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Time-dependent analysis of post-tensioned concrete structures

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Summary

This article explains the time-dependent parameters critical to the performance of post-tensioned concrete structures, as treated in the design codes and practiced by design engineers, with an emphasis on recent developments in the field. Following an introduction to the application of time-dependent analysis in post-tensioned construction and to the technical requirements for its successful utilization, the treatment of time-dependent parameters in model codes is presented. The discussion then introduces the basics for the practical implementation of creep, shrinkage, aging of concrete and relaxation in prestressing in the context of code requirements. The application of laboratory-generated data is also discussed, for materials not adequately covered in the codes such as high performance concrete. Finally, the paper outlines a modern analytical technique for the structural modeling of prestressing tendons and its advantages over previous methods. The method yields an integrated solution, inclusive of all time-dependent phenomena. The new method eliminates the need for a separate analysis to determine immediate and long-term losses in prestressing because the losses become an integral part of the overall structural analysis.

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In both pre- and post-tensioned structures, time-dependent analysis becomes necessary if the effects of time, changes in the structural system, and/or high construction loads impact the performance and safety of a structure during its construction and when complete. In many instances, the need for time-dependent analysis is prompted by stringent deformation requirements or an unusual structural geometry[1-5**].

A prime example of the type of construction requiring time-dependent analysis is the balanced cantilever bridge (Fig. 1). In the balanced cantilever bridge, the moments encountered during the construction phase are greater over the supports than the corresponding moments in the completed bridge (Fig. 1b). In addition, the construction requires strict deformation control, in order to avoid excessive misalignment of the cantilever tips at closure (Fig. 1c).

Time-dependent analysis is required for many special post-tensioned structures. The necessity for a detailed time-dependent analysis, be it for a bridge or

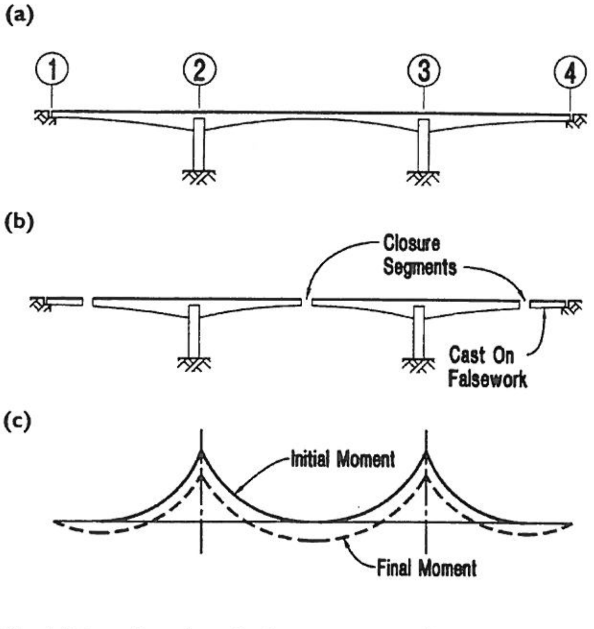

Fig. 1 Balanced cantilever bridge construction: (a) span arrangement, (b) construction phase arrangement, (c) redistribution of moments

Abbreviations
AASHTO = American Association of State Highway and Transportation Officials
ACI = American Concrete Institute
CEB = Comité Euro-International du Béton

Terminology
C_u = ultimate concrete creep strain coefficient
e = constant in shrinkage formula
E_i = concrete modulus of elasticity
ε_s = concrete shrinkage strain
f = constant in shrinkage formula
f'_c = concrete cylinder strength
f'_{cu} = concrete cube strength
f_{si} = initial stress in prestressing tendon
f_{pu} = ultimate strength of prestressing steel
f_s = final stress in prestressing tendon
K = constant in shrinkage formula
S_u = ultimate concrete shrinkage strain coefficient

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any other special structure, depends upon one or more of the following features.

