TN455_restraint_cracks_safety_abridged_021616

IMPACT OF RESTRAINT CRACKS ON STRENGTH[1] OF POST-TENSIONED MEMBERS

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CONTENTS

1 MEMBER SHORTENING; PREREQUISITE TO PRECOMPRESSION

In actual construction, post-tensioned members such as floor slabs and beams are supported on walls and columns. These supports can restrain the free shortening of the member when the tendons are stressed. Unless the member is allowed to shorten, it will not receive the full amount of precompression from the stressed tendons. In theory, if the supports prevent any shortening (Fig. 1-1b), the entire post-tensioning force will be diverted to the supports, leaving the member with no precompression. Failure to account for restraint from the supports can lead to cracking. Apart from possible aesthetic objections, these restraint cracks can cause leakage, and expose the reinforcement to the corrosive elements. More importantly, restraint cracks can reduce the contribution of the post-tensioning tendons to the strength capacity of the member.

The extent of the restraint cracking in a post-tensioned member depends on a number of factors, including the stiffness of the supports. Figure 1-1 illustrates two extremes. In part (a) a post-tensioned member on very flexible supports shortens under the precompression, forcing the supports to follow the member's movement. This can result in cracking of the supports. At the other extreme, a member on very stiff supports will be restrained against in-plane shortening and can develop restraint cracks as it shortens.

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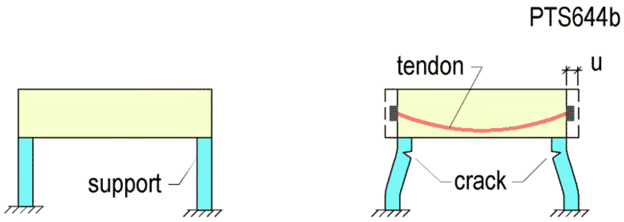
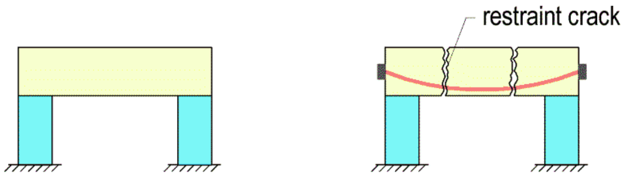


(i) elevation (ii) deformed member

(a) Flexible supports



(i) elevation (ii) cracked member

(b) Stiff supports

FIGURE 1-1 Effects of Support Restraint on Member Cracking
Where supports are flexible, cracking is likely to occur at supports;
stiff supports lead to cracking in the slab

Cracks resulting from the restraint of the supports are typically long (Fig. 1-2); they extend through the entire depth of the member; and occur at axially weak locations, such as where the reinforcement is terminated, or at reduction in concrete area.

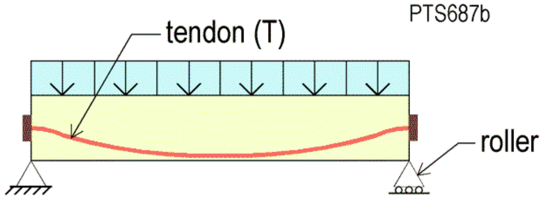
FIGURE 1-2 Example of a Restraint Crack (P751)
Restraint cracks are generally long, wide, extend through the depth of the member and
occur at axially weak locations, such as where the reinforcement ends.

Cracking due to restraint from the supports is generally most pronounced at the first level of a structure, due to the restraint from the foundation; there is less cracking at higher levels. Experienced design engineers are aware of the possibility of restraint cracking and its consequences; they use a number of measures to allow the post-tensioned member to shorten, while minimizing the effects of cracking in either the member or its supports.

