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Unified Minimum Flexural Reinforcement Requirements for Reinforced and Prestressed Concrete Members



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This study was undertaken to simplify, and to the extent possible, to unify ACI 318 minimum flexural reinforcement requirements for reinforced and prestressed concrete members. A major goal was to eliminate the feature of current provisions for prestressed concrete members in which the minimum reinforcement requirement increases with increasing levels of prestress. The concept of increasing minimum reinforcement requirements for prestressed members with increases in the level of prestress is not supported by the research results reviewed in this study, or by the analytical investigations reported herein. This study includes review of relevant research reports, review of existing ACI 318 code provisions for minimum flexural reinforcement, and review of the minimum flexural reinforcement provisions of the CEB-FIP Model Code for Concrete Structures. On the basis of this review, new unified formulas and essentially unified code provisions are proposed for minimum flexural reinforcement of reinforced and prestressed concrete members. The formulas are based on the tensile strength of both prestressed and nonprestressed reinforcement (as contrasted with the previous formula for reinforced concrete based on yield strength of nonprestressed reinforcement). The new formulas include consideration of variations in concrete strength.

Keywords: applied loads; bonded tendons; cracking load; cracking moment; flat plates; legal loads; minimum flexural reinforcement; precast sections; prestressed concrete; reinforced concrete; structural research; supplementary bonded reinforcement; T-beams; unbonded tendons.

1—INTRODUCTION*

This study of minimum reinforcement requirements in reinforced and prestressed concrete members was undertaken with the purpose of simplifying, and to the extent feasible, unifying the minimum reinforcement requirements for both reinforced and prestressed concrete.

The opening paragraph of the ACI 318-71 commentary to section 10.5 is as follows:

“Section 10.5.1 is concerned with beams, which for architectural or other reasons, are much larger in cross section than required by strength considerations. With very small reinforcement ratios, the computed bending moment as a reinforced concrete section becomes less than that of the

corresponding plain concrete section computed from its modulus of rupture. Failure in such a case is quite sudden.”

In regard to this commentary suggestion of possible sudden flexural failure of reinforced (or prestressed) concrete members, it is first noted that there are no published reports of such failures in buildings or bridges, and the authors are unaware of any failures of this type in practice over the past 40 years. Second, the possibility of sudden failure of reinforced or prestressed flexural members observed in a limited number of cases in laboratory tests, has been associated with loads which were a substantial multiple of the design (factored) loads calculated on the basis of the reinforcement in the members. Accordingly, there is considered to be no basis in research or experience to suggest that sudden failure of grossly under-reinforced prestressed or nonprestressed flexural members will occur at the design loading associated with the reinforcement in the member.

Under the ACI Building Code, flexural members are designed to support factored legal loads (from Model Building Codes) as cracked reinforced or prestressed concrete sections. Minimum reinforcement requirements based on the cracking moment represent an alternative loading system unrelated to the legally specified loading. The loading related to the cracking moment is only a function of section geometry and concrete strength for reinforced concrete members. For prestressed concrete members, the cracking load is also a function of the amount of prestressed reinforcement and its location.

The alternative loading based on the cracking moment may be a large multiple of the legally specified loading. For canti-

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levered conventionally reinforced concrete T-beams with the flange in tension, the loading resulting from the cracking moment criteria may be more than thirty times the legally specified loading. The ACI Building Code places an upper limit on the amount of additional reinforcement necessary to satisfy cracking moment loading requirements by accepting reinforced concrete section with an area of tensile reinforcement at least one-third greater than required by analysis (Section 10.5.3), and by accepting prestressed members with shear and flexural strength at least twice that required (Section 18.8.3). However, this paper illustrates that the discrepancy between the one and one-third factor for nonprestressed reinforced concrete sections, and the factor of two specified for prestressed concrete, is not technically justified.

Review of research on minimum reinforcement requirements for nonprestressed reinforced concrete members reveals that, due to strain hardening of the reinforcement, the capacity of grossly under-reinforced members usually exceeds the design capacity by a factor of about 1.5 to 1.75 (the ratio f_s'/f_y). While strain hardening is not generally considered in design of nonprestressed flexural members, it is considered appropriate to use strain hardening with respect to proportioning of minimum reinforcement (to resist loads associated with the cracking moment which are much higher than design loads for grossly under-reinforced members.)