- The constituent parts of the structure are called upon to carry loading in a configuration and through a construction-phase structural system different from that of the completed structure. An example is the balanced cantilever construction scheme, where cantilevering is during the construction phase only.
- The construction loading on the structural components results in stresses exceeding those of the completed structure. Hence, the design of the components, including the prestressing, is in part controlled by the construction method. In many cable-stayed bridges, the design of the deck depends on the construction equipment used, and is controlled through construction-phase support and loading conditions.
- Early loading greatly amplifies the time-dependent concrete deformations in the structure. Early-age loading of concrete, often within the first 24–48 hours after casting, leads to high deformation values which must be carefully evaluated for control of deflection and camber in the completed structure. It is not uncommon nowadays in balanced cantilever construction for the cast segments to be loaded within three days of their construction.
- The structure undergoes significant changes in its structural system during its erection. For example, spliced-precast-prestressed girder bridges are generally assembled with interim supports. As simply supported members, they carry their selfweight. After splicing and the addition of the topping, the structure resists the live loading through the composite action of the precast and the cast-in-place parts.
- The method of construction greatly influences the initial stresses in the completed structure, to the extent that the analysis of the completed structure without regard to its construction scheme becomes irrelevant. Consider the span-by-span construction of a two-span bridge made continuous over the common support. The selfweight moment at the interior support is primarily governed by the method of construction.
- Most retrofit projects involve the addition of fresh concrete, external or internal prestressing, and recently developed synthetic fabrics (strips). Mixed material properties, in conjunction with the interaction of the freshly placed concrete with the retrofitted components and the subsequent redistribution of loading among the new and existing components, require time-delayed analysis specific to segmental construction.

Time-dependent parameters

Early loading, coupled with high construction stresses which may exceed service level conditions, bring into focus the importance of deformation control in segmentally constructed bridges. In concrete bridge construction, short- and long-term control of stresses and deflections are customarily achieved through the application of prestressing and post-tensioning tendons. *Creep*, *shrinkage*, and *aging* effects of concrete, as well as *relaxation* of prestressing steel, are the primary factors in the delayed displacement of a concrete structure[6,7]. These time-dependent parameters are an essential consideration in the design and analysis of segmentally erected, prestressed concrete structures. Computational predictions will deviate significantly from the actual response of a prototype if these long-term effects are not realistically evaluated and allowed for in the analysis.

Creep is a stress-originated effect and is dependent on, among other factors, the entire load history of the concrete element. In contrast, shrinkage is a non-stress-originated factor. It is due primarily to reduction in water content and change in volume caused by carbonation over time. The primary effect of concrete aging is an increase in the concrete modulus of elasticity. The time-dependency and superimposed interactions of these effects results in complex behavior under loading that is difficult to predict. For example, a loading applied at an early stage will cause a deformation much different from a loading of the same magnitude applied at a later stage. Of the three parameters stated above, the prediction of the impact of the creep component is by far the most difficult and complex since its value is a function of the entire load history of the structure.

Temporal effects on prestressing add another layer of complexity to the analysis. Time dependency directly related to prestressing is due entirely to the relaxation phenomenon of prestressing steel which is similar in nature to the creep component of concrete. The relaxation of prestressing steel is defined as the decrease in tendon force with time under constant deformation. Relaxation of prestressing steel is not the only contributor to prestressing losses over time. Creep, shrinkage and aging of concrete also affect tendon forces and must be accounted for in any time-dependent analysis of prestressing losses.

Creep, shrinkage and aging of concrete and relaxation of prestressing steel may be classified as time-dependent, material-inherent parameters. The other class of time-dependent parameters relates to construction scheduling and configuration of the structure. Segmentally erected, prestressed concrete bridges often follow repetitive cycles of construction operations. The shorter is the cycle period, the younger is the cast-in-place concrete age at loading or prestressing, and the larger are the concrete time-dependent effects due to creep, shrinkage, aging and relaxation. A short construction cycle, in turn, leads to a higher degree of stress redistribution through the construction sequence and in the completed structure. Composite construction, consisting of casting new concrete against aged components in segmental erection, adds to the complexity in prediction of the structural response.

Analysis requirements

Due to the special features of segmental construction, the design technology to use for a time-dependent analysis must have the following analytical capabilities (see Box, below).

Conceptual treatment

The following discussion reviews the design parameters of specific importance to segmental construction and their conceptual treatment in building codes and practice.

