2-2 View of a Typical Floor (Jaleh floor)

### X.3.3 Analytical Requirements for Floors with Irregular Geometry

Using the terminology of the Finite Elements Method, concrete floors form a continuum and act as a “plate” element as opposed to a “stick” element. Thus, the internal formulation of the software used for the analysis of concrete floor vibrations should be based on plate assumptions. [Aalami, 2014]. Finite element formulations that model a concrete floor slab as a grid or use other variations of stick elements are not adequate. Also, it is not necessary to use shell elements for vibrational analysis of floor slabs, membrane effects become important for thin metal plates with span to depth ratio of 80 or above [Aalami, 1984].

### X.3.4 Sub-Structuring for Vibration Analysis

When the area of interest is limited to a small part of the slab, as in the preceding example, the computational effort can be reduced by modeling just that part of the slab. To arrive at a reliable response, the sub-region modeled should extend beyond the area of interest by at least one panel in each direction. This will minimize the impact of the boundary conditions at the perimeter of the sub-region. This technique is illustrated by an example in Section X9.1B.

## X.4 STEP 2 – IDENTIFY THE EXCITING FORCE AND ESTIMATE ITS MAGNITUDE

The primary sources of vibration are the foot fall of walking individuals and the rhythmic action of a group.

### X.4.1 Foot fall

In the absence of more detailed information, a pace of two steps per second (2 Hz) is recommended when doing a vibration analysis for foot fall. For an open plan office, the average pace of an individual is generally about 2.1 steps per second (2.1 Hz). This is a reasonable upper limit for the design of corridors and large circulation areas. For low-height partitioned office spaces and labs, a pace of 1.8 Hz is recommended [TR43, 2005]. Walking rates above 2.5 Hz are uncommon.

Although the force of vibration depends on the individual’s style of walking, it is generally estimated as a fraction of their weight, based on their pace. Figure X.4.1-1 offers an approximate guideline adapted from reference [TR43, 2005]. The graph is used to determine the fraction of the individual’s weight, expressed in terms of a Dynamic Load Factor (DLF).

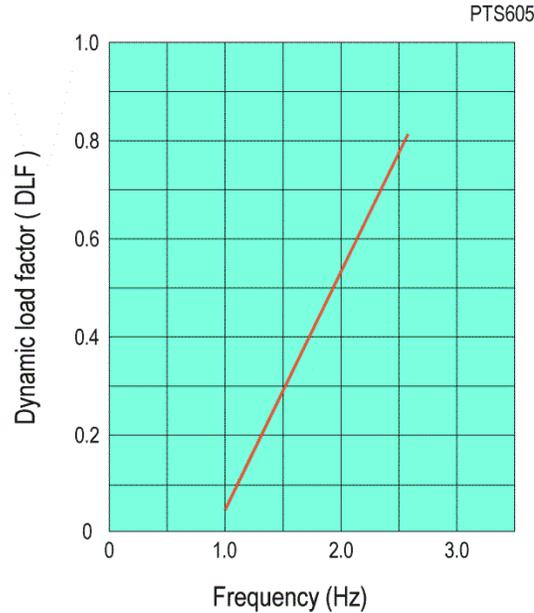


FIGURE X.4.1-1 Dynamic Load Factor for First Harmonic of Walking Force

As an example, for an individual weighing 150 lb (667 N), the associated DLF from Fig. X.4.1-1 is 0.53. This translates to an exciting force  $P_o$  of:

$$P_o = 0.53 \times 150 = 79.5 \text{ lb (354 N)}$$

#### X.4.2 Rhythmic Activities; Aerobics

Design for vibrations caused by rhythmic activities is summarized in reference [Naeim, 1991]. The following is an excerpt from Naeim's work, extended and modified to be compatible with the empirical relationships used in this Technical Note for concrete floors.

Coordinated rhythmic activities, such as dancing, audience participation in arenas and concert halls, and most importantly aerobics, can result in undesirable levels of vibration. Multi-purpose facilities that have an aerobics gym and office space on the same floor pose the most difficult vibration design task. For rhythmic activities, resonant or near resonant behavior can result in significant dynamic amplification and hence human discomfort.

Table X.4.2-1 provides suggested parameters to use for assessing vibrations from rhythmic activities. The last column gives the unit force for the effective floor area. The effective floor area is defined in Section X.6.

TABLE X.4.2-1 Suggested Design Parameters for Rhythmic Activities (T184)

Activity	Forcing frequency Hz	Weight of participants $w_p$ (psf) (kN/m <sup>2</sup> )	Dynamic load factor $DLF$	Dynamic load intensity $P_o = DLF w_p$ psf (kN/m <sup>2</sup> )
Dancing	1.5 – 3.0	11.1* (0.531)	0.5	5.55 (0.266)
Lively concert or sports event	1.5 – 3.0	30.0** (1.44)	0.25	7.50 (0.359)
Aerobics	2.0 – 4.0	3.6*** (0.172)	1.5	5.40 (0.258)

Average weight of a person assumed 150 lb (79.5 kg)

\* assuming each dancing couple taking 27 ft<sup>2</sup>; (2.51 m<sup>2</sup>);

\*\* assuming 5 ft<sup>2</sup>; (0.464 m<sup>2</sup>) per person

\*\*\* assuming 42 ft<sup>2</sup>; (3.90 m<sup>2</sup>) per person

### X.4.3 Impact of Foot Fall on Sensitive Equipment

The National Institutes of Health [NIH, 2012] recommends using the following walking frequencies to evaluate the impact of vibration resulting from foot fall on sensitive equipment in hospitals or laboratories.

- ❖ Walking pace for a closed corridor (walls on both sides and doors on either or both sides) - 1.5 Hz
- ❖ Walking pace for open or “ghost” corridor (wall on one side, with or without doors, or end of laboratory benches) - 1.25 Hz
- ❖ Walking for cross aisles (between laboratory benches) - 1 Hz.