Review of research on prestressed concrete members with very small reinforcement ratios reveals that loss of load capacity at the time of cracking was observed only on members with unbonded tendons without supplementary bonded reinforcement. Although none of the members with unbonded tendons which exhibited a loss of load capacity at the time of cracking failed in a sudden manner, sudden failure might have taken place in a limited number of cases if the tests had been conducted using a pressure controlled rather than a strain controlled loading device (in which case the load would not have been reduced at the time of cracking). Behavior of members reinforced only with unbonded tendons has not been of practical relevance since publication of ACI 318-71 which required use of supplementary bonded reinforcement in such members. In tests of members containing unbonded tendons and supplementary bonded reinforcement, no loss of load capacity was observed at the time of cracking. With regard to minimum reinforcement requirements of prestressed members with bonded tendons (pretensioned or post-tensioned), it is significant that no loss of load capacity was observed at the time of cracking in tests of 33 bonded post-tensioned members with reinforcement ratios as low as 0.101 percent.

Application of the minimum reinforcement provisions of Section 18.8.3 of ACI 318-95 based on 1.2 times the cracking load results in larger minimum reinforcement requirements as the level of prestress increases. Since the cracking load is already a multiple of the design load for grossly under-reinforced members, use of the 1.2 factor on the cracking load is considered to be unwarranted. In any case, this provision is particularly unfortunate for continuous post-tensioned members which often have areas with very high residual compressive stress at service loads, with corresponding high cracking loads. Research on prestressed concrete members does not support the concept that minimum flexural reinforcement requirements should be increased in proportion to increases in the level of prestress. This paper demonstrates that this concept can be eliminated from ACI minimum reinforcement requirements with safety against the possibility of sudden flexural failure. Furthermore, this paper presents essentially unified minimum reinforcement provisions for nonprestressed and prestressed flexural members.

2—REVIEW OF UNIVERSITY OF ILLINOIS TESTS OF REINFORCED CONCRETE BEAMS¹

Three beams, designated N-1, N-2, and N-3, were tested in the referenced series with a very small percentage of tension reinforcement and a relatively high concrete strength. The beams were 6 inches wide and 12 inches deep. Beams N-1 and N-2 contained one Number 3 Grade 40 rebar, and Beam N-3 contained one Number 4 Grade 40 rebar. The strength of the rebar was 70 ksi. The University of Illinois report discussing these tests states: "The beams were designed in such a manner that the tension reinforcement would be stressed up to or beyond its yield point when it was called upon to resist the load corresponding to the load causing cracking."

A comparison of the test capacity and the factored load capacity of the three beams in accordance with ACI 318-89 is presented in Table 1. A graphical presentation of the test results in comparison with the ACI 318-89 capacities of the beams is presented in Fig. 1 (Fig. 25 from the University of Illinois test report). The moment capacity of beams N-1 and N-2 indicates that the full f_s' , or full strain hardening of the steel was developed. In the case of beam N-3, full strain hardening did not develop due to a bond failure.

While these beams did not fail suddenly at the time of cracking, the failures might have been sudden if the test machine had been such that the load was not reduced at the time of cracking (load controlled rather than strain controlled testing device). The cracking loads for beams N-1 and N-2 were, respectively, 1.83 and 2.54 times the design (factored) load permitted by ACI 318-89 on the basis of the reinforcement

Table 1—Comparison of test capacity and factored load capacity of three beams in accordance with ACI 318-89

Beam	Dead load, lb/lf	Service LL permitted by ACI 318-89, lb/lf	Measured capacity*	1.4D + 1.7L ACI 318-89	Measured capacity
					1.4D + 1.7L
N-1	75	144	719	350	2.05
N-2	75	144	742	350	2.12
N-3	75	314	964	639	1.51 [†]

* Equivalent uniform applied load plus 75 lb/lf DL

† Bond failure

provided. The probability of loadings of these multiples of design load in a building (as distinct from a member in a laboratory testing machine) is considered to be very remote. Fortunately, for most flexural members, concern about even such remote possibilities of sudden failure can be eliminated with reasonable economy by use of a simple equation for minimum flexural reinforcement.