Design technology requirements for time-dependent analysis of segmental construction

- **The ability to model the construction of the structure as it is being installed. This includes addition of structural components, and fasteners, and the addition or deletion of temporary members**
- **The capability to start the construction at several independent locations and at different times, with each location following its own schedule; and finally to bring the components into an integrated, completed structure**
- **The ability to handle the addition and deletion of temporary supports**
- **The ability to introduce externally applied displacements to control deformations**
- **Provisions for inclusion of the stiffness of construction equipment, such as launching girders and form travelers, when the attachment of the equipment to the bridge components affects the stiffness of the structure under construction**
- **The capability to model both pre- and post-tensioning, for bonded and unbonded construction and mixed systems**
- **The capability to model external tendons**
- **The capability to model cable stays**
- **Full capability to model different concrete materials, based on model codes or laboratory tests performed for the structure under construction; the ability to model as many material properties as the prototype details demand**
- **Authentic modeling of creep, and shrinkage**
- **Allowance for aging of concrete**
- **Determination of losses in prestressing due to friction, seating loss, stress relaxation, creep, shrinkage and aging of concrete**

AGING OF CONCRETE

Aging of concrete generally affects its strength and its modulus of elasticity. Concrete generally gains strength as it ages. In addition, its immediate response to applied loading changes because its modulus of elasticity increases continuously with time, resulting in an increased stiffness. As concrete ages, the rate of increase in strength decreases and the time-strength curve is assumed to be asymptotic to an upper bound value.

Concrete strength f'_c

Concrete compressive strength, f'_c, at any given day is generally expressed in terms of its nominal strength at 28 days. The ACI[8**] and CEB[9**–12] model codes provide the recommended relationships for expressing the gain in concrete strength versus time.

Modulus of elasticity $E_i(t)$

The modulus of elasticity of concrete at any given time, $E_i(t)$, is approximated in terms of its strength at the same time. Consequently, the expression for gain in concrete strength with time assumes special importance, since it indirectly controls the modulus of elasticity, and hence the displacement response of a structure under applied loading. Refer to ACI[8**] and CEB[9**] for recommended approximations for modulus of elasticity versus time.

CREEP

Creep strain is defined as the change in strain under sustained stress with constant humidity and temperature. Creep is a stress-originated strain. The rate of creep decreases to zero over time and is only partially recoverable. Creep is generally responsible for large deformations and significant redistribution of stress.

The code recommended relationships express the creep strain, for any given time, as a fraction of the 'ultimate creep coefficient' (C_u). The ultimate creep coefficient is the maximum creep attained at time infinity for a laboratory specimen under controlled and constant conditions. Further, creep is assumed to be a linear function of stress. The prediction models[8**,9**] limit the parameters to those normally known to a structural engineer. The parameters include the characteristic compressive strength, the dimensions of member(s), the ambient relative humidity, the concrete age at loading, and the duration of loading.

SHRINKAGE

Shrinkage strain is not stress related. It is the deformation under zero loading and constant temperature. Shrinkage of concrete is primarily due to loss of water upon drying (drying shrinkage) and volume change due to carbonation (carbonation shrinkage). The shrinkage rate decreases to zero over time. In most cases the shrinkage strain is greater than

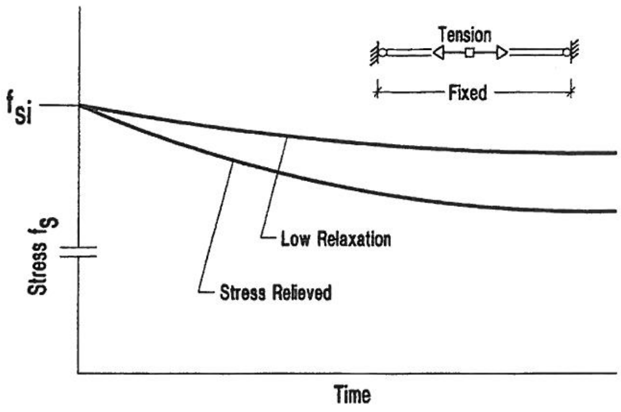

Fig. 2 Stress loss in prestressing due to relaxation

the strain due to applied loading, and therefore assumes special significance in design. Relationships describing shrinkage are usually expressed in terms of the 'ultimate shrinkage coefficient' (S_u). Consult the ACI[8**] and CEB[9**] codes for relevant relationships.