From Fig. X4.1-1, the exciting force for a 150-pound individual walking at 1.5 steps per second in a corridor next to a laboratory would be:

$$P_o = 0.3 \times 150 = 45 \text{ lb (0.20 kN)}$$

## X.5 STEP 3 – SELECT THE FLOOR’S DYNAMIC PARAMETERS

The parameters of a concrete floor that impact its vibration response are its geometry, boundary (support) conditions, weight, modulus of elasticity, damping, post-tensioning, and cracking.

### X.5.1 Geometry and Boundary Conditions

Since concrete floors are a continuum, beams, openings, changes in thickness, steps and other features that define the geometry of a floor will impact the floor’s stiffness and need to be faithfully modeled for the vibration analysis to arrive at reliable frequencies and mode shapes. If the model used for the analysis is limited to one level of a multi-story building, it is only necessary to include one level of supports below the floor and one level above. The connections to the floors above and below can be considered rigid on the assumption that connections between the supports and floor slabs are monolithic.

Generally, it is not necessary to include the post-tensioning tendons in the analytical model of a post-tensioned slab unless the analysis is intended to account for slab cracking. The impact of cracking is discussed in Section X.5.10.

### X.5.2 Weight

Force of vibration is related to the “mass” of the vibrating object which is expressed as ( $W/g$ ), where “ $W$ ” is the weight of the slab plus any object fixed to the floor that will follow the floor’s displacement, and “ $g$ ” is the gravitational acceleration taken as  $32.2 \text{ ft/sec}^2$  ( $9.81 \text{ m/sec}^2$ ). Applied forces that do not affect the structure’s stiffness are not included. In addition, the mass of any vehicles parked on a slab is not included, since when excited, the inflated tires are likely to cause the vehicles to vibrate in a different mode than the supporting slab. The force of a footfall will only excite a limited area of a large floor system. Section X.6 defines the floor area to be used for calculation of the effective weight in the peak acceleration formula

### X.5.3 Modulus of Elasticity

Concrete’s modulus of elasticity for vibration (its dynamic modulus) is higher than its modulus of elasticity under static loads, in particular when high strength concrete is used. For vibration analysis, a value 20 % to 25% higher than the static modulus is recommended.

### X.5.4 Damping

Damping has an inherently high variability and is difficult to determine before a floor is placed in service. The recommended values from reference [Allen, D.E., et al] are 2 to 3% for bare concrete floors and 5 to 8% for occupancies with full height partition walls. Damping factors suggested in this reference are listed in Table X.5.4-1.

TABLE X.5.4-1 Recommended Damping Factors for Various Occupancies (T185)

Occupancy	Damping factor $\beta$
Bare concrete floor	0.02
Furnished, low partition	0.03
Furnished, full height partition	0.05
Shopping malls	0.02

### X.5.5 Post-Tensioning

When there is axial compression from an externally applied force (Fig. X.5.5-1b), an out-of-plane displacement of the compressed member will result in an applied moment ( $P\Delta$ ) in addition to the compression. The applied moment reduces the flexural stiffness of the member and consequently lowers its natural frequency. This phenomenon does not apply to a post-tensioned member, regardless of whether the post-tensioning tendon is internal or external to the member. The displacement of a post-tensioning tendon is locked to that of the member (Fig. X.5.5-1 c and d). Thus, when the member deflects, the member and the tendon displace by the same amount, keeping the eccentricity of the tendon constant. Hence, the vibration design of a post-tensioned member is the same as that of its conventionally-reinforced alternative. This approach is valid unless cracking is to be

accounted for. Post-tensioned slabs generally crack less and consequently exhibit greater stiffness when compared to similar conventionally-reinforced slabs.

*As stated by Pavic [Pavic et al, 2001]— modeling the effects of prestressing on concrete member, including floors is an area of confusion. Some authors, such as Clough and Penzien [Clough et al, 1993] maintain that prestressing, as a compressive axial load, has the ability to reduce natural frequencies. They argue that the second-order ( $P\Delta$ ) effect of the axial force reduces the elastic stiffness of the member. However, this phenomenon cannot be a result of axial forces from prestressing. In prestressed members, including floors, where the prestressing tendons are in intimate contact with the surrounding concrete, the tendon profile follows the deformed shape of the member. No additional eccentricity between the tendon and the centroid of the member can be created. This applies to both bonded and unbonded post-tensioning.*

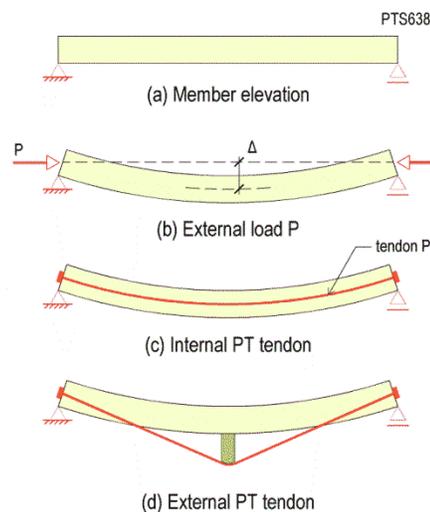


FIGURE X.5.5-1 P-delta Effect for PT and Externally Loaded Members

### X.5.6 Extent of Cracking

Cracking in concrete reduces the flexural stiffness of a floor and, consequently, lowers its natural frequencies. In conventionally reinforced concrete design, cracking is generally allowed. For span-to-depth ratios of 30 or larger, a 25% reduction in stiffness is reasonable.

Under ACI 318, allowable in-service tensile stresses for post-tensioned, two-way floors are somewhat low, leading to designs with no or limited cracking. Hence, no reduction in stiffness is required.

Designs based on the European code EC2 [EC2, 1994] and other major non-US codes allow cracking in post-tensioned floors under service conditions. Depending on the “design crack width”, a reduction in stiffness for vibration analysis may be necessary. In the absence of more detailed information, a reduction in stiffness of 25% can be assumed for a design crack width of 0.2 to 0.3 mm (0.008 – 0.012 in.)