In summary, these three test beams illustrate that full strain hardening can be anticipated in grossly under-reinforced non-prestressed rectangular beams. For this reason, such beams inherently provide reserve (not considered in design) factors of safety with reference to design loads in accordance with the ratio of f'_s/f_y . For the grades of reinforcement commonly used for flexural members, these ratios are:

	f_y	f'_s	f'_s/f_y
	ksi	ksi	
Grade 40	40	70	1.75
Grade 60	60	90	1.50

3—REVIEW OF UNIVERSITY OF ILLINOIS TESTS ON PRESTRESSED CONCRETE MEMBERS

General review of tests

Among the primary research resources in the early development of prestressed concrete technology in the U.S. were two reports published by the University of Illinois: "Strength in Flexure of Bonded and Unbonded Prestressed Concrete Beams" by J. Warwaruk, August 1957,² and "Strength and Behavior in Flexure of Prestressed Concrete Beams," by J. Warwaruk, M. Sozen, and C. P. Siess, September 1960³ (These reports were the basis for University of Illinois Engineering Experiment Station Bulletin 464, by J. Warwaruk, M. Sozen, and C. P. Siess, 1962, which is more generally available in technical libraries).

A total of eleven simple span beams (9-ft span) with nominal dimensions of $b = 6$ in., and $h = 12$ in., were tested at the University of Illinois under the work reported in the August 1957 report. In reference to this discussion of minimum flexural reinforcement requirements, the most significant specimen in this series was the pretensioned beam J-3 which had no supplementary bonded reinforcement and a reinforcement ratio, ρ (in percent), for the prestressing steel of 0.166. The value of M_u/M_{cr} for this simple beam was 1.61. The minimum ρ value for prestressed beams proposed in this study is 0.200 (see Section 9).

Tests of 82 simple span prestressed beams of the same span and nominal dimensions as used for the tests discussed above are referenced in the Sept. 1960 report. The eleven beams tested in Ref. 2 are included in the test data discussed in this report. In the narrative summary on the tests in this report, the last paragraph is as follows: "If a bonded or an unbonded beam is simply supported or if its various critical sections are reinforced in the same proportion to the 'elastic moments' for a possible condition of loading, it is desirable that

$$M_u \geq M_{cr} \quad (83)$$

at each section in order to avoid a sudden failure immediately after cracking. This sets automatically a limit on the minimum amount of reinforcement in relation to the shape and size of the beam, the quality of the concrete, and the level of

prestress. *Simply-supported beams* [emphasis added] for which Eq. (83) is not satisfied are quite undesirable because failure may occur without any warning. It should be noted that Eq. (1) may be satisfied by increasing the amount of supplementary reinforcement as well as the amount of prestressed reinforcement."

One of the 82 beams discussed in Ref. 3, an unbonded beam with no bonded reinforcement (identified as OU34.034) had a ratio of M_u/M_{cr} (measured values) of 0.96. As is the case with all tests in this series designated "OU," this beam does not comply with the minimum bonded reinforcement requirements of ACI 318 for members with unbonded tendons. (Minimum bonded reinforcement provisions for members with unbonded tendons were first included in ACI 318-71, and remain unchanged for one-way members to this date.)

Deflection behavior

Review of measured and computed midspan deflection for the 82 tests reported in Ref. 3 discloses that none of the beams failed without ample warning deflection. The largest ratios of $\Delta_{Fail}/\Delta_{Cr}$ occurred in the most lightly reinforced beams. However, all of these tests were conducted with strain controlled devices. The failures of some of the unbonded beams which lost load capacity at the time of cracking might have been sudden if a load controlled testing device had been used.

Load in prestressed test specimens with bonded reinforcement ratios as low as 0.101 and 0.107 percent increased after cracking, and there was no indication that any member with a larger amount of reinforcement would fail suddenly at any load without a large amount of warning deflection.

Proportioning of test specimens with respect to applied loads

It is considered important to note that the reinforcement of the University of Illinois test beams was not proportioned in reference to any standard for the load factor required under the applied loads. Beams with various amounts of bonded or unbonded reinforcement were simply tested to failure and the behavior was reported. In this context, the load-deflection curves for tests including beam OU34.034 are presented in Fig. 2 (Fig. 44 from the University of Illinois report). Beam OU34.034 had a ratio of M_u/M_{cr} of 0.96.