LOSS OF STRESS IN PRESTRESSING

Relaxation in prestressing

Prestressing tendons lose a fraction of their initial stress with time due to the relaxation phenomenon. Most commercially available prestressing steel is classified as either low relaxation or stress relieved. Low relaxation tendons lose less prestressing force over time than the stress relieved variety (Fig. 2).

In time-dependent analysis the prestressing steel is treated as a linear elastic material subject to time-dependent strains of relaxation. The following equation[13**] is generally used for the calculation of tendon stress due to relaxation under constant strain.

$$f_s = \{1.0 - [(\log t)/c]*[(f_{si} / f_{sy}) - 0.55]\}*f_{sy} \qquad (1)$$

where f_s = steel stress at time t, f_{si} = initial steel stress, f_{sy} = 0.001 offset yield stress, c = constant (= 10 for stress relieved strand, = 45 for low relaxation strand), t = time in hours after stressing.

Stress loss in prestressing due to other time-dependent parameters

The stress in prestressing steel is more significantly affected by the creep, shrinkage and aging of concrete than by relaxation effects. Further, concrete strains due to applied loading and temperature changes also affect the stress in a tendon. The importance of these indirect losses on prestressing steel stresses, and the necessity that they be considered in the computations, is recognized in design codes[5**,14,15]. However, there are no specific procedural recommendations for their evaluation although there are several approximate procedures commonly used by design engineers[16,17]. Recent developments in the structural modeling of tendons have provided a reliable means of performing stress loss calculations. The latest development in tendon modeling, 'discrete modeling', is outlined in the next section.

Practical implementation

It is important to recognize that, for practical implementation of time-dependent analysis, the model codes and the consultant community must adopt assumptions which simplify the computations, yet guarantee the performance of the prototype within the acceptable prediction accuracy of engineering designs. The code prescribed implementation consists of:

- the determination of the applicable design parameters, and
- the procedure for their implementation in design.

The determination of design parameters is discussed next.

DETERMINATION OF DESIGN PARAMETERS

Code models and recommendations

Characteristic values

For convenience in practical design, the response of structural concrete to instantaneous and long-term effects is, by convention, described in terms of three characteristic values, f'_c, C_u and S_u. These values are typically obtained through laboratory-controlled, standardized tests, although model codes do make provisions for approximating their values. When applied to a prototype, the characteristic values are adjusted using coefficients which represent the deviation of the prototype from the laboratory conditions.

The most common characteristic value is the compressive strength of concrete at 28 days, measured using either a standard cylinder (f'_c), or a standard cube (f'_{cu}). As a first approximation, for normal weight and normal strength concrete, the cylinder strength is assumed to be 80% of the cube strength of the same material.

For shrinkage, the ultimate shrinkage coefficient (S_u) is defined as the maximum shrinkage strain attained at time infinity for a laboratory specimen under controlled conditions.

The ultimate creep coefficient (C_u) is defined as the maximum strain per unit of applied stress, attained at time infinity in a laboratory specimen, when the specimen is loaded at a specific age and retained under constant stress thereafter for the entire duration of the test. ACI[8**] expresses C_u for the specimen loaded at age 7 days. The CEB code[9**] defines C_u for loading at age 28 days. Obviously, for the same specimen the two values will be different. As a first approximation, for normal concrete, the ultimate creep coefficient determined using the ACI code is typically 40% higher than the associated value from the CEB code.

Since for most prototypes it is not practical to have an experimentally obtained, ultimate creep and shrinkage coefficient, building codes propose empirical relationships to estimate C_u and S_u. The relationships are based on the composition of the concrete mix. A frequently used alternative is the selection of values derived from the observed performance of a similarly constructed structure.

Variation of creep and shrinkage strain with time

The deformation of a prototype is governed by the manner in which creep and/or shrinkage build up over time. In addition to recommendations for estimation of ultimate creep and shrinkage coefficients, model codes[8**,9**] also suggest expressions (shape functions) for the manner in which creep and shrinkage build up with time. The ultimate coefficient and the shape function enable the designer to estimate the values of creep and shrinkage at different times.

