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Reviews the application of load balancing in a broader context as the emerging standard for analyzing post-tensioned buildings. Terminology, concepts, and current procedures used in the extended scope of load balancing are presented and the governing relationships are introduced and discussed. The redistribution of elastically computed moments due to limited joint plastification is examined and numerical examples illustrate the application of load balancing to more complex structures and the importance of faithful representation of balanced loading.

Keywords: flexural strength; limit state design; **load-balancing method;** moments; plasticity; **post-tensioning; prestressed concrete;** prestressing; serviceability; **structural analysis;** structural design.

Load balancing was introduced by T. Y. Lin[1] as a simple yet powerful alternative method for analyzing prestressed members. It has been widely accepted in North America, where today it is the predominant method of analyzing post-tensioned members. Applying load balancing to complex geometries has not been discussed in the literature, leaving some investigators and engineers unclear regarding its scope and generality. The lack of a common base for the terminology and concepts used in load balancing is a growing problem for consulting engineers due to an increased application of post-tensioning in commercial and residential buildings.

The emergence of load balancing as the principal method of analyzing post-tensioned buildings and its refinements when applied to complex structures calls for a restatement of the concept in its broader context.

This paper offers an illustrative and consistent overview of the principles and the associated corollaries of load balancing. Terminology is clarified and procedures for treating more complex and general geometries are given. The application of load balancing to both the serviceability and strength aspects of prestressed members is covered and several numerical examples are presented.

As the concept was initially proposed, prestressing was viewed primarily as an attempt to balance a portion of the load on the structure, hence the name "load balancing." The load-balanced structure was then regarded as a nonprestressed member with a reduced loading to which a precompression due to prestressing must be superimposed.

Fig. 1, a continuous beam on simple supports, can be used to illustrate the definitions and concepts of load balancing in its extended scope. The beam is post-tensioned with a constant force P. The tendon has a reversed parabolic profile with two inflection points in the interior span and one in each of the exterior spans. The low points of the tendon are at midspans. The horizontal component of prestressing force P along the tendon is considered constant. The support conditions, the tendon profile selected, and the assumption of constant force are arbitrary and do not affect the definitions and the concepts presented here.

BALANCED LOADING FOR SIMPLE CONDITIONS

For the purpose of analysis, remove the tendon in Fig. 1 from its duct and replace it with the forces the tendon exerts on the structure when in place. Fig. 2 illustrates this separation, with Fig. 2(a) showing the free-body diagram of the beam with tendon removed. The loading shown in this diagram is defined as "balanced loading." In this case, it is comprised of upward and downward forces resulting from parabolic tendon segments (Fig. 3) as well as a constant compression force P. For clarity, the supports and other loads such as self-weight are not included in the diagram, since these do not affect the definition of balanced loading. Loadings in Fig. 2(a) and (b) are equal and opposite to one another. The uniform loading W in Fig. 2(b) is drawn with the tendon as the base line.

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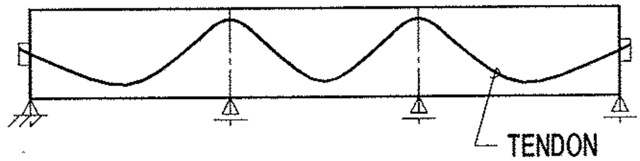


Fig. 1—Three-span post-tensioned beam

Fig. 2(b) is the free-body diagram of the removed tendon. The forces shown on the tendon are equal to but opposite those acting on the beam; both systems are in self equilibrium. The removed tendon [Fig. 2(b)] is always statically determinate, since by definition it is a flexible member capable of sustaining a tensile force only. The beam [Fig. 2(a)] may or may not be statically determinate; its determinacy depends on its support conditions, which are not shown.

Both of the force systems shown in Fig. 2(a) or (b) are considered balanced loading; the balanced loading consists of the presentation of all the forces due to prestressing that act on the member with tendon removed.

LOAD-BALANCING METHOD

The load-balancing method for prestressed members is one in which the analysis and design of the structure is conducted by representing the prestressing forces through their balanced loading as illustrated in Fig. 2. The balanced-loading presentation is used for both serviceability and strength considerations.

Primary moments

In Fig. 4, the beam of Fig. 2 has been cut a distance a from the left anchorage. Note that supports and reactions are not shown in the figure; these may or may not be present in the actual structure. The actions at the cut are a concentric compression P, a moment M_p, and a shear V_x, all due to the balanced loading shown in Fig. 2.

The moment M_p acting at this section, which is necessary to maintain equilibrium of balanced loading, is defined as the primary moment. From Fig. 4(a)

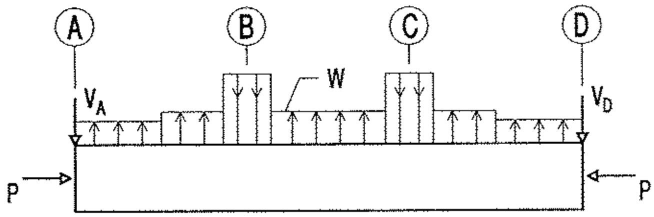
$$M_p = \int (wdx)x + V_A a \qquad (1)$$

where

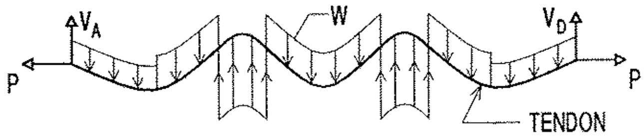
M_p = primary moment
w = intensity of balanced loading at distance x
V_A = vertical component of tendon force at anchorage
a = distance of the cut section from anchorage

The vertical component of tendon force V_A is given from the free-body diagram of the first segment of tendon between the anchorage and its low point, as illustrated in Fig. 3.