The factored live load capacity of Beam OU34.034 based on stress in unbonded tendons specified by equation (26-7) of ACI 318-63 (when the cracking load criteria was introduced) was computed to be 3.95 kips. The factored live load capacity of Beam OU34.034 based on stress in the unbonded tendons specified by Equation (18-4) of ACI 318-89 was computed to be 4.76 kips. The capacity of Beam OU34.034 in the test was found to be 6.604 kips. The ratio of test capacity to code capacity factored live load for this beam is 1.67 based on ACI 318-63, and 1.39 based on ACI 318-89, as shown on Fig. 2. This result indicates that the beam in this test series with the least satisfactory behavior with respect to the cracking moment provided a substantial margin of safety with respect to legally specified loads for the member. In reality, Beam OU.34.034 could not have been legally used in 1989 since it did not contain the minimum bonded nonprestressed reinforcement specified since ACI 318-71.

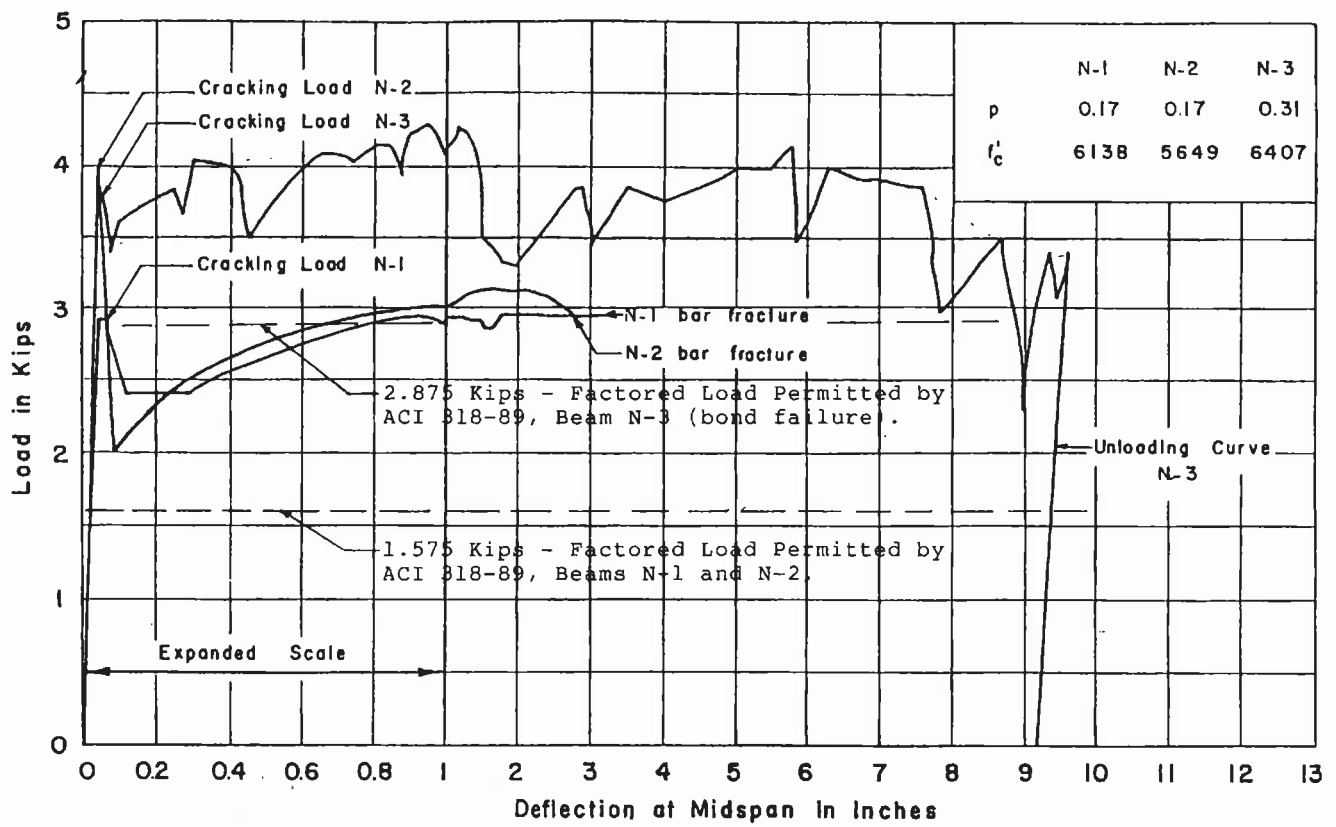


Fig. 1—Load-deflection diagrams for University of Illinois beam series N¹

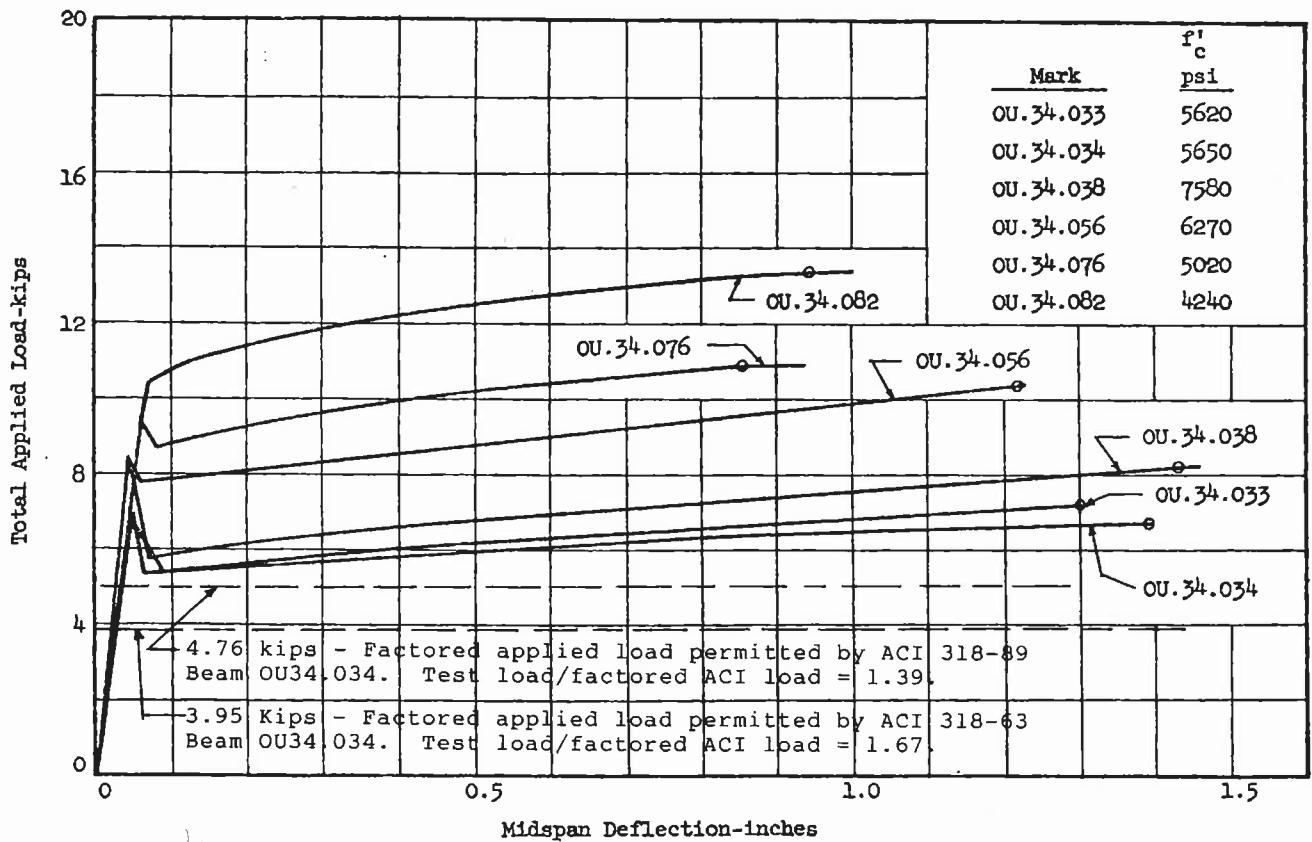


Fig. 2—Load-deflection curves for beams of series OU.34³

Location of reinforcement in unbonded members without bonded reinforcement

The average h/d ratio for the 26 beams tested without bonded reinforcement (designated “OU” in Appendix B) was 1.58 and the values ranged from 1.44 to 1.73. The h/d values for the 33 bonded “OB” post-tensioned members averaged 1.39, and ranged from 1.26 to 1.51. For the unbonded “OU” beams, the average d was 7.59 in., and for the bonded “OB” beams, the average d was 8.63 inches. Since the required reinforcement is a function of the value of the h/d ratio squared, the location of reinforcement in the “OB” beams with bonded prestressed steel was 1.3 times more effective than the reinforcement in the “OU” beams with unbonded prestressing steel. This is considered to be a contributing factor to the undesirable load-deflection behavior (loss of load capacity at the time of cracking) of a few of the lightly reinforced “OU” beams.