Laboratory models

It has become routine to determine time-dependent design parameters experimentally, especially where high performance concrete is involved. Tests are performed with the goal of recreating the actual conditions in the prototype, using material samples taken from the prototype or from the same sources as those used for the prototype.

Shrinkage

Shrinkage response is obtained through testing of specimens from the prototype that are kept under simulated prototype conditions. Readings are normally made for observation times ranging from 30 to 60 days. Extrapolation of those readings becomes necessary to determine shrinkage response beyond the time range of the laboratory tests. Expressions derived from the model code, such as the following[8**], extrapolate the laboratory readings to the estimated ultimate shrinkage coefficient (S_u).

$$\varepsilon_s(t) = K\{(t - t_o)^e / [f + (t - t_o)^e]\} * S_u \qquad (2)$$

where $\varepsilon_s(t)$ = shrinkage strain at time t, S_u = ultimate shrinkage strain coefficient, K = constant determined from fitting the laboratory readings to the expression, f = see 'K', e = see 'K'.

Creep

In using laboratory-generated values to investigate the creep response of a structure three measurements are sought. These measurements are:

- the instantaneous response of the specimen to the applied loading, referred to as the elastic response,
- the creep deformation of the specimen with time, and
- the ultimate deformation of the specimen.

Unlike shrinkage, where one specimen from each concrete mix can be adequate to generate data for the shrinkage response of the prototype, for creep analysis the number of laboratory tests necessary depends on the number of principal loading applications that must be resisted by the prototype.

The simple case involves the response of a structure to a single and instantaneous application of loading. An example of this case is the creep response of a cast-in-place box girder bridge after the removal of shoring. The impact of selfweight activated at the time of shoring removal is the one-time principal loading responsible for the primary creep of the structure. For this condition, a single laboratory test is adequate, provided the time of loading of the laboratory specimen coincides with the day of removal of shoring in the prototype.

Multiple test specimens are required when the sustained loading on a structure is not applied at one time[3]. A good example is the cast-in-place balanced cantilever bridge discussed earlier (see Fig.1). The construction scheme requires that the cantilever sections resist incrementally increasing loads with time as they are extended into the span. For this condition, several laboratory tests are required to capture the creep effects due to the multiple loadings at various concrete ages that are a consequence of the construction scheme.

Prestressing relaxation tests

Almost all prestressing wires come with the manufacturer's data on relaxation. For any given prestressing material, the stress loss depends primarily on the initial stress of the loaded strand or wire. The stress losses can vary by several times, depending on the metallurgical composition and the manufacturing process of the strand or wire.

To obtain the relaxation characteristics of a wire, the wire is stretched to a given strain and locked in position. With constant temperature and fixed initial strain, the loss of force in the wire is measured after a given time (typically, 1000 hours). A typical set of results for a wire tested at three stress levels is given in Table 1.

The relaxation coefficient, c, may be computed from the following expression. The test results provide the parameters of the right-hand side of the expression.

$$c = [\log(t)/(1 - f_s/f_{si})] * [1.053(f_{si}/f_{pu}) - 0.55] \qquad (3)$$

Table 1 Prestressing steel relaxation test results

Wire	Ratio of initial stress to breaking stress f_{si}/f_{pu}	Loss in stress (%) f_s/f_{si}	Time, t (hours)
1	0.80	12	1000
2	0.70	8	1000
3	0.60	4.5	1000

f_{si} = initial stress in prestressing tendon; f_{pu} = ultimate strength of prestressing steel; f_s = final stress in prestressing tendon

The coefficient, c, obtained from eqn (3) is then used in the expression to predict stress losses at times other than the times of the laboratory measurements[3].

IMPLEMENTATION PROCEDURE

Creep implementation: stress history

Among the time-dependent parameters of concrete, only creep effects are a function of the entire stress history of the structure. An efficient implementation protocol for the prediction of creep strains covering the entire stress history has been developed[18**,19**] and successfully implemented by previous investigators[20,21*]. The procedure is based on the principle of superposition, illustrated schematically in Fig. 3. The constituents of the superposition are creep curves obtained from single loading conditions taken at various concrete ages.