Similarly, taking moments about Point O in Fig. 4(b) yields



(a) Free-body diagram of beam after removal of tendon



(b) Free-body diagram of tendon

Fig. 2—Force system between tendon and beam.

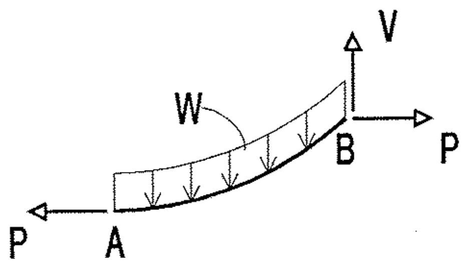


Fig. 3—Free-body diagram of a tendon section between its low point (A) and point of inflexion (B)

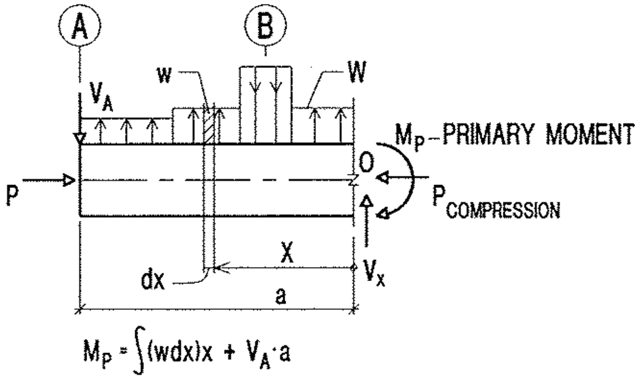
$$Pe = \int (wdx)x + V_A a \qquad (2)$$

where P is the component of post-tensioning in the direction of the member, and e is the distance from the tendon centroid to the neutral axis of the member.

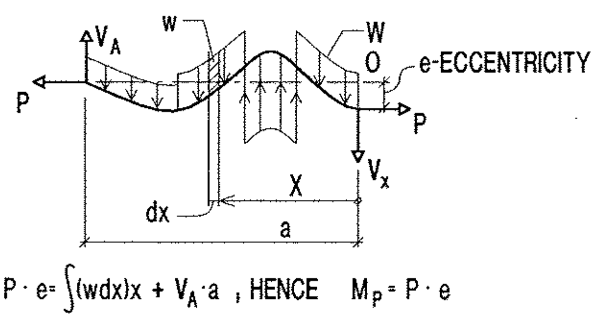
Since the balanced loading w is the same in both figures, the right-hand sides of Eq. (1) and (2) are identical, from which

$$M_p = Pe \qquad (3)$$

Note that the definition of primary moment, as illustrated by its application to the beam, is independent of the member's support conditions and nonprestressing loads.

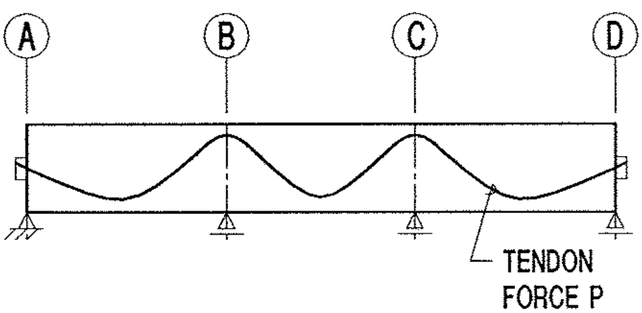


(a) Free-body diagram of cut beam

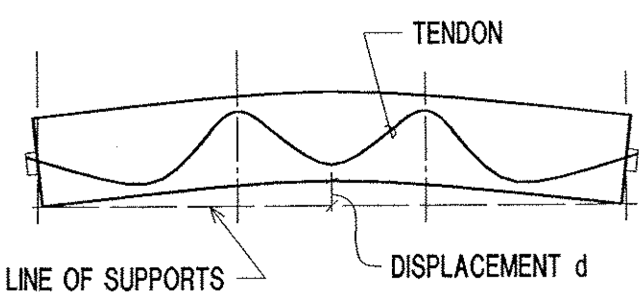


(b) Free-body diagram of cut tendon

Fig. 4—Primary moment M_p *at distance* a *along the member*

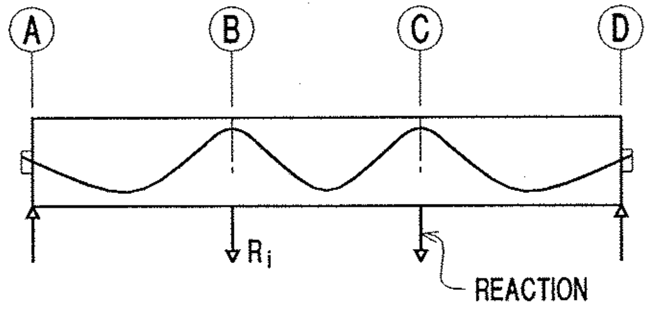


(a) Three-span post-tensioned beam

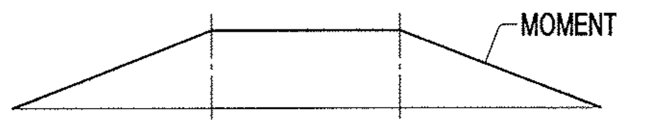


(b) Beam detached from supports

Fig. 5—Beam subject to post-tensioning load only



(a) Prestressing reactions in beam on support (secondary reactions)