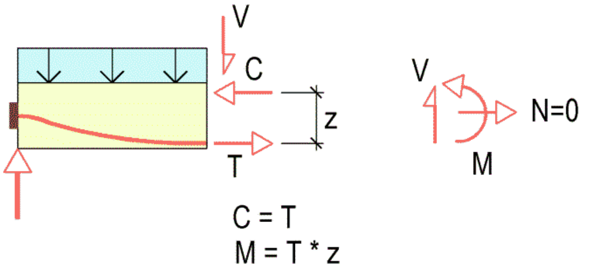
This paper investigates the impact of restrain cracks on the bending strength of post-tensioned members. It concludes that these cracks adversely affect the design strength of the members, thereby reducing their safety factor. The paper also explores the difference between the response of members reinforced with unbonded and bonded tendons to restraint cracks.

2 IMPACT OF SUPPORT RESTRAINT ON FLOOR STRENGTH

Figure 2-1 illustrates the mechanism by which post-tensioning tendons contribute to the strength of a member in the absence of support restraints. This will be contrasted to the case in Fig. 2-2 – where the member is subject to support restraints.



(a) No support restraint



(i) Resistance at cut (ii) Force demand

(b) Free body diagram of a beam section

FIGURE 2-1 Post-Tensioned Member with no Support Restraint to Shortening. Part (b-ii) shows the force demand on the cut section; part (b-i) shows the resistance provided at the cut section to meet the demand.

For the member shown in Fig. 2-1, the strength demand at a section (part b-ii) consists of moment (*M*), shear (*V*) and axial force (*N*). The demand actions *M*, *V* and *N* are in static equilibrium with the forces acting on the severed segment of the member. For the safety of the structure, the resistance that can develop at the face of the cut by the forces *T*, *C* and *V* should not be less than the demand actions *M*, *N* and *V*.

Since the member is on rollers, the reaction at the support (part b of the figure) is limited to a vertical force. There are no externally applied horizontal forces on the cut segment. From the equilibrium of the forces, the net axial force on the face of the cut will be zero ($N = 0$). Hence, the resisting forces need to counteract the moment, M and shear force, V only.

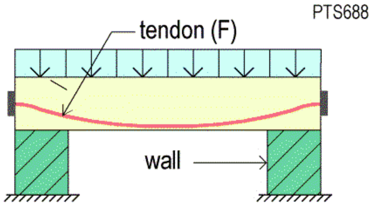
The resistance to the demand moment M at the section is developed by the tendon force T and the compression force C in the concrete:

$T = C$ (Exp 2-1)
$M = Tz$ (Exp 2-2)

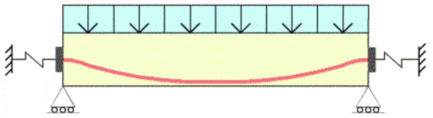
Where z is the moment arm of the forces at the face of the cut. In this case, where there is no restraint to shortening from the supports, the entire tendon force T is available to resist the demand moment M.

Consider now the case shown in Figure 2-2, where a post-tensioned member is attached to supports that restrain the member's shortening. For this figure and what follows the definitions given below are used.

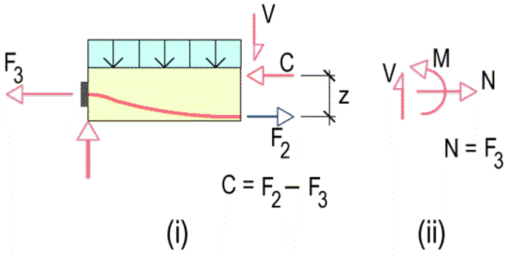
F = force in the tendon at ultimate limit state (strength condition);
$F2$ = force in the tendon in service condition;
$F3$ = restraint of support in service condition.



(a) Member with restraining walls



(b) Simplified model of member with axial restraint



(c) Free body diagram of severed member

FIGURE 2-2 Post-Tensioned Member with Support Restraint
The restraint of the support is modeled with the spring shown at the supports of the member in part (b). F3 in part (c) is the restraint of the support (this is the force in the spring shown in part b)

At tendon stressing the supports shown absorb a part of the post-tensioning force, marked F_3 in part (c) of the figure. The amount of the force F_3 that is diverted to the supports depends on the stiffness of the supports; the remainder of the post-tensioning force results in precompression in the member. For ease of visualization, the member is modeled as shown in part (b). The springs attached at each end of the member represent the restraint of the supports to the shortening of the member.[3]