Summary of observations on University of Illinois tests of prestressed members

On the basis of the above discussions, the following observations are considered to be important points with reference to this discussion on minimum reinforcement requirements for prestressed concrete members:

1. No loss of load capacity at the time of cracking was observed for any beam with bonded prestressed or non-prestressed reinforcement.
2. None of the beams failed without large warning deflections. However, it is acknowledged that sudden failure might have occurred in a few beams with an unbonded tendon and no supplementary bonded reinforcement if a load controlled testing device had been used.
3. The unbonded beam with the least satisfactory ratio of M_u/M_{cr} (OU34.034) had a test capacity of 1.67 times the capacity calculated (permitted) on the basis of Equation (26-7) of ACI 318-63, and 1.39 times the capacity calculated on the basis of Equation (18-4) of ACI 318-89. This beam had no supplementary bonded reinforcement.
4. The tests do not support concerns about sudden failure of prestressed members with bonded prestressed reinforcement ratios as low as 0.101 percent.
5. The h/d ratios used for members with unbonded tendons and lack of supplementary bonded reinforcement contributed to the less satisfactory load-deflection behavior of lightly reinforced unbonded members at the time of cracking.

4—BACKGROUND ON ACI 318 MINIMUM REINFORCEMENT REQUIREMENTS

The minimum requirements for prestressed concrete were first introduced in Section 2609 of the 1963 Code (Section 18.8.3 in more recent editions of the Code). The language of Section 2609 was as follows:

“(c) The total amount of prestressed and unprestressed reinforcement shall be adequate to develop an ultimate load in flexure at least 1.2 times the cracking load calculated on the basis of a modulus of rupture of $7.5\sqrt{f'_c}$.”