(b) Secondary moments due to prestressing support reactions

Fig. 6—Secondary reactions and moments

Secondary moments and sectional strength

Consider a precast post-tensioned beam [Fig. 5(a)] resting freely on its side on a flat bed prior to erection. Tendon forces cause the beam camber as indicated by the curved soffit in Fig. 5(b). The camber is due to the flexing of the beam under the action of the previously described primary moments M_p. The beam must be forced down to be straightened before it can be tied to the aligned supports [Fig. 5(a)]. In actual prestressed members, this may be achieved through self-weight. The forces necessary at the supports to hold the beam in its designated position are called secondary reactions. Fig. 6(a) shows the secondary reactions of the beam held on simple supports.

In cast-in-place members, the sequence is reversed so that the beam is cast while positioned on the supports. When tendons are stressed, additional reactions develop at the supports. These reactions are due to the restraint of the supports to free deformations of the prestressed member caused by prestressing. The reactions developed at the supports are called the secondary actions. The free-body diagram of the beam detached from its supports is shown in Fig. 6(a). Since, in this example, the supports are assumed hinged, no moments are developed at these locations. The reactions at the supports shown in Fig. 6(a) are due to post-tensioning only. These, being the only forces on the beam, must form a self-equilibrating system. That is to say, the sum of secondary reactions must be zero

$$\sum R_{sec} = 0 \quad (4)$$

$$\sum M_{sec} = 0 \quad (5)$$

where R_{sec} is the secondary reaction and M_{sec} is the secondary moment. In this example, there are no secondary moment reactions at the supports.

Fig. 6(b) illustrates the distribution of secondary moments in the beam resulting from the secondary actions of Fig. 6(a).

Pursuing the same example, in which only the post-tensioning forces are being reviewed, observe in Fig. 7(a) that at any section along the beam, the secondary reactions induce a secondary moment M_{sec} and a secondary shear V_{sec}. There is no resultant horizontal force at the cut section for the roller-support example considered. From the statics of the free-body diagram of the cut beam, the secondary moment and shear are given by the following relationships

$$V_{sec} = \Sigma R_i \quad (6)$$

$$M_{sec} = \Sigma R_i X_i \quad (7)$$

The secondary shear and moment shown in Fig. 7(a) at the cut section are sustained by forces developed in concrete and reinforcement over the cross section. At the flexural limit state, the moment is assumed to be resisted through a compression block and a tensile force as shown in Fig. 7(b),from which

$$C = T \quad (8)$$

$$M_{sec} = Tz = Cz \quad (9)$$

in which C is total compression force; T is the combined tension force due to prestressing and nonprestressed reinforcement; and z is the internal lever arm of the section.

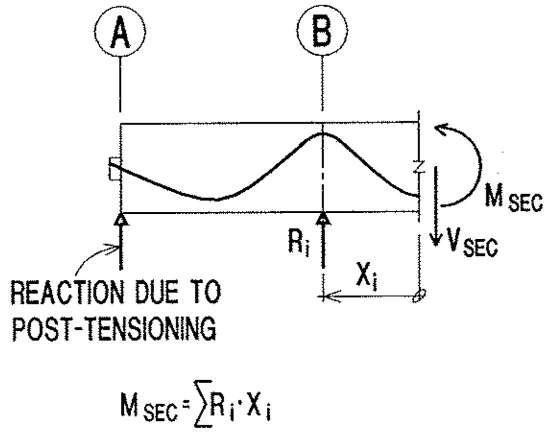
It is evident that at limit state the section must develop an internal resistance associated to secondary moment only. If, in addition to prestressing, other loads such as dead and live loading are also present, the resistance developed by the section must account for the combined moments caused by all the loads. The combination of moments stipulated in ACI 318 for gravity conditions is

$$M_n = \frac{(1.4\, M_d + 1.7\, M_\ell + M_{sec})}{\phi} \quad (10)$$

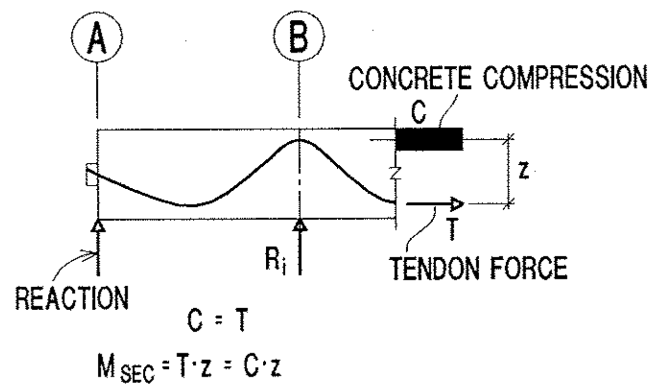
where

M_n = nominal strength of section
ϕ = strength reduction factor
M_d = moment due to dead loading
M_ℓ = moment due to live loading

The secondary moment is not factored. First, the parameters governing its value are more accurately known, as they are not subject to the extent of variations pertinent to dead and live loads. Second, in most cases, secondary moments counteract the moments due to dead and live loading. Hence, an increased load factor for the secondary moments is often not conservative.