The discussion followed for the member in Fig. 1-1 will be used here to determine the contribution of the tendon force to the safety of the member. Part (c) of Fig. 2-1 is the free body diagram of the left segment of the member. The demand actions at the face of the cut are once more the moment M, shear V, and axial force N. In this case, however, from the equilibrium of the forces in the horizontal direction we have:

$$N = F_3 \qquad \text{(Exp 2-3)}$$

Thus, in addition to the moment M and shear V, there is a net axial tension F_3 that must be resisted by the actions developed at the face of the cut. From the equilibrium of the forces on the cut segment:

$$C = F_2 - F_3 \qquad \text{(Exp 2-4)}$$

Hence, the resisting moment at the face of the cut will be:

$$M \simeq F_2 z - F_3 e \qquad \text{(Exp 2-5)}$$

Where e is the distance between the force F_3 and the centroid of the compression force C. The approximation sign (~) is used, since the force F_3 acts at the interface between the support and the member, but for the current discussion, it is shown at the centroid of the member, with the restraint modeled as a spring.

In summary, when a member is restrained at supports, the post-tensioning force available to resist the demand moment M is reduced. The amount of reduction, in this example F_3, depends on the stiffness of the restraining supports.

The preceding is a simplification of the mechanism for development of resistance in a post-tensioned member, intended to present the concept. With increase in applied load, there will be an increase in tendon strain, which in turn results in an increase in tendon force. At ultimate limit state, the force in the tendon (F) is thus $F_2 + \delta F_2$, where δF_2 is the increase in tendon force due to local strain. The amount of the increase depends on whether the tendon is bonded or unbonded. For bonded tendons, the increase is local and can bring the tendon's stress to its ultimate strength (f_{pu}). For unbonded tendons the increase is typically considerably less.

To illustrate the concept, in the following the extreme condition of large support restraint is examined. In this condition, the entire post-tensioning is diverted to the supports, leading to cracks through the depth of a member. Non-prestressed reinforcement helps to control crack width and crack dispersion. To highlight the interaction of prestressing and the restraint of the supports, in the following the contribution of non-prestressed reinforcement is not accounted for.

3 UNBONDED TENDONS; RESTRAINT CRACKS AND SAFETY

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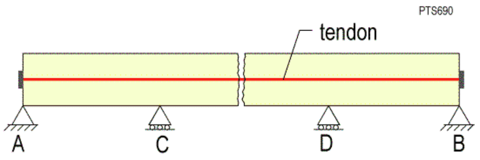
[3] There will also be a moment at the end of the member due to the shift of the restraining force (F_3) at the support from the support/member interface to the centroid of the member shown in part (b). This moment is not shown in the figure, since its presence does not impact the current discussion.

Figure 3-1 shows a member reinforced with unbonded tendons with a single crack that extends through the depth of the section (part a). The crack is from shrinkage of concrete and the full restraint of the supports A and B to free shortening of the member. Supports C and D are shown as roller supports.

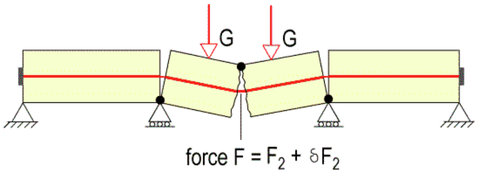
For simplicity and presentation of concept, tendons are shown along a straight line; selfweight and external loads are not shown. Note that in part (a) of the figure, the tendon retains its force (F_2) across the cracked section, but there is no compressive force on the face of the crack, since the member is assumed fully fixed against shortening at its end supports. The entire tendon force is diverted to the supports A and B. The force F_2 in tendon at service is the same as the restraint of support F_3 (not shown in the figure)

An idealized partial free body diagram of the left segment of the member for the post-tensioning forces is shown in part (c) of the Figure.