This code provision appears to reflect the results of the University of Illinois tests of prestressed concrete members discussed in the previous section of this report. This is the first explicit statement in the code relating reinforcement re-

quirements to the cracking load calculated on the basis of the modulus of rupture. Minimum reinforcement requirements for reinforced concrete explicitly based on cracking moment or cracking load were not introduced until ACI 318-95.

At the time of the University of Illinois tests of prestressed members discussed above, and in ACI 318-63, there were no code provisions requiring minimum amounts of bonded reinforcement to be used in members with unbonded tendons. Minimum bonded reinforcement requirements for members with unbonded tendons were introduced in ACI 318-71. In Section 8 of this report, it will be shown that the minimum bonded reinforcement requirements for members with unbonded tendons are comparable to the minimum reinforcement requirements for reinforced concrete members in Section 10.5 of ACI 318-95.

In the 1983 code, an upper limit on the amount of reinforcement required by Section 18.8.3 was introduced permitting a waiver of the 1.2 times cracking load requirement for members with “shear and flexural strength at least twice that required by Section 9.2.” An upper limit of reinforcement “at least one-third greater than required by analysis” has appeared in Section 10.5 since ACI 318-63.

5—REVIEW OF MINIMUM REINFORCEMENT REQUIREMENTS OF THE CEB-FIP MODEL CODE FOR CONCRETE STRUCTURES

The third and fourth editions of the *CEB-FIP Model Code for Concrete Structures*⁴ contain simple, unified provisions for the minimum area of longitudinal reinforcement for reinforced concrete and prestressed concrete. These requirements are as follows for reinforced concrete members:

“9.2.2—BEAMS

9.2.2.1—Longitudinal reinforcement

A minimum area of longitudinal bonded reinforcement should be provided to avoid brittle failure in case of unforeseen loss of concrete tensile strength.