(a) Resultant moment and shear at Section A



(b) Assumed internal force distribution at limit state

Fig. 7—Sectional actions due to prestressing and the corresponding internal distribution of forces

The preceding discussion demonstrates that, for strength considerations, it is only the secondary moments that affect the computations and not the primary or balanced moments.

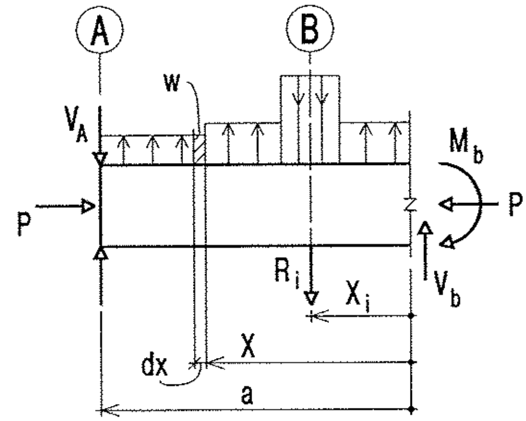
Balanced moments and serviceability

In the cut beam of Fig. 7(a), the tendon is in place. In the load-balancing method of analysis, however, it is assumed that the tendon is removed from its duct and substituted by forces it exerted when in place. Removing the tendon from the cut beam of Fig. 7(a) yields the free-body diagram of Fig. 8. This illustration is the combination of the forces due to the removed tendon from Fig. 4(a) and the support reactions of Fig. 7(a). From the preceding definitions, the forces acting on the beam are the balanced loading and the associated secondary reactions. At distance a from the support, the actions are balanced moment M_b, the balanced shear V_b, and the axial loading P.

From equilibrium of the beam

$$M_b = \left[\int (wdx)x + V_A a \right] + \sum R_i X_i \quad (11)$$

Substituting from Eq. (3) and (7)



$$M_b = \left[\int (wdx)x + V_A \cdot a\right] + \sum R_i \cdot X_i$$

$$= M_P + M_{SEC}; \quad \text{OR}$$

$$M_b = P \cdot e + M_{SEC}$$

Fig. 8—Balanced moment M_b *as sum of primary and secondary moments*

$$M_b = M_p + M_{sec} \qquad (12)$$

which leads to the relationship of balanced moment being equal to the sum of primary and secondary moments

$$M_b = Pe + M_{sec} \qquad (13)$$

Prestressed members are required to be checked for serviceability under working conditions and strength for safety against collapse. The primary considerations for serviceability are stress checks in concrete for crack control and deflection calculations. The load-balancing method model shown in Fig. 8 is used for serviceability checks. Since the tendon is removed, the moment M_b and the axial loading P must be resisted by the stresses developed on the concrete section and any available supplementary reinforcement. At this stage, the removed tendon's cross-sectional area and its position on the cross section are no longer a consideration.

The preceding modeling and computational procedure have the advantage that they convert the analysis of a prestressed member to that of a nonprestressed substitute, to which an additional set of loading (balanced loading) is applied. Thus, calculation routines and computer programs commonly used by design engineers for the treatment of beams and frames in flexure may be used directly to analyze prestressed structures using load balancing.

The beam model of Fig. 8 is also used for deflection calculations without loss of accuracy. The only difference between this model and the prototype is the absence of the tendon in the model. Since the tendon, in the idealized case, has no flexural stiffness, its deletion from the beam does not alter the flexural response of the remainder of the section that is used for deflection computations. Strictly speaking, straining of the beam alters the distribution of prestressing and, hence, affects the beam's flexural performance. But the changes are not of practical significance and do not diminish the effectiveness of the balanced-loading method as a powerful analysis technique for deflection calculations.

FLEXURAL STRENGTH AND REDISTRIBUTION OF MOMENTS

Commonly, the internal actions along a member are determined from elastic analyses using gross cross-sectional geometries. The actions at the critical sections are factored and combined to give a design moment, such as expressed by Eq. (10). The strength of a section is evaluated against this design moment using internal force diagrams as illustrated in Fig. 7(b). The load-balancing method, which uses elastic analysis, falls into this category. The elastic solutions obtained from the balanced loading directly yield the secondary actions that are contributory to ultimate strength factored moments and shears.

The other method of determining sectional actions in indeterminate structures is the plastic method of analysis. The plastic method has been used extensively in Europe for steel structures, and to some extent for concrete frames, for over three decades. In this method, the formation of an adequate number of plastic hinges results in a collapse mechanism and thereby the failure of the structure. The distributions of moments and shears of the collapse mechanism are computed and used to evaluate the strength adequacy of sections to which they apply. The resulting sectional actions are typically different from those derived from an elastic analysis; however, the treatment of sections for ultimate strength is the same as in the previous method using the internal force presentation of Fig. 7(b).

The plastic method of analysis has not been applied widely in the U.S. to concrete structures in general or to post-tensioned members in particular. The primary reason is that the plastic method results in redistribution of elastic moments, and thereby nonelastic rotations at critical locations, to an extent that in most cases is in excess of the limits considered acceptable for concrete structures. The acceptable limits for redistribution of moments are set forth in the codes.[2]

Another reason for a lack of interest in applying plastic methods to post-tensioning is that prestressed members must be reviewed for service stresses in addition to the regular strength requirements of nonprestressed members. An elastic analysis is the prerequisite of service stress checks; hence, the plastic analysis would pose an additional computational effort. It is not a substitute for elastic analysis, as is the case of nonprestressed frames.