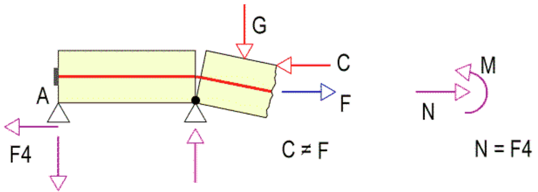
With increase in the applied load, the member will develop hinge lines at the locations marked in Fig. 3-1b. The downward displacement of the slab prior to collapse will elongate the tendons along their length, resulting in an increase (δF_2) in the tendon force. The initial tendon force at location of through crack under service condition (F_2) will increase to its final value F as shown in part (c) of the figure. The impending failure mechanism re-establishes contact between the two sides of the crack, where a compressive force C will develop. For unbonded tendons, the increase in tendon force across the crack prior to failure will be partially transferred to the supports A and B[4] because although the member itself is restrained against movement, the tendon can slide within its sheathing. At the ultimate state, the restraint force from the supports increases to F_4.



(a) Member with through restraint crack



(b) Failure mechanism



(c) Partial free body diagram

(d) Force demand at crack

FIGURE 3-1 Failure Mechanism and Partial Force Diagram of Member with Through Crack

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[4] If the member length is longer than is common in building construction, the increase in tendon force can be absorbed by the increase in friction, but this seldom occurs in practical conditions.

In part (a) the force at restrained support A is F_3, The tendon force at the crack gap is F_2. Since the gap extends through the depth of the member, $F_2 = F_3$. In part (b) the stretching of tendon increases its force at the crack from the service condition F_2 to F, as shown in part (c)

The force demand (design values) at the crack will be the axial tension N, moment M and shear force V shown in part (d) of the figure. The axial tension N equals F_4, the restraint of the support at point A at Ultimate Limit Sate from equilibrium of forces along the member $N = F_4$.

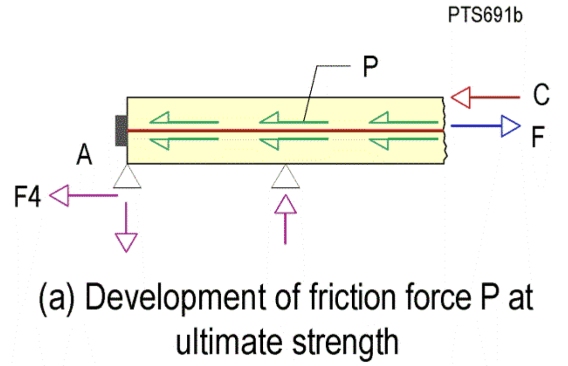
The demand actions N and M at the cracked section are resisted by the increase in tendon force across the crack resulting from the displacement of the member and the compressive force (C) developed at the newly established contact surface.

The relationships are:

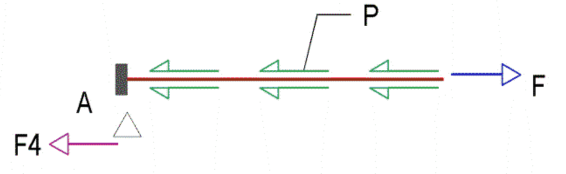
$C = F - F_4$ (Exp 3-1)
$M = C z$, (Exp 3-2)
$M = (F - F_4) z$ (Exp 3-2)

Where z is the lever arm between the centroids of the tension and compression forces, and F is the force in the tendon at the crack. Note that the tensile force in the tendon that contributes to the resistance capacity of the cracked section is the difference between the force in the tendon at the crack (F) and the restraint of the support (F_4).

The partial free body diagram of the horizontal forces for the left segment of the member of Figure 3-2 is shown in Fig. 3-3. The figure shows the contribution of the friction forces (P) between a strand and its sheathing in developing the compression force (C). It is shown in the following that the compressive force C that can develop across the crack prior to the collapse of the member is limited to the friction force (P) that builds up between a strand and its sheathing (part b of the Figure). This is based on the initial premise that the support restraint at A is large enough to prevent the shortening of the member.