Commentary (Notes)

If a specific study is not carried out in this respect, the area of longitudinal tensile bonded reinforcement provided should be at least taken equal to:

- $0.0015 b_t d$ for steel grades S400 (58,000 psi) and S500 (72,500 psi).
- $0.0025 b_t d$ for steel grade S220 (31,900 psi).

where b_t is the average width of the concrete zone in tension.

in a T-beam, if the neutral axis in the ULS is located in the flange, the width of the latter is not taken into account in evaluating b_t .”

In the third edition of the CEB-FIP Model Code (1978), the $0.0015b_t d$ criteria also applied to members reinforced with prestressing steels. The notes to the 1978 CEB-FIP Model Code on slabs states that the minimum reinforcement requirements for beams also applies to slabs.

Comparison of the $0.0015b_t d$ criteria to the results of the University of Illinois test data for prestressed beams discussed in Section 3 reveals that all members with this amount of bonded reinforcement had satisfactory margins between the measured values of M_u and M_{cr} .

Comparison of the minimum reinforcement requirements of the CEB-FIP recommendations and ACI 318-95 for 15 reinforced concrete beam designs is presented in the following section of this report.

6—COMPARISON OF MINIMUM REINFORCEMENT REQUIREMENTS FOR EXAMPLE REINFORCED CONCRETE BEAM DESIGNS

To provide a basis for comparison of minimum reinforcement provisions for non-prestressed members, 15 designs were prepared of various applications, as follows:

1. Three simple span beams with spans of 9, 15, and 20 ft.
2. Five cantilever T-beams with section depths ranging from 24 to 50 in.
3. Five simple span T-beams with the same section properties as the cantilever T-beams.
4. Two rectangular cantilevers with depths of 12 and 36 in.

In the process of preparation of these reinforced concrete beam designs, it was noted that the CEB-FIP minimum reinforcement requirement appeared to be deficient in some cases. For this reason, the proposed unified ACI minimum reinforcement requirement (for concrete strength of 4000 psi) was increased about $\frac{1}{3}$ above the CEB-FIP value, resulting in the following equation:

$$A_{s, \min} = 0.0020b_w d$$

The parameter b_w was selected in preference to the CEB-FIP parameter b_t to simplify the calculation process. To include the concrete strength, 0.0020 was divided by the square root of 4000, and $\sqrt{f'_c}$ was added to the equation giving:

$$A_{s, \min} = 0.000032\sqrt{f'_c} b_w d$$

The above equation reflects the use of Grade 60 reinforcement. As discussed in a recent article,⁵ strain hardening can be considered to be fully developed for reinforced concrete members with $\rho < 0.25\rho_b$. To include the reinforcement strength, the equation is multiplied by 90,000, the tensile strength of Grade 60 reinforcement, and f_{su} (tensile strength of reinforcement) is added as a denominator to the equation:

$$A_{s, \min} = 2.88\frac{\sqrt{f'_c}}{f_{su}} b_w d$$

As a conservative approximation, the equation is rounded off to provide the following equation proposed for use by ACI:

$$A_{s, \min} = 3.0\frac{\sqrt{f'_c}}{f_{su}} b_w d$$

Table 3 presents a comparison of minimum reinforcement requirements using the CEB-FIP criteria (with web width for b_p , the ACI 318-95 criteria, and the proposed ACI criteria for

Table 3—Comparison of minimum reinforcement requirements: Example reinforced concrete beam designs

Example	CEB-FIP criteria	M_u^2	M_{cr}	ACI 318-95	Proposed ACI
	in. ²				
SSB-1	0.054	0.056	0.073	0.11 (0.076) [†]	0.072
SSB-2	0.090	0.11	0.106	0.189 (0.113) [†]	0.120
SSB-3	0.126	0.175	0.14	0.265 (0.138) [†]	0.17
TBC-1	0.86*	0.51	3.08	0.67 [‡]	0.67
TBS-1	0.86	3.16	1.35	1.82 (1.41) [†]	1.15
TBC-2	0.86*	0.80	5.02	1.00 [‡]	1.00
TBS-2	0.86	4.91	1.43	1.82 (1.49) [†]	1.15
TBC-3	0.60*	0.75	2.85	1.00 [‡]	0.96
TBS-3	0.61	4.63	1.02	1.29 (1.10) [†]	0.82
TBC-4	0.39*	0.38	2.23	0.51 [‡]	0.51
TBS-4	0.39	1.61	0.70	0.83 (0.76) [†]	0.53
TBC-5	0.26*	0.24	1.37	0.36 [‡]	0.36
TBS-5	0.26	1.11	0.46	0.56 (0.48) [†]	0.35
RC-1	0.62	0.47	0.67	0.63 [‡]	0.83
RC-2	0.19	0.21	0.24	0.28 [‡]	0.25