Recognizing, on the one hand, the economy associated with the plastic method of analysis and, on the other hand, the undesirable rotations inherent in fully materializing its potential, most codes permit a limited amount of redistribution of elastic moments in an effort to secure some of the savings of the plastic method while maintaining a safe and serviceable structure. It is noteworthy that ACI 318, Canadian Code CAN3-A23.1,[3] and British Code BS:8110[4] all set a maximum limit of 20 percent on redistribution of moments in prestressed members calculated using elastic methods.

There have been several discussions regarding the secondary actions and the redistribution of moments in prestressed concrete members.[5-8] In the plastic method of analysis, hinges are formed in the indeterminate structure one after the other until the structure becomes determinate, at which condition the secondary actions vanish. On this premise, some engineers disregard the secondary moments in factoring and combining the elastically obtained values to arrive at the design moments.

This is not consistent with the accepted design practice of allowing only limited plastification for concrete frames. It has been shown experimentally[5] that within the allowable range of redistribution, secondary moments are fully present. Increase in prestressing force due to added strain in tendons, caused by joint rotations, prolongs the fall in secondary moments. T. Y. Lin[7] concludes that, regardless of the method of calculation used, secondary actions produced by prestressing shall be included in the calculation used to compute ultimate load capacity of a continuous prestressed member. If full plastic hinges develop, the computed value of secondary actions will be zero. If plastic hinges do not develop fully, secondary actions will lie between the values determined by elastic analysis and zero. The ACI Building Code clearly permits a limited plastification.

ACI 318-89 Commentary Sections 18.10.3 and 18.10.4 imply that secondary moments shall not be redistributed at all, that these should be included with their full values in computing design moments, and that the limited redistribution applies to dead and live loadings only. The background to ACI's commentary appears to be the test results in Reference 5 in which, by design, no secondary moments were present. Therefore, the conclusions from the tests were formulated to be limited to dead and live loading. But since the capacity of a section to accommodate limited plastification depends on the geometry and reinforcement of that section, and the extent of plastification precipitated is governed by the magnitude of total applied moment and not the composition of that moment, it seems reasonable that the limit of redistribution be set for the total moment of the section. This interpretation would bring the ACI Building Code in line with the corresponding Canadian and British codes, where the maximum 20 percent redistribution limit applies to the entire applied moment.

To provide context for the discussion of secondary actions at limit state, in particular for engineers dealing with the load-balancing method and elastic analyses, consider the two-span symmetrical beam of Fig. 9, which reviews the application of load balancing to the plastic method of design.

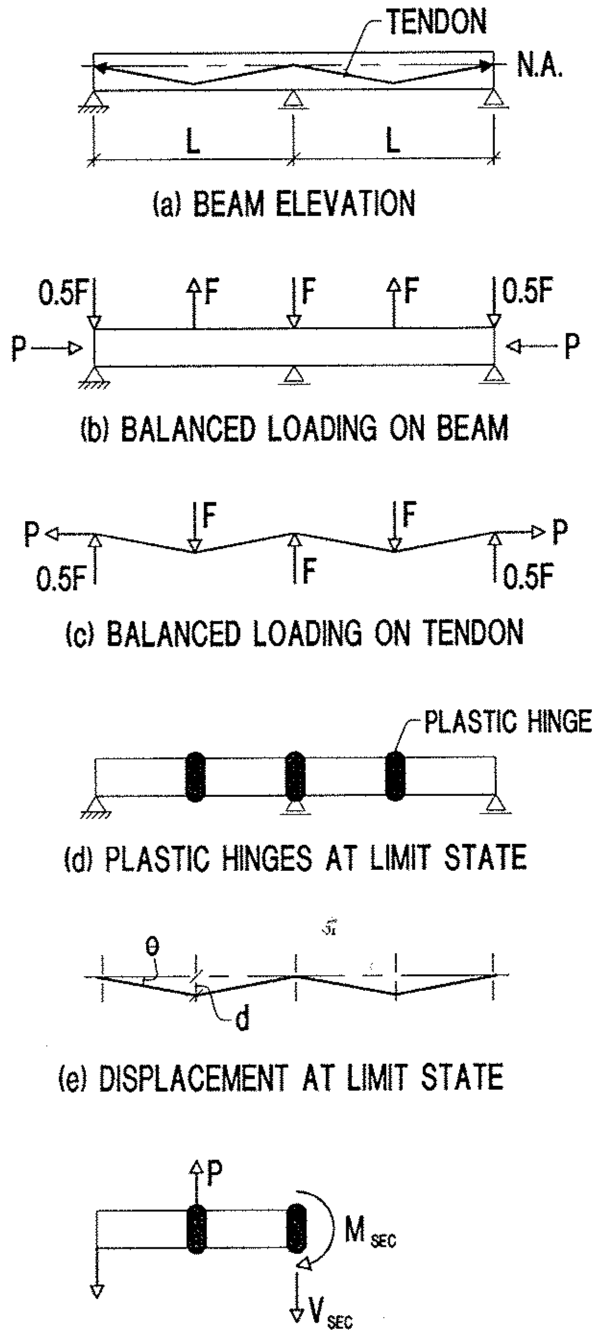


Fig. 9—Secondary actions at limit state

For simplicity and illustration of concepts, only the forces due to post-tensioning are considered. The tendon is harped, as shown in Fig. 9(a). Fig. 9(b) shows the balanced loading on the beam with the tendon removed and (c) illustrates the free-body diagram of the tendon. Fig. 9(d) is a likely hinge formation for the combined actions of dead, live, and prestressing loads; the associated mechanism is shown in (e). This mechanism need not correspond to the balanced loading diagram of (b), nor does it have to be the true failure mechanism for the present discussion.