(a) Development of friction force P at ultimate strength



(b) Free body diagram of horizontal forces

FIGURE 3-3 Development of Friction Force P at Ultimate Strength

Part (a) shows the segment prior to collapse, where force C develops at contact area between the two sides of the crack.

From part (b) of the figure:

$P = F - F_4$ (Exp 3-3)

From part (a) of the figure:

$C = F – F_4$ (Exp 3-4)

Where F_4 is the restraint from the support at the ultimate limit state. Therefore,

$C = P$ (Exp 3-5)

To arrive at the upper- bound for the moment that can possibly develop at the crack, the tendon is assumed to be stretched to its rupture force, recognizing that this is impractical for unbonded tendons, before a member can be considered "failed."

The force F in the tendon is calculated as:

$F = A_{ps}\ f_{pu}$ (Exp 3-6)

Where A_{ps} is the tendon cross-sectional area and f_{pu}[5] is its specified strand strength (commonly 270 ksi; 1860 MPa). The tendon force F will decrease along the tendon length due to friction between the tendon and its sheathing. For a given tendon profile and friction coefficients, the stress loss due to friction can be calculated with the following equation:

$P_x = P_j e^{-(\mu\alpha + Kx)}$ (Exp 3-7)

Where,

P_x = stress in tendon at distance x from the point of application of force to tendon;
P_j = stress in tendon at the point of application of force;
μ = coefficient of angular friction (/radian);
α = total angle change of the strand in radians from the stressing point to distance x;
x = distance from the stressing point; and
K = coefficient of wobble friction (/ft[6]; /m).

Once the friction force P and hence the compressive force C across the crack are determined, the design capacity of the section is known. Note that in an actual structure, the contribution of the non-prestressed steel across the crack must be included in the calculations; the compressive force C will be resisted by both the tendons and the non-prestressed reinforcement. The capacity of the section will depend on the location and the magnitude of the tendon force and the location and amount of the non-prestressed reinforcement.

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[5] In practice f_{pu} is unlikely to materialize for unbonded tendon. The current discussion is a hypothetical case for an upper-bound limit

[6] The dimension of the wobble coefficient K includes the coefficient of friction. K is μ (average of unintended change in angle per unit length of tendon.

In the general case, a restraint crack is likely to break the member into two non-equal lengths as illustrated in the example of Fig. 3-4. For static equilibrium of the member shown, the restraining forces (F_4) on each side of the crack must be equal. Thus the friction force (P) that can be sustained across the crack is that from the segment with the smaller friction loss – typically the shorter side of the member. Concluding with $C = P$, the moment capacity is:

$M = Pz$ (Exp 3-8)

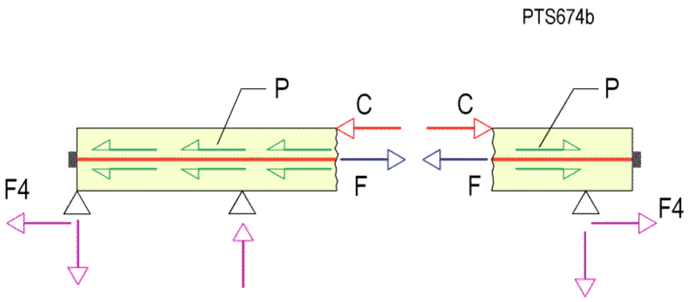
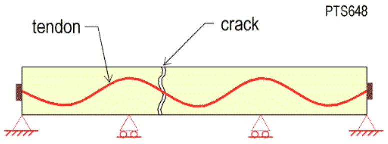
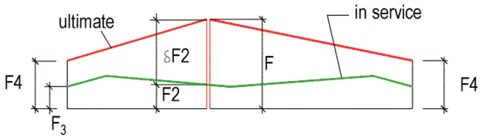


FIGURE 3-4 Partial Free Body Diagram of a Non-Symmetrical Member Cracking

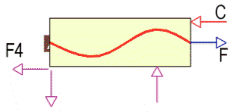
In summary, for the conditions discussed, the maximum tensile force that will be available to develop a resisting moment at the crack is limited to the friction that develops between the tendon and its sheathing at ultimate limit state. This is further illustrated in Fig. 3-5. In this Figure $F_2 = F_3$ is the in-service tendon force at the location of through crack prior to the application of added load and establishment of the compression force C (refer to Fig. 3-1).