## X.6 STEP 4 CALCULATE THE APPLICABLE WEIGHT OF THE VIBRATING FLOOR

The empirical formulas that are used to estimate the resulting peak acceleration of a floor were derived for steel floors consisting of linear members serving a relatively narrow tributary. When applying the empirical formulas discussed in Section X.7 to concrete floors, there needs to be an adjustment to the effective panel width for weight computations. In theory, the extent of the area that will be mobilized by an exciting force depends on the magnitude of the force. A foot fall excites a smaller area than a hall filled with a group engaged in aerobic activity. However, for the empirical formula, the area of floor used to calculate peak acceleration should be the panel where the exciting force is applied, as shown in Fig. X.6-1

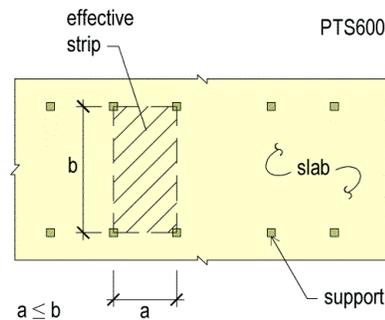


FIGURE X.6-1 Effective Strip for Evaluation of Peak Acceleration (PTS600)

## X.7 STEP 5 CALCULATE PEAK ACCELERATION AND MAXIMUM VELOCITY

When the objective of a design or investigation is to determine the occupants' perception of vibration and their objection to its intensity, the floor's peak acceleration ( $a_p$ ) resulting from the foot fall and the associated frequency ( $Hz$ ) are used. However, when evaluating the impact on equipment that is sensitive to vibration, the maximum velocity from the floor's vibration must be considered.

### X.7.1 Peak Acceleration

A floor's peak acceleration is derived from its natural frequency and the exciting force, among other parameters. The following is the commonly used relationship [ATC, 1999; AISC/CISC 1997]:

$$\frac{a_p}{g} = \frac{P_0 e^{-0.35f}}{\beta W} \quad (\text{Exp X.7.1-1})$$

Where,

- $a_p$  = peak acceleration;
- $f$  = natural frequency;
- $g$  = gravitational acceleration [32.2 ft/sec<sup>2</sup>; 9.81 m/sec<sup>2</sup>];
- $P_0$  = the exciting force;
- $W$  = effective weight of the panel and the superimposed dead load; and

$\beta$  = modal damping ratio.

### X.7.2 Maximum Velocity

The following equation is used to calculate the maximum velocity from an excitation:

$$v = \frac{a}{2\pi f} \quad (\text{Exp X.7.2-1})$$

Where,

$a$  = peak acceleration [  $a_p$  ] ;

$f$  = frequency; and

$v$  = maximum velocity.

The values of peak acceleration and frequency are those determined from the preceding Sections.

## X.8. STEP 6 – EVALUATION

The procedure for evaluating floor vibrations depends on whether the objective is determine the occupant's perception of the vibration and objection to it, or the impact on vibration-sensitive equipment.

### X.8.1 Perception and Objection to Vibration

**A. Foot fall of Individuals:** In general, humans are most sensitive to vibration for frequencies between 4 to 8 Hz; larger accelerations can be tolerated at higher or lower frequencies. Perception of vibration, and whether or not it is annoying or objectionable is highly subjective and values vary from reference to reference. The Canadian Steel Code [CAN3-S16.1-M84, 1984] defines the “perceptibility” for the common case of foot fall for various levels of floor damping, as shown in Fig.X.8.1A-1. The Applied Technology Council [ATC, 1999] has published a number of graphs showing the threshold of human sensitivity to vertical vibration; the graph for foot fall is reproduced in Fig. X.8.1A-2. The ATC graphs are more commonly used than the Canadian Steel Code. Other references state somewhat different values.

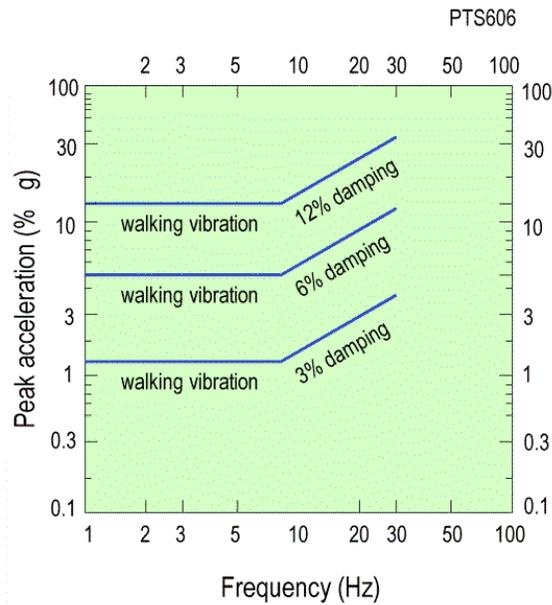


FIGURE X.8.1A-1 Vibration Perception Scale for Foot Drop (CAN3)

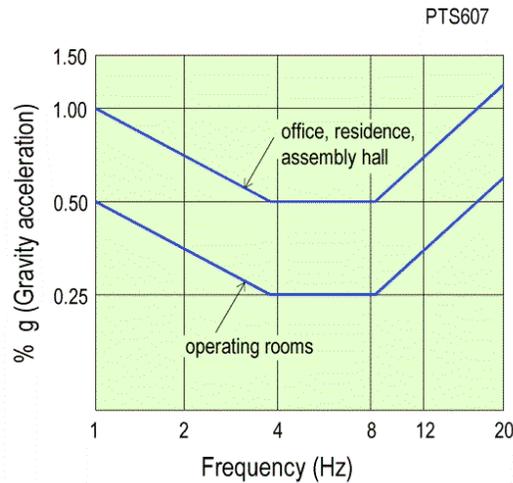


FIGURE X.8.1A-2 Threshold of Human Sensitivity to Vertical Vibration; (ATC)

A less frequently used equation is that suggested by Allen and Murray [Allen, D.E., et al 1993]. The equation calculates the minimum natural frequency ( $f_{min}$ ) that is required to avoid a perception of disturbing vibration caused by walking. The equation is:

$$f_{min} \geq 2.86l_n \left( \frac{K}{\beta W} \right) \quad (\text{Exp X.8.1-1})$$

Where,

$f_{\min}$  = minimum frequency of the floor system;

$K$  = a constant, given in Table X.8.1-1;

$l_n$  = natural logarithm;

$W$  = effective weight of area of floor panel affected by the point load (heel drop); and

$\beta$  = modal damping ratio [Table X.5.4-1].