The work equation for the plastic method is set up by applying the forces of the equilibrium systems shown in

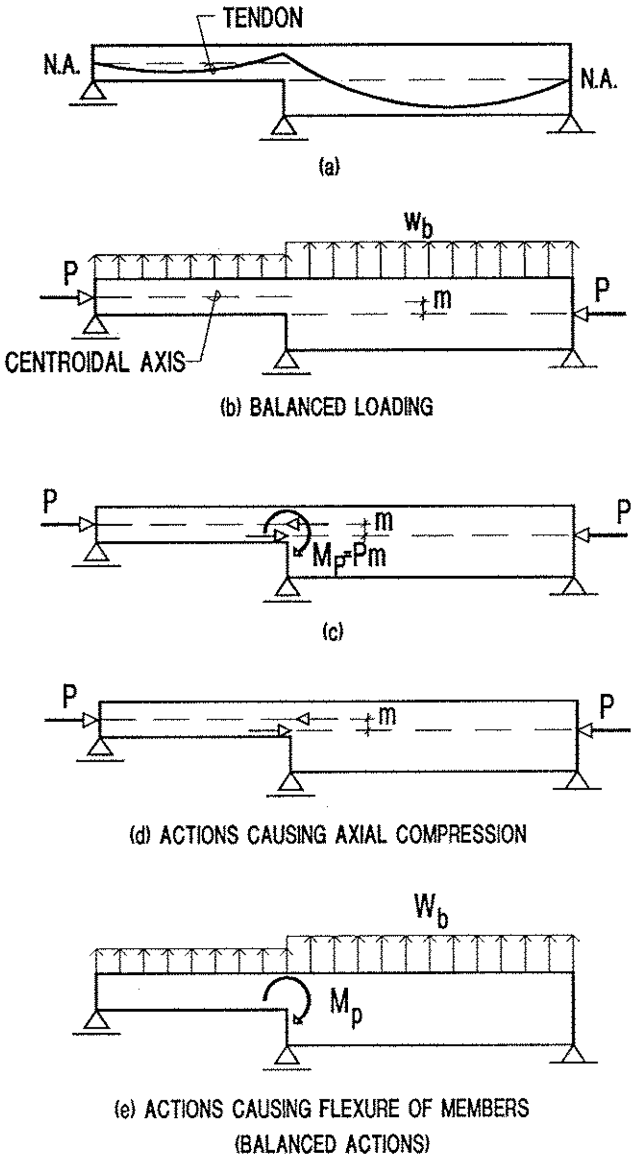


Fig. 10—Balanced loading in continuous members with changes in thickness

Fig. 9(b) and (c) to the displacements shown in (e). For the symmetrical half shown in Fig. 9(f), note that the work contributions of the force systems (b) and (c) to the displacements shown in (f) are zero, since any action in (b) is accompanied by an equal and opposite action in (c). Hence, M_{sec} and V_{sec} will be zero.

The objective of the preceding was to illustrate that the load-balancing method may be applied in setting up work equations for the plastic method and results in the known conclusion of zero secondary actions. The critical issue is that, in setting up the work equations, the contribution of the removed tendon must also be included.

LOAD BALANCING AND SHIFT IN THE CENTROIDAL AXIS

A change in cross-sectional geometry of a member along its length results in a shift in the centroidal axis of that member at the location of change in geometry. Prestressed members that are analyzed by the load-balancing method require consideration of a concentrated moment at each shift in the centroidal axis. Application of these moments in the load-balancing method as well as their significance are illustrated in the numerical examples that follow. It is concluded that including the concentrated moments in the analysis of prestressed members is essential to obtain acceptable design solutions.

In the two-span beam shown in Fig. 10(a), the continuous tendon is anchored at the neutral axis (NA) at each end. The tendon is a single parabola in each span with the location of the low point determined by calculation. This differs from the previous examples in which the low points were assumed at midspans.

The loading exerted by the prestressing tendon, when in place, is entered in Fig. 10(b), in which the tendon is assumed removed from its duct. Note that the in-plane post-tensioning forces P acting at the edges of the beam do not align, since the centroidal axes of the two spans are shifted by a distance m over the central support. The "balanced loading" shown in Fig. 10(b) consists of the upward forces W_b and the in-plane forces P. P may be represented by the force and couple system shown in Fig. 10(c). Note that the couples M_p are in self-equilibrium.

Using the presentation of Fig. 10(c), the balanced loading [Fig. 10(b)] may be grouped into two parts [Fig. 10(d) and (e)]. In-plane forces shown in Fig. 10(d) cause uniform axial compression. They do not induce bending. Both W_b and M_p [Fig. 10(e)] must be considered as applied loading in analyzing post-tensioning moments.

The concentrated moment discussed occurs at each location of shift in centroidal axis of a member along its length.

Example 1—Change in thickness of different spans

Fig. 11 shows a two-span beam of uniform width [14 in. (356 mm)], but of different depths. The associated balanced loading is shown in Fig. 11(b) and (c). For example, the balanced loading on the first half of Span 1 for a drape of 1 in. (25 mm) is given by $W = (8 \times 90 \times 1)/(12 \times 15^2) = 0.267$ kips-ft (3.897 kN/m).