Relaxation in prestressing: stress history

In a prototype, strains in the prestressing strands are not constant. Creep, shrinkage and external loads cause additional changes in strain over time. To incorporate the impact on stress relaxation of these additional force variations over time, the computation of stress in strand is made subject to an applied strain history[17]. The procedure is simplified by assuming that all non-relaxation changes in tendon force occur at discrete time steps (Fig. 4).

Aggregated stress losses in prestressing

Many approximate methods have been in use for the computation of stress losses in prestressing tendons, with due allowance to non-relaxation factors, such as creep[16,22]. The approximations were necessitated by shortcomings in the structural modeling of prestressing tendons[3]. New analytical modeling techniques[3,21*] can now greatly limit the approximate nature of stress loss computations. The principal features of a new modeling technique, referred to as 'discrete modeling', are described next.

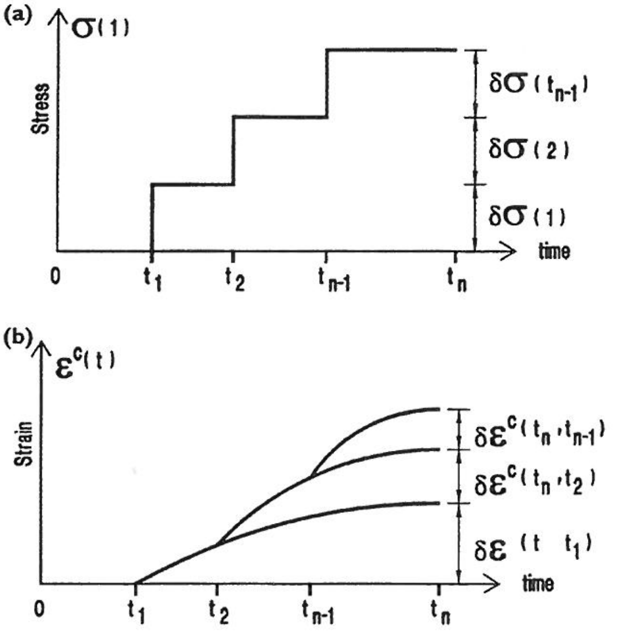

Fig. 3 Linear superposition of creep strains: (a) stress history, (b) total creep strain

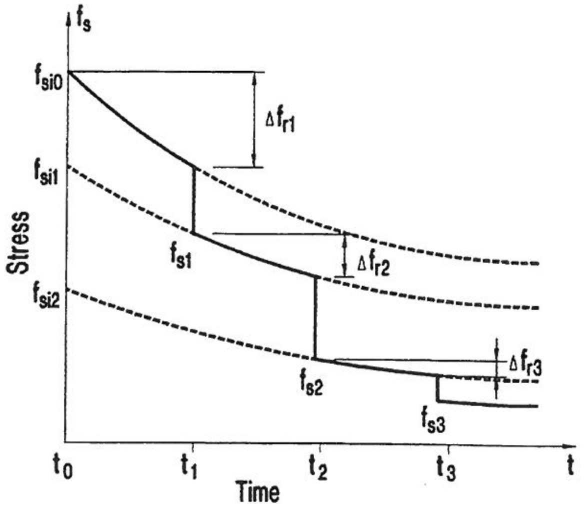

Fig. 4 Model of tendon stress relaxation

In contrast to the equivalent force method, in which the effects of a tendon are represented by the forces it exerts on the structure, the discrete method incorporates a scheme whereby each tendon, or section thereof, is represented by a finite element, in a manner similar to that of the remainder of the structure. Each tendon is modeled as a discrete finite element with the constraint that its deformation must remain compatible with the deformations of the concrete within which it is housed (internal tendons) or to which it is connected (external tendons). This method forces the tendon to experience the same deformation as the surrounding structural element to which it is bonded. Consequently, the deformations due to time-dependent factors, such as creep, are faithfully imparted to the tendon force as they occur. This procedure eliminates the question of stress loss computation as an independent operation.

Fig. 5 illustrates a concrete segment from a prototype represented as a finite element, together with a tendon segment passing through it. In the general case, more tendons may pass through a concrete segment, in which case each tendon segment will be treated as an independent element, such as the one illustrated in Fig. 5. The ends of the tendon segment (marked as tendon nodes) and the nodes of the concrete segment containing the tendon are assumed to be rigidly connected (Fig. 5b). The tendon segment is modeled as a truss element. Consequently, any change in tendon force in the segment shown becomes a linear function of the change in distance between its two ends (tendon nodes).