The above relationship is applicable to floors framed with steel members, since as discussed in section X3.2, the first natural frequency will not necessarily govern for concrete floors on irregular supports .

TABLE X.8.1A-1 Constant  $K$  for Minimum Acceptable Frequency (T186)

Occupancies	$K$	
	kip	kN
Offices, residences, assembly halls	13	58
Shopping malls	4.5	20

**B. Foot fall from Group Activities:** Higher accelerations can be tolerated for collective activities, such as dance and aerobics. The recommended evaluation chart by ATC is shown in Fig. X.8.1B-1. Use of this chart is illustrated in Section X.9.5B.

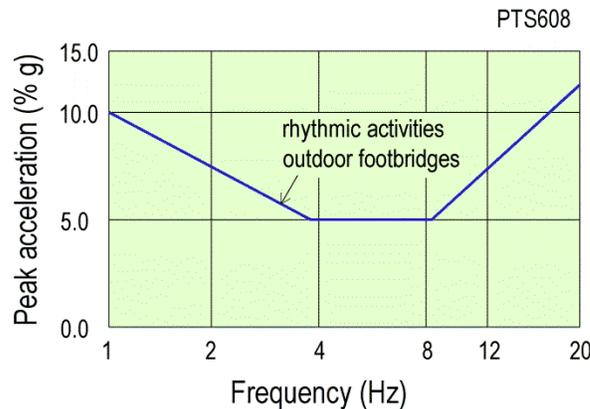


FIGURE X.8.1B-1 Threshold of Human Sensitivity to Vertical Vibration From Rhythmic Activities (ATC)

**X.8.2. Evaluation of Foot Fall on Vibration-Sensitive Equipment**

The National Institutes of Health [NIH, 2012] recommend acceptable maximum velocity values for proper operation of critical hospital spaces and laboratory rooms. An excerpt from the NIH recommendations is listed in Table X.8.2-1.

TABLE X.8.2-1 Recommended Maximum Velocity Values (NIH, 2012) (T187)

Function	Maximum velocity micro meter/sec	Maximum velocity Micro in/sec
General laboratory	50	2000
Ordinary surgery	25	1000
Bench scope up to 100X mag	50	2000
Bench scope up to 400X mag	25	1000
Electron microscope greater than 30,000X; mass spectrometer	6	240
Electron microscope up to 30,000 mag	12	480

## X.9 NUMERICAL EXAMPLES

A single level of a multi-story concrete structure is used to discuss the various sources of vibration-inducing forces and their evaluation.

### X.9.1 Definition and Specifics of the Floor System

A three-dimensional view of the floor is shown in Fig. X.9.1-1. The panel selected for investigation is shown in Fig. X.9.1-2.

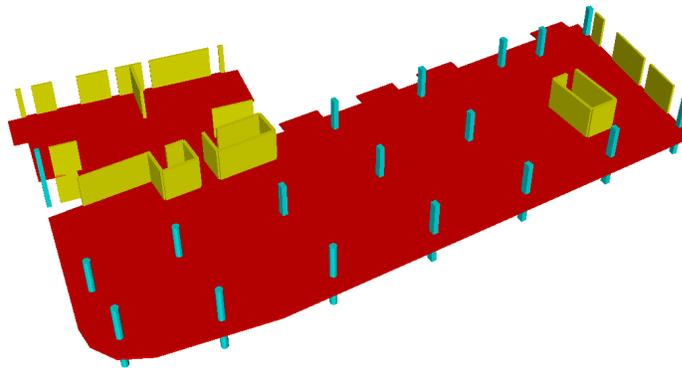


FIGURE X.9.1-1 A Typical Floor of Parissa Building (P250)

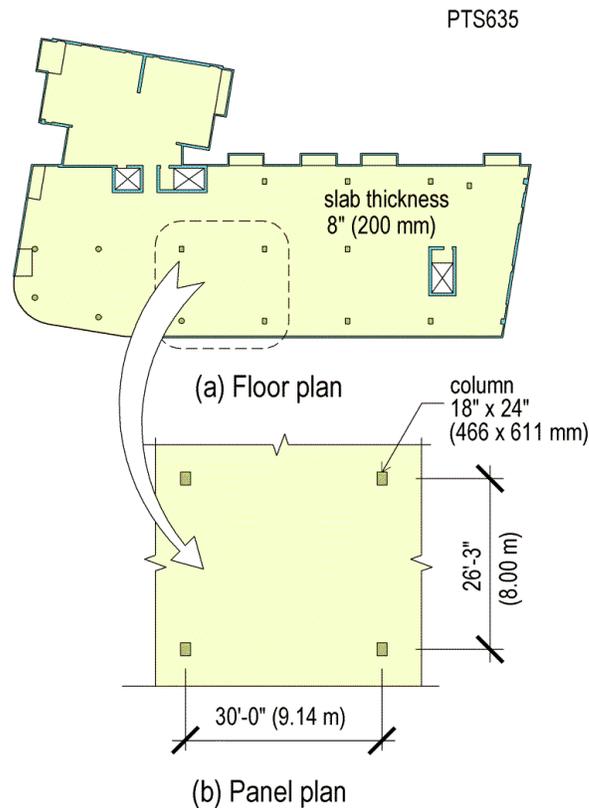


FIGURE X.9.1-2 Floor Panel Selected for Investigation

Other parameters of the floor are:

Slab thickness	= 8 in (203 mm)
Superimposed dead load	= 20 psf (1 kN/m <sup>2</sup> )
Concrete strength $f'_c$	= 5000 psi (33.8 MPa)
Poisson's ratio ( $\nu$ )	= 0.2
Unit weight of concrete ( $W_c$ )	= 150 pcf (2400 kN/m <sup>3</sup> )
Static modulus of elasticity ( $E_c$ )	= 4,284 ksi (29,564 MPa)
Dynamic modulus of elasticity ( $1.2E_c$ )	= 5,144 ksi (35,477 MPa)

### X.9.2 Step 1: Determine the Natural Frequencies

The floor's natural frequencies can be calculated with either of the options presented earlier: (i) frequency tables and (ii) software designed for vibration analysis.