Note that there is a 5 in. (127 mm) shift between the centroidal axes over the second support. This precipitates a moment equal to 37.5 kips-ft [$(90 \times 5)/12 = 37.5$ kips-ft (142.5 kN/m)].

Two solutions are obtained for the post-tensioning moments, one with and one without accounting for the concentrated moment of 37.5 kips-ft (142.5 kN-m) acting at the second support. Fig. 12 shows the results of the analysis for the case in which the concentrated moment is included. Fig. 12(a) shows the reactions due to post-tensioning. The secondary and the balanced moments are shown in Fig. 12(b) and (c), respectively.

The results of the computations in which the neutral axis is disregarded are shown in Fig. 13. Comparing the two figures indicates that the results of the two solu-

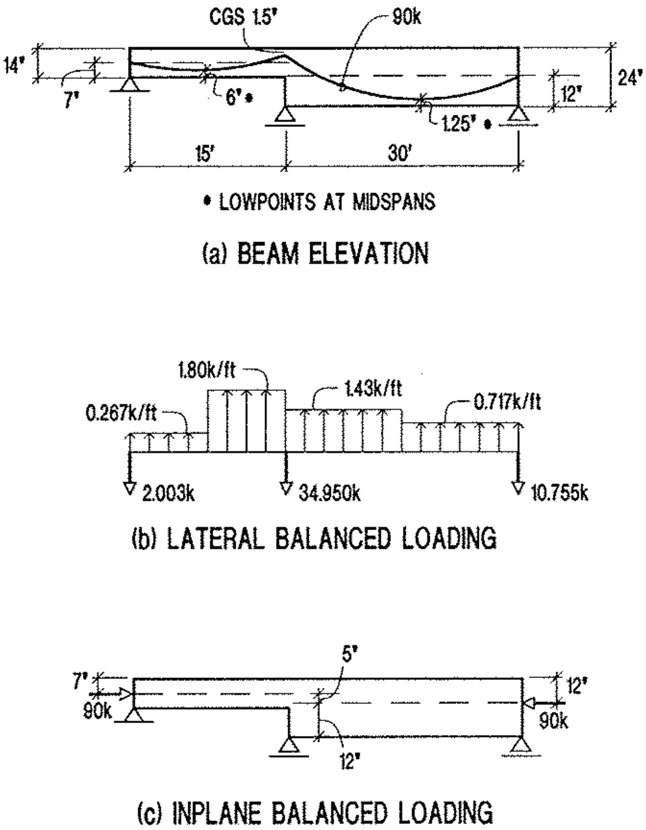


Fig. 11—Post-tensioned beam and balanced loading

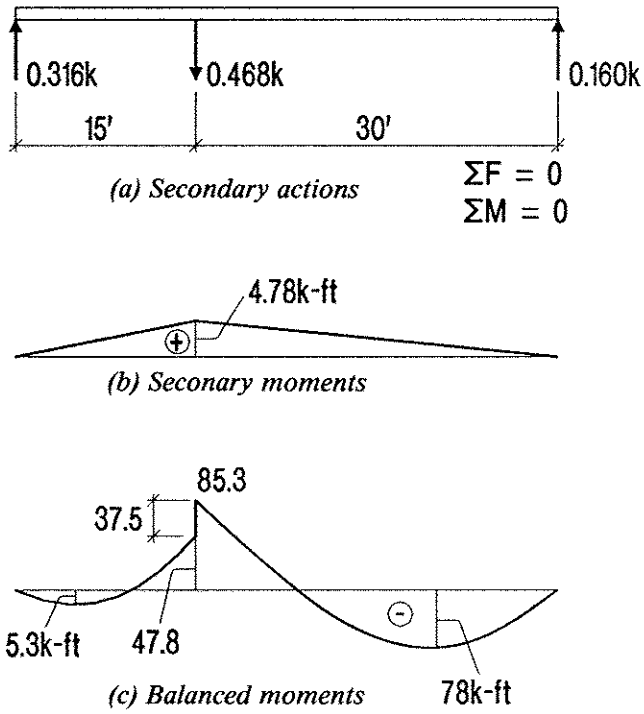


Fig. 12—Shift in neutral axis accounted for

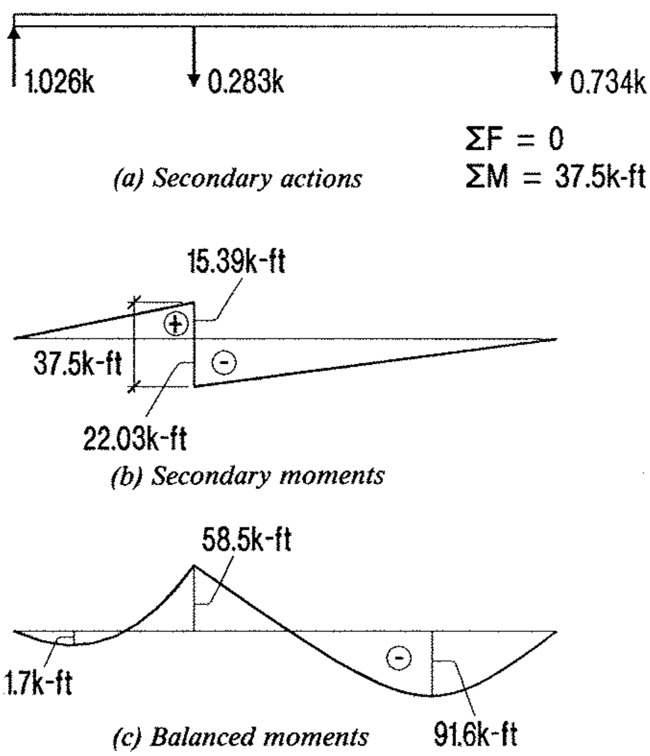


Fig. 13—Shift in neutral axis not accounted for

tions differ significantly both in magnitude and distribution of moments, and that the moment due to shift in centroidal axis may not be disregarded.