The imposed compatibility between the tendon nodes and the corresponding nodes of the concrete frame, in conjunction with the assumption that plane

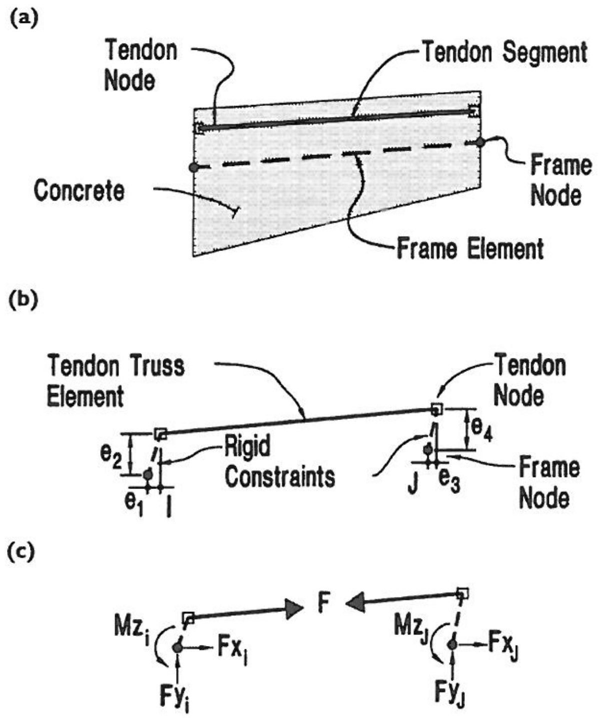

Fig. 5 Discrete modeling of prestressing tendon as idealized truss element: (a) segment (element) elevation, (b) tendon segment geometry, (c) tendon segment degrees of freedom

sections remain plane, force the length of the tendon segment to be directly linked to that of the concrete segment. Thus, deformations in concrete due to creep, shrinkage, aging strain, temperature, and applied loading are all imparted to the tendon through the tendon segment's immediate elastic and long-term relaxation characteristics. Likewise, through imposition of static equilibrium at each element face, a change in tendon force will directly affect the actions on the concrete section. Using this approach, long-term stress losses and the interaction between tendons and concrete become implicit in the analysis. Consequently, the question of computation of prestressing losses for the purpose of obtaining a global solution becomes redundant.

EXTRACTION OF ACTIONS AND DISPLACEMENTS

By convention and practice, and based on the historical development of time-dependent analyses, AASHTO[15], as well as several other design codes, requires that the impact of each of the time-dependent parameters on the performance of the structure be evaluated independently, and be included in the design with its own load factor.

However, this requirement is in direct contradiction with the discrete modeling technique described in the preceding section, in which the time-dependent losses are inherent in the analysis. The contradiction is due to the time lag between the development of analytical technology and the updating of code requirements. Therefore, for code compliance and for the practice of engineers who are used to treating each of the losses separately, an algorithm was developed which extracts the losses due to each of the time-dependent factors from the results of the integrated solution. The algorithm's approach is the reverse of the traditional approach, in which the components are computed and then added to give the integrated solution. Details of this algorithm are given in ref.[3].

Concluding remarks

Current analysis technology can predict the time-dependent effects of post-tensioned and prestressed structures with a much-improved degree of accuracy over previous methods. For structures where camber, alignment and stress control are critical, a time-dependent analysis using current methods provides superior results.

Currently, model codes[8**,9**] provide definitions of time-dependent factors and expressions for their evaluation under ideal conditions. These formulations have now been implemented into current practice in the analysis of actual structures, where loading history and construction sequence are typically far more complex than the ideal conditions envisaged in codes. Where the latest practice extends beyond the scope of the code, such as in construction with high performance concrete, new analysis technology together with time-dependent parameter values verified from laboratory tests, provide the foundation for good design predictions even in the most advanced construction schemes.

New analysis techniques can provide an integrated solution in which the impact of each of the time-dependent parameters on the performance of the structure is fully accounted for without the need for a separate analysis.

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