**A. Use of Frequency Tables:** Using the frequency tables requires engineering judgment to predict the shape of the governing mode, which is not always apparent in floors with a complex layout. For the current example, we assume the geometry of the construction is regular enough for the first mode to govern, so a mode shape in the form of a cylinder as shown in Fig. X.9.2A-1 is likely to be the right choice.

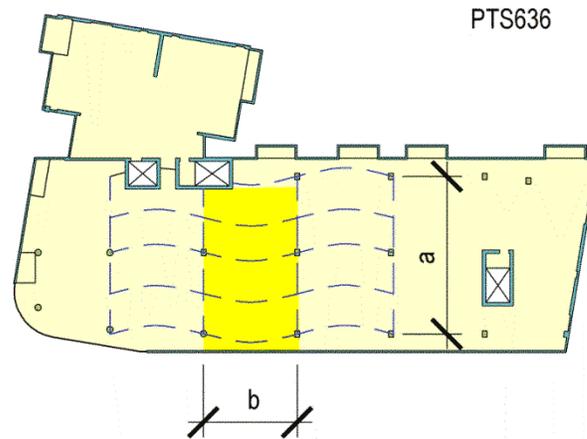


FIGURE X.9.2A-1 Shape of Probable First Mode of Vibration

Next, we need to assume the boundary conditions for use in Table X.3.1-1. The cylindrical shape assumed is associated with the simply supported panel shown in the first row of the table. To calculate the frequency for the first mode, side  $a$  (the longer side) is approximated as twice the length of the panel. This value can be approximated, as it does not significantly affect the result.

$$a = 2 \times 26.25 = 52.50 \text{ ft (16.00 m)}$$

$$b = 30 \text{ ft (9.14 m)}$$

From Table X.3.1-1

$$f = \frac{c}{a^2} \phi$$

Where,

$$c = \sqrt{\frac{Eh^3}{12(1-\nu^2)}} \times \frac{g}{q}$$

Using 150 lb/cu ft weight of concrete and 20 psf superimposed load, weight per square inch of the slab area  $q$  is:

$$q = \frac{150 \times 8}{12 \times 144} + \frac{20}{144} = 0.833 \text{ psi (5.54} \times 10^{-3} \text{ MPa)}$$

$$c = \sqrt{\frac{5144 \times 1000 \times 8^3}{12(1-0.2^2)}} \times \frac{32.2 \times 12}{0.833} = 325,653 \text{ in}^2/\text{sec (2.10} \times 10^8 \text{ mm}^2/\text{sec)}$$

$$\phi = 1.57(1 + \gamma^2) = 1.57[1 + (52.5/30)^2] = 6.38$$

$$f = (325,653/630^2)6.38 = 5.23 \text{ Hz}$$

**B. Application of Vibration-Specific Software:** The continuum nature of concrete floors requires specialty software, particularly if the layout is complex. For expediency, design engineers tend to use the same three-dimensional model that was generated for the floor's structural design and select the governing frequency. For this example, Fig. X.9.1B-1 illustrates the discretization of the floor using finite elements. Figure X.9.1B-2 shows the floor's first three modes of vibration and the associated frequencies.

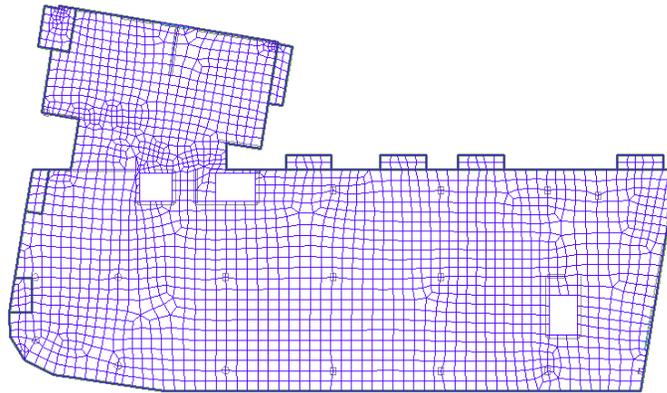
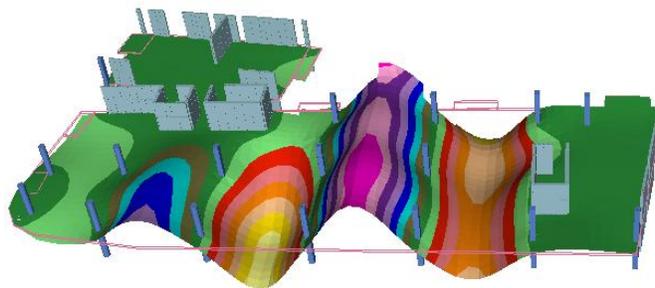
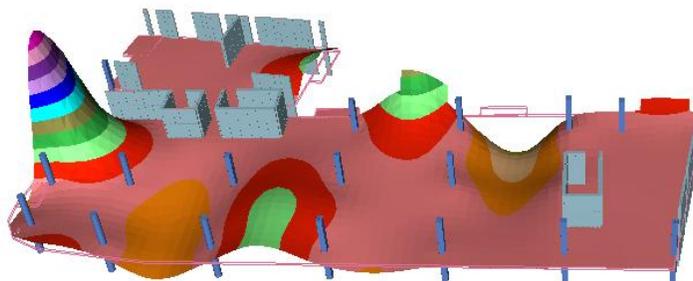


FIGURE X.9.1B-1 – Discretization of the Entire Floor<sup>3</sup> (P255)