(a) Elevation of cracked member



(b) Tendon force diagram at strength and serviceability limit states



(c) Section forces at crack (ULS)

FIGURE 3-5 Member with Unbonded Tendons; Force Diagrams and Service and at Strength Limit State
In part (b) the tendon fore in service F_2 at the location of crack is equal to the support restraint F_3. At strength limit, the tendon force at location of crack is increased to F.

In the preceding diagram, the force ($F - F_4$) is the force that will be available to resist applied moments at ultimate limit state – the moment capacity of the section. The force ($F - F_4$) is the friction force between the strand and its sheathing.

Further, it is concluded that when members reinforced with unbonded tendons experience excessive support restraints, the friction between a tendon and its sheathing plays a role in the strength available from the tendon at the member's ultimate strength capacity. The larger the friction force between a tendon and its sheathing, the greater will be the available tendon strength to resist applied loads.

4 BONDED TENDONS; RESTRAINT CRACKS AND SAFETY

Members reinforced with bonded tendons develop a larger moment capacity at locations of restraint cracks compared to members that are reinforced with the same amount of unbonded post-tensioning. There are three reasons.

First, bonded tendons can typically develop their specified strength (f_{pu}) prior to failure, whereas members reinforced with unbonded tendons tend to undergo large deflections, and reach failure due to crushing of concrete or excessive deflection, before tendons reach their specified strength (f_{pu}). Consequently, ACI 318[7] [ACI 318, 2011] and EC2 [EC2, 2004] specify a significantly lower permissible stress (f_{ps}) for unbonded than bonded tendons for flexural capacity design of concrete members. Depending on the span dimensions for unbonded tendons, ACI 318 limits the increase in tendon stress at ultimate strength to between 30 to 60 ksi (206 to 413 MPa), whereas in EC2 the increase is limited to merely 100 MPa[8] (15 ksi). This is about 7 to 9 % gain in strength over service condition, leaving about 30% of a tendon's strength untapped at member failure.

Second, for members reinforced with bonded tendons, the increase in demand moment at a point results in an increase in the tendon force at the same location. This local increase in tendon force is not compromised by the restraint of the supports. On the other hand, for unbonded tendons – as outlined in the previous sections – support restraints can diminish the effectiveness of local increase in tendon force in resisting an applied moment. This is explained in detail in what follows.

Third, compared to unbonded tendons, for bonded tendons the higher friction force between the strand and the sheathing at stressing works advantageously at the strength limit state of a cracked section.

Consider the member with a bonded tendon shown in Fig. 4-1. Let the restraint from the supports be large enough to cause cracking as shown in part (a). The force in the tendon at the time of grouting follows essentially the friction diagram shown in part (b). Let the force in tendon at location of crack in service condition be F_2. For the static equilibrium of the arrangement shown, F_2 is equal to the restraint of the support (F_3) while the gap at the crack is open. An increase in the applied moment at the crack location will tend to elongate the tendon locally leading to an increase in the tendon force by δF_2 (part c of the Figure).

The demand actions at the location of crack (part d of the Figure) are moment M, axial force N and shear V. From equilibrium of the forces N *is* equal to F_3, the force due to restraint of the support.

The tensile force available to resist the demand actions at the crack location is:

$$T = F_2 + \delta F_2 - F_3 \qquad \text{(Exp 4-1)}$$

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[7] ACI 318-11 Section 18.7

[8] EC2 Section 5.10.8(2)