Example 2—Nonprismatic slab

Fig. 14 shows the cross section of a typical interior span of a mat foundation with thickened strips beneath the columns. The columns are 18 in. (457 mm) round and spaced 47 ft 6 in. (14.48 m) apart in the direction of the thickened strip. There is a shift in the centroidal axis at the junction between the slab and the thickened strip. Fig. 14(b) shows the post-tensioning moment from two analyses, one with due consideration to the concentrated moments at the changes in cross section and one without. Note that the moments at support from the two analyses are each about 150 kips-ft (203.4 kN/m), but of different signs. The two solutions are widely different.

The inclusion of the concentrated moments at the location of changes in centroidal axis is essential both for serviceability and strength calculations.

CONCLUDING REMARKS

From the presentation of the load-balancing method for analyzing prestressed members, load balancing is shown to be a simple and general method that can be applied to structures with complex geometries and loading.

In the treatment of complex geometries, the moments caused by changes in the centroid of sections must be included in the analyses for acceptable solutions.

The load-balancing method may be applied to limit state conditions. It yields the known result of zero secondary actions.

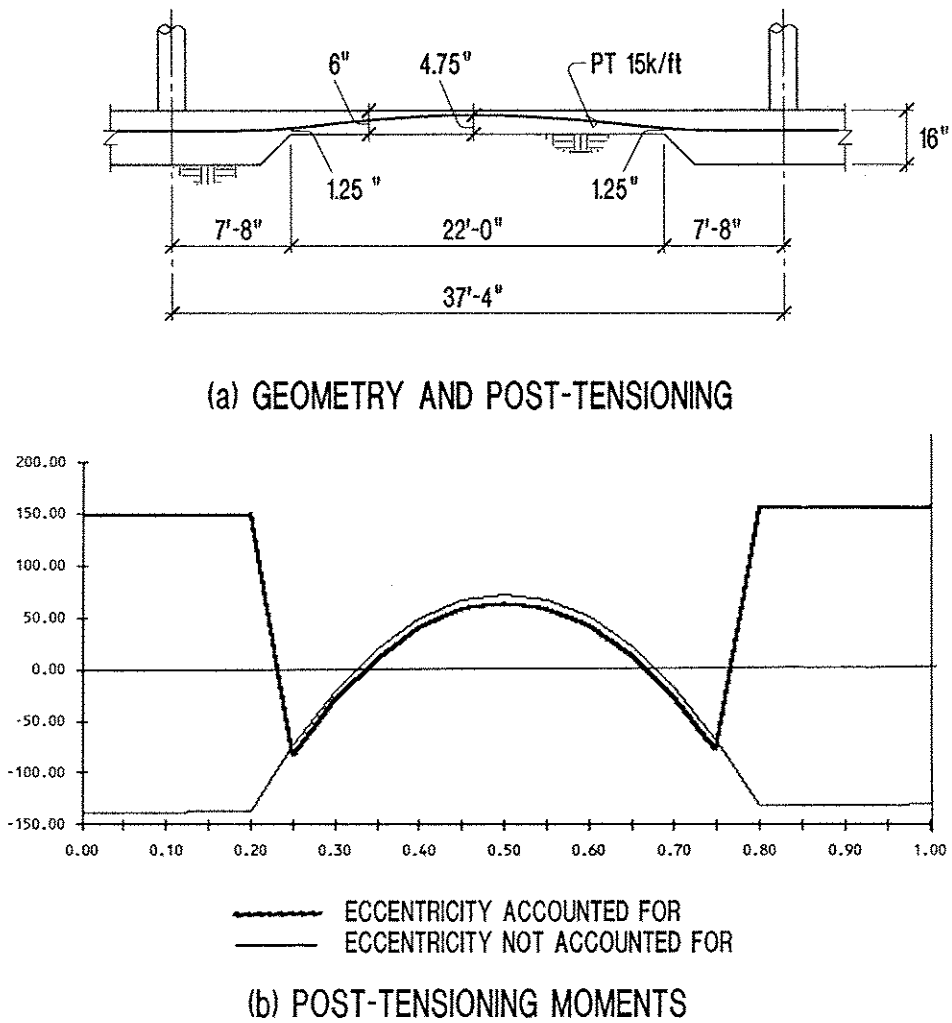


Fig. 14—Typical interior span of mat foundation

It is recommended that in line with the Canadian and British codes, the redistribution of moments due to limited plastification of joints should apply to the total moment at a joint, not only to its dead and live load components.

NOTATION

- a = distance of the cut section from anchorage
- C = total compression force
- e = distance from tendon centroid to neutral axis of member
- M_n = nominal strength of section
- M_d = moment due to dead loading
- M_l = moment due to live loading
- M_p = primary moment
- M_{sec} = secondary moment
- P = component of post-tensioning in direction of member
- R_{sec} = secondary reaction
- T = combined tension force due to prestressing and nonprestressed reinforcement
- V_A = vertical component of tendon force at anchorage
- w = intensity of balanced loading at distance x
- z = internal lever arm of section
- ϕ = strength reduction factor

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