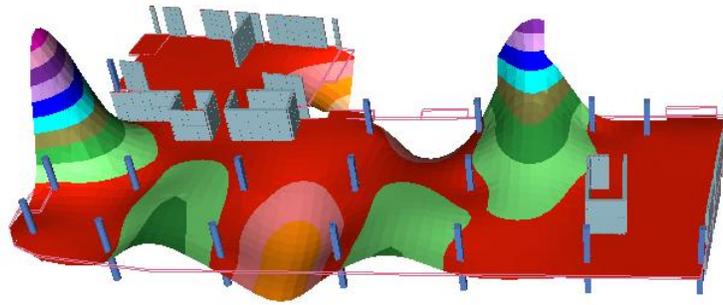


(a) First mode – frequency 5.97 Hz (P388)



(b) Second mode – frequency 6.33 Hz (P742)

<sup>3</sup> From Floor-Pro software; [www.adaptsoft.com](http://www.adaptsoft.com)

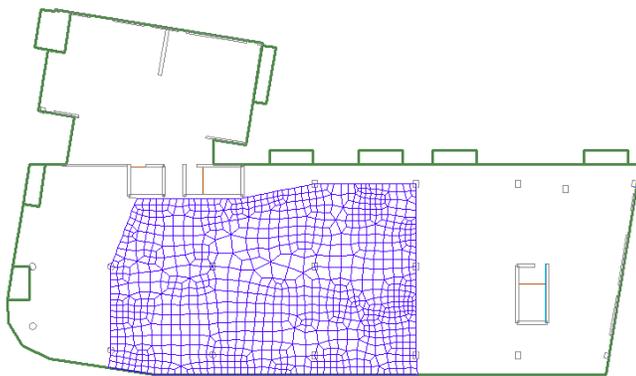


(c) Third mode – frequency 6.44 Hz (P743)

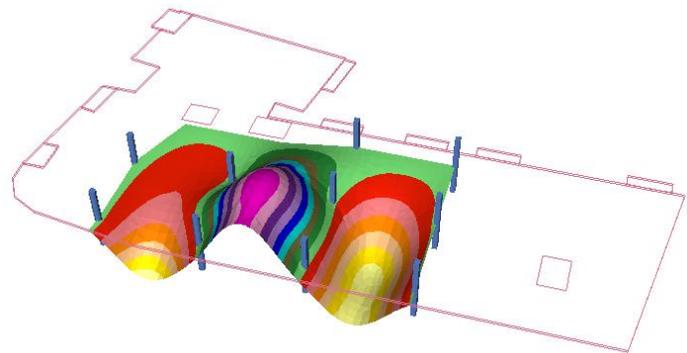
FIGURE X.9.1B-2 First Three Modes and Frequencies

It can be seen that the first mode excites the area of interest and its shape is indeed close to the cylindrical deformation assumed for use of the frequency table.

For complex floor layouts, such as the one shown in Fig.X3.-2, it can be advantageous to limit the analysis to a sub-region of the floor that includes the area of interest. The model should extend at least one panel beyond the area of interest on each side, to minimize the impact of the boundary conditions at the perimeter of the sub-region. Fig. X.9.1B-3 shows the sub-region for the vibration investigation of the current example and the outcome of the analysis.



(a) Selection of a sub-region (P744)



(b) First mode of vibration – Frequency 6.07 Hz (P745)

FIGURE X.9.1B-3 Selection of a Sub-region and its First Mode of Vibration

### X.9.3 Step 2: Identify the Exciting Force

In most instances the exciting force to be evaluated is from human activities. The more frequent cases are covered in the following.

**A. Foot Fall in Residential/Office Environments:** For common conditions, a 150-pound (667 N) individual, taking 2 steps per second (a normal walking pace) can be assumed. Referring to Fig. X.9.2A-1, the dynamic load factor is 0.53. Hence:

$$P_o = 0.53 \times 150 = 79.5 \text{ lb (354 N)}$$

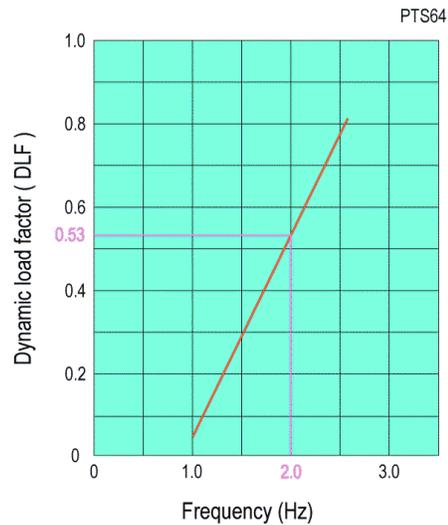


FIGURE X.9.2A-1 Dynamic Load Factor for Two Steps per Second

**B. Impact Force from Aerobic Activities:** Using Table X.4.2-1, it is assumed that each person occupies an average of 42 ft<sup>2</sup> (3.90 m<sup>2</sup>) and weighs 150 lb (79.5 kg), with a dynamic load factor of 1.5. Hence, the intensity of the load over the floor area will be:

$$\text{Average intensity} = 1.5(150/42) = 5.36 \text{ psf (0.258 kN/m}^2\text{)}$$

**C. Impact on Sensitive Laboratory Equipment:** For hospitals and laboratories with sensitive equipment, using the National Institutes of Health recommendation (see Section X.4.2), a walking speed of 1.5 steps per second is assumed. From Fig. X.4.1-1, the associated DLF is 0.28. Hence,

$$P_o = 0.28 \times 150 = 42 \text{ lb (0.187 kN)}$$

### X.9.4 Step 3: Calculate the Floor's Dynamic Parameters

The floor system's geometry was defined earlier and used to determine the floor's natural frequencies. At this stage, the parameter of interest is the applicable weight of the floor to be used for estimating the peak acceleration. As outlined in Section X.6.1, regardless of whether the excitation is due to the foot fall of a single individual or a group activity, a single panel representative of the region of interest will be considered. In this case, the dimensions of the panel of interest are: 26.25 × 30 ft (8.00 × 9.14 m).

$$W = (26.25 \times 30)[(8/12)0.15 + 0.02] = 94.50 \text{ k (420.36 kN)}$$

### X.9.5 Steps 5 and 6; Calculate the Response to Vibration and Evaluate

In the following, the impact of the exiting force  $P_o$  for each of the three sources of vibration discussed above will be evaluated:

**A. Foot drop in residential/office environment:** Per Section X.9.3A the exciting force  $P_0$  is equal to 79.5 lb (0.354 kN).