Since at location of crack $F_2 = F_3$, the available force (T) to resist the induced moment will be equal to δF_2. Likewise from equilibrium of forces the compression force C is

$$C = (F_2 + \delta F_2) - F_3 = \delta F_2 \qquad \text{(Exp 4-2)}$$

The moment that can be developed at the crack M is equal to:

$$M = Cz = \delta F_2 z \qquad \text{(Exp 4-3)}$$

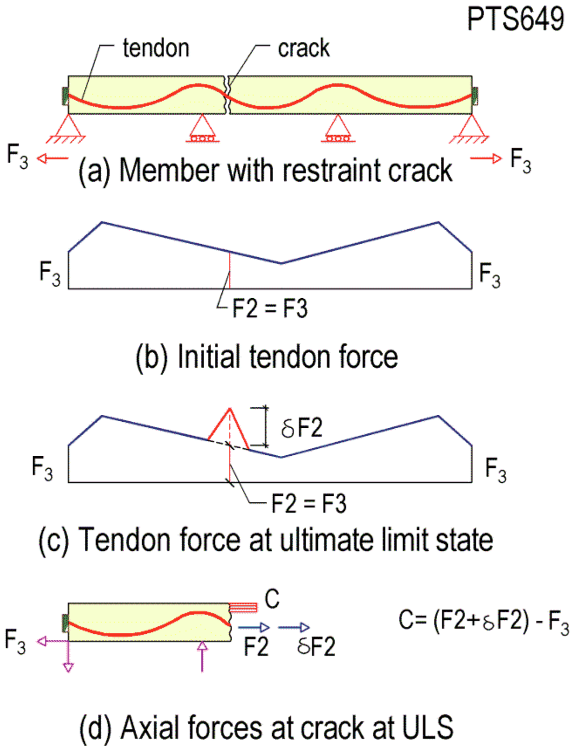
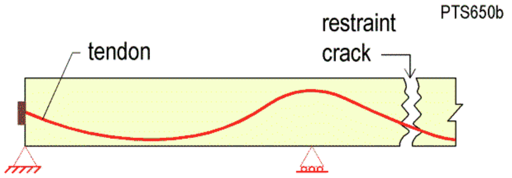


FIGURE 4-1 Member with Bonded Tendon and Restraint Crack; Forces at Strength Limit

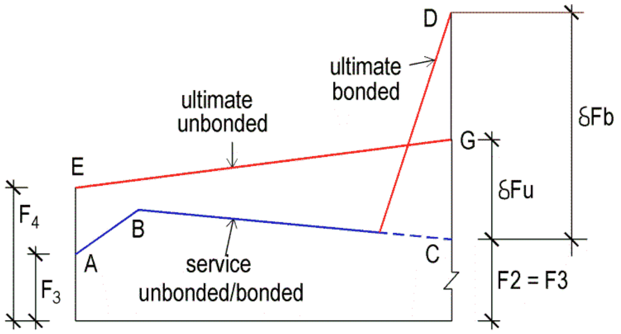
5 COMPARISON BETWEEN UNBONDED AND BONDED SYSTEMS

Figure 5-1 compares the performance of a member reinforced with unbonded post-tensioning to that of a member with bonded post-tensioning with respect to the impact of restraint cracks on member safety. Referring to the figure, the net force (T) from the post-tensioning tendons available at the crack to resist the demand moment is:

Unbonded: $T = (F_2 + \delta F_u) - F_4$ (Exp 5-1)
Bonded : $T = \delta F_b$ (Exp 5-2)



(a) Member elevation with restraint crack



(b) Tendon force at service and ultimate limit states

FIGURE 5-1 Comparative Distribution of Force in Tendon at Ultimate Limit State for Bonded and Unbonded post-tensioning Systems.

The diagram shows the distribution of force in tendon for the segment to the left of the crack. Line ABC is the force in tendon in service condition. It is influenced by the friction and seating loss at stressing. For illustration of concept, it is assumed to be the same for bonded and unbonded systems and represent the service condition. For moment capacity at crack location there will be a local increase in tendon force for the bonded system marked by point D. The tendon force available to resist the demand moment is equal to local increase in tendon force (δF_b) shown by CD. At strength limit, the distribution of force in the unbonded tendon will be governed by the re-alignment of friction force along the tendon length from line ABC to line EG. The available force to resist the moment demand will be $(F_2 + \delta F_u) - F_4$,

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