Peak acceleration,  $a_p$  :

$$\frac{a_p}{g} = \frac{P_0 e^{-0.35f}}{\beta W}$$

Where,

$f$  = natural frequency = 5.97 Hz

$\beta$  = damping factor = 0.03

$$\frac{a_p}{g} = \frac{79.5 e^{-0.35 \times 5.97}}{0.03 \times 94500} = 0.00367 = 0.37\%$$

The entry values of the applicable natural frequency (5.97 Hz) and the peak response acceleration relative to gravitational acceleration (0.37 %) are checked against the ATC chart and shown in Fig. X9.5A-1

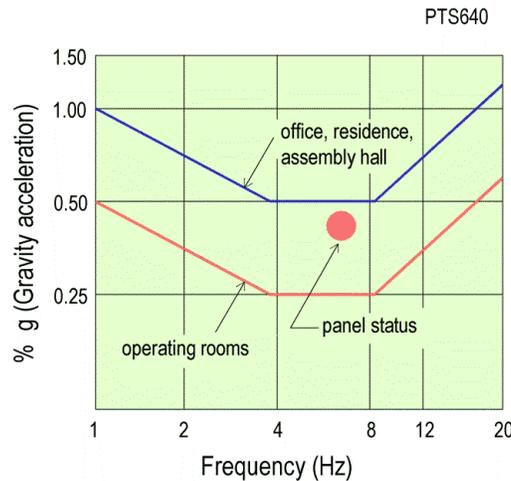


FIGURE X9.5A-1 Floor's Vibration Response and its Evaluation

The calculated value for the status is below the threshold of human sensitivity but above the threshold for sensitive activities. The design is acceptable for residential and office buildings, but not for hospital operating rooms.

**B. Impact of Force from Aerobic Activities:** The exciting force from aerobic activities was calculated to be 5.36 psf in X9.3B. Assuming that the entire floor is covered by people engaged in aerobics activities, the exciting force will be:

$$P_0 = 5.36 \times 26.25 \times 30 = 4221 \text{ lb ( 18.78 kN)}$$

$$\frac{a_p}{g} = \frac{P_0 e^{-0.35f}}{\beta W}$$

$$\frac{a_p}{g} = \frac{4221 e^{-0.35 \times 5.97}}{0.03 \times 94500} = 0.184 = 18.4\%$$

With the entry values of the governing natural frequency (5.97 Hz) and the peak response acceleration relative to gravitational acceleration (18.4 %), Fig. X9.5B-1 indicates that the level of excitation is – as expected – well above the acceptable limits (not shown – falls outside the chart). A column-supported slab with a span-to-depth ratio of almost 40 is too thin for aerobic activities.

One option is to increase the slab thickness. Consider a slab thickness of 11 in. (280 mm). Reanalyzing the floor system with all other parameters kept the same, gives the frequency of the first mode as 9 Hz. Continuing with the evaluation, we get:

$$W = (26.25 \times 30)[(11/12)0.15 + 0.02] = 124.03 \text{ k (551.71 kN)}$$

$$\frac{a_p}{g} = \frac{4221 e^{-0.35 \times 9.00}}{0.03 \times 124030} = 0.049 = 4.90\%$$

With a frequency of 9.00 Hz and peak acceleration of 4.90% g, the marked location in Fig. X9.3B-1 indicates that the new slab thickness will provide an acceptable design for the aerobic activities.

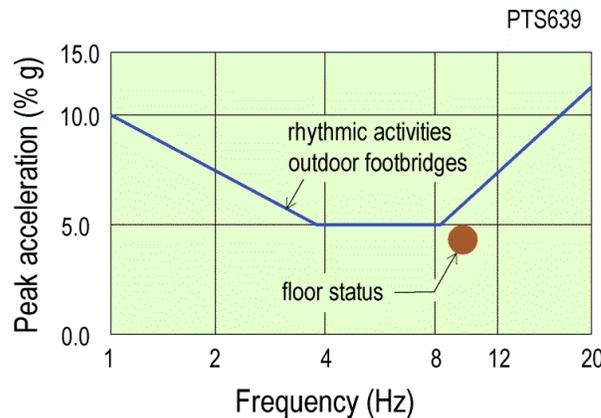


FIGURE X9.3B-1 Evaluation Chart for Aerobic Activities

**C. Impact on Sensitive Laboratory Equipment:** The impact of vibration resulting from foot fall on sensitive equipment is evaluated through the maximum velocity of the slab, which can be derived from the maximum

○

acceleration. The calculation of the maximum acceleration is the same as described in section X9.5A for impact in a residential or office environment.

The maximum acceleration is given by:

$$\frac{a_p}{g} = \frac{P_0 e^{-0.35f}}{\beta W}$$

The maximum velocity  $v$  is given by:

$$v = a_p / 2\pi f$$

Substituting for the acceleration in the velocity equation gives

$$v = \frac{P_0 g}{2\pi\beta W f e^{0.35f}} \quad (\text{Exp X9.3B-1})$$

From X4.3,  $P_0 = 45 \text{ lb}$  (0.20 kN)

From X9.2, the first natural frequency  $f = 5.97 \text{ Hz}$

$$g = 384 \text{ in/sec}^2 \text{ (9,81 m/sec}^2\text{)}$$

$$P_0 = 45 \text{ lb}$$

$$W = 94500 \text{ lb (420.36 kN) from X9.4, Step 3}$$

$$\beta = 0.035 \text{ for a densely equipped laboratory}$$

Substituting the values in Exp X9.3B-1, leads to

$$v = \frac{45 \times 384}{2 \times 3.14 \times 0.035 \times 94500 \times 5.97 e^{0.35 \times 5.97}} = 0.017 \text{ in/sec} = 17000 \text{ micro in/sec (0.43 mm/sec)}$$

From Table X.8.2-1, the calculated velocity is much higher than what is considered appropriate for sensitive laboratory equipment so a thicker slab is necessary. A 12 in. (250 mm) slab will result in a maximum velocity of .0024 in/sec = 2400 micro inches per second (0.061 mm/sec). This is probably acceptable for a general laboratory.

## **X.10 DETAILED ALLOWANCE FOR CRACKING**

For a simplified first approximation, allowance for cracking of a slab is made through a reduction in its overall stiffness, as described in Section X5.6. The following outlines the detailed analysis for a closer approximation a detailed analysis must be done after the slab reinforcement has been calculated and detailed. The status of a concrete floor regarding cracks, crack width, and stiffness loss depends on the location and amount of

reinforcement as well as the load on the structure. Although crack initiation is primarily a function of the concrete's material properties and the extreme fiber tensile stresses, once a crack is initiated, the extent of the crack and the loss in stiffness at the crack location depend on the availability and configuration of the reinforcement at that location. For this reason, the floor should be subject to its service load (also referred to as its sustained or quasi-permanent loading), when it is being evaluated for the effects of vibration. The detailed post-cracking vibration analysis of a concrete floor must be done with commercially-available software – it does not lend itself to manual calculation<sup>4</sup>. The following is an overview of the process.

- ❖ Generate a 3D structural model of the slab that includes details of its geometry and material properties.
- ❖ Enter all the reinforcement that is shown on the “as-built” construction document into the structural model. If the floor has not been built yet, enter the reinforcement shown on the most recent design drawings. This includes the bars that are necessary by computation, as well as those that are added for detailing. Each bar should be entered at its correct location, length, orientation and height.
- ❖ If the slab is post-tensioned, enter each post-tensioning tendon at its correct position, orientation and force. As discussed above, post-tensioning does not impact the vibration characteristics of a floor system, if the analysis is based on the uncracked regime. However, if cracking is going to be accounted for in the design, the pre-compression resulting from the post-tensioning plays a key role in limiting the crack depth and the resulting loss in stiffness.
  - If the post-tensioning tendons are unbonded, only the precompression from the tendons is significant for the post-cracking vibration response of the floor. Because there is no bond between the concrete and prestressing steel, the steel is not mobilized. The physical presence of an unbonded tendon does not affect the cracking response at the crack location.
  - If the post-tensioning tendons are bonded, the prestressing steel will participate at the local level in the characteristics of the crack and the loss in stiffness. This should be represented in the analytical model.
- ❖ Apply the sustained load to the structure to simulate the floor under service conditions.
- ❖ Analyze the structure and determine the distribution of bending and axial forces in the slab, as well as the distribution of stresses at the extreme fiber.
- ❖ Determine the locations in the slab where the calculated moment exceeds the cracking moment. At these locations, the slab will crack. The depth of the crack and the resulting loss in stiffness depend on the amount, location and orientation of both the non-prestressed and prestressed bonded reinforcement. Determine the loss of stiffness at each location. For the Parissa slab used as an example in the preceding section, the loss of stiffness under service conditions is given in Fig. X.10-1.
- ❖ Analyze the floor for vibration, using the reduced stiffness.

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<sup>4</sup> ADAPT Builder, [www.adaptsoft.com](http://www.adaptsoft.com)

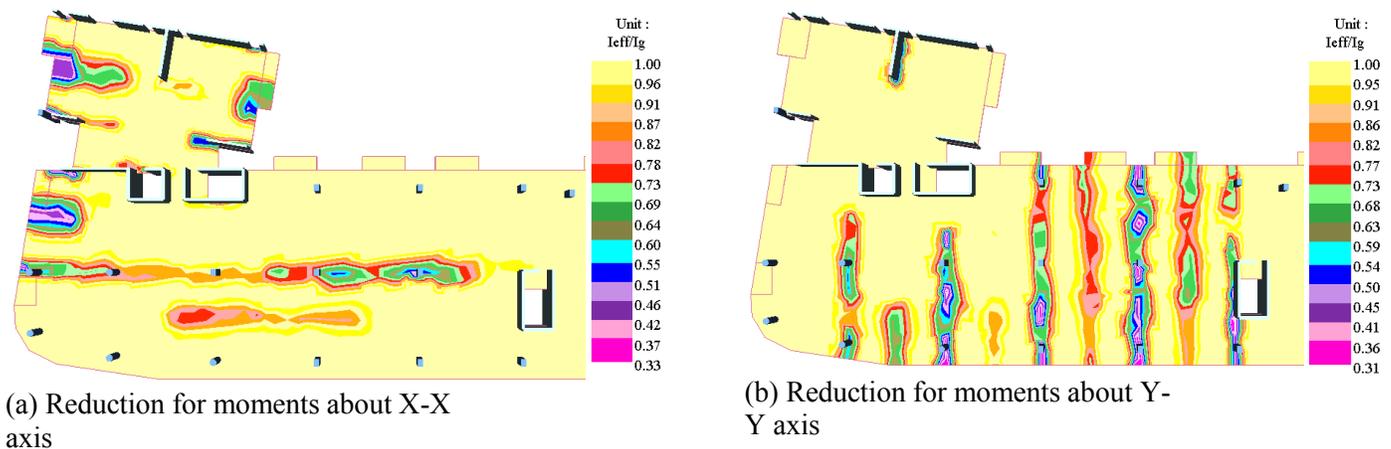


FIGURE X10-1 Reduction of Stiffness due to Cracking Cracked/Uncracked Stiffness Ratio (P254; P253)

- ❖ Perform a vibration analysis based on the stiffness of the slab after cracking. The slab stiffness will vary widely because of the cracking.

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