* CEB-FIP criteria based on web width for b_p

† ACI 318-95 area of steel in parentheses results from use of section-specific section properties to obtain ACI Code Eq. (10-3)

‡ Section 10.5.3 requirement of $1.33M_u$ reinforcement governs all cantilever cases

Grade 60 reinforcement. The “proposed ACI” reinforcement areas in Tables 3 and 4 are based on the equation:

$$A_{s, \min} = 0.000032\sqrt{f'_c} b_w d$$

The use of the generalized, unified equation,

$$A_{s, \min} = 3.0\frac{\sqrt{f'_c}}{f_{su}} b_w d,$$

would provide 4 percent more minimum reinforcement than the values shown in Tables 3 and 4.

Table 4 presents the results of capacity calculations performed with a computer program using non-linear material properties. Considering additional capacity available due to strain hardening, the proposed ACI equation values for minimum reinforcement as shown in Table 4 provide satisfactory ratios of $\phi M_{nsh}/M_u$ or $\phi M_{nsh}/M_{cr}$ for each of the 15 designs. Note that very high steel strains are obtained in all cases except a few of the T-beam cantilever designs with higher ρ values.

Table 4—Example design capacities considering nonlinear material properties and strain hardening

Type	Data	ACI prop. $A_{s,min}$	$\rho \times 10^3$	M_u	ϕM_n	M_{cr}	ϕM_{nsh}		$\frac{\phi M_{nsh}/\phi M_u}{\phi M_{nsh}/\phi M_{cr}^*}$ ft. kips	Strain $\times 10^3$	
				ft. kips			Approx.	Accurate		Concrete	Steel
Rectangle	SSB-1	0.07	2	1.48	1.91	1.94	2.86	2.76	1.42	-3.00	47.46
	SSB-2	0.12	2.00	4.95	5.30	4.78	7.95	7.69	1.61	-3.00	47.46
	SSB-3	0.17	2.02	10.81	10.52	8.89	15.78	15.23	1.71	-3.00	47.23
T-section	TBC-1	0.67	1.15	108.38	141.71	629.84	211.47	211.61	1.95	-2.05	50.00
	TBS-1	1.15	0.25	677.34	247.78	289.35	371.36	369.03	1.28	-0.83	50.00
	TBC-2	1.00	1.72	168.02	210.82	999.23	316.24	292.34	1.74	-3.00	44.23
	TBS-2	1.15	0.17	1050.44	248.04	307.29	372.06	365.50	1.19	-0.75	50.00
	TBC-3	0.96	2.32	111.75	144.42	435.52	216.43	193.99	1.74	-3.00	42.60
	TBS-3	0.82	0.22	698.44	125.20	154.56	187.64	183.73	1.19	-0.68	50.00
	TBC-4	0.51	1.89	36.45	48.47	204.92	72.7	70.88	1.94	-3.00	48.22
	TBS-4	0.53	0.22	158.2	52.15	68.31	78.23	76.20	1.12	-0.79	50.00
	TBC-5	0.36	2.00	22.01	35.01	126.97	52.51	50.27	2.28	-3.00	46.67
	TBS-5	0.35	0.24	109.15	34.77	44.90	52.16	51.75	1.15	-0.83	50.00
	Rectangle	RC-1	0.83	2.00	73.25	126.83	102.46	190.24	183.33	1.79	-3.00
RC-2		0.25	1.98	9.9	11.63	11.38	17.45	16.97	1.49	-3.00	48.22

* $\phi M_{nsh}/\phi M_u$ is used for cantilevered T-beams; $\phi M_{nsh}/\phi M_{cr}$ is used in all other cases

7—CONSIDERATION OF CRACKING MOMENT IN TERMS OF APPLIED LIVE LOAD

Minimum reinforcement requirements for prestressed elements are expressed in Section 18.8.3 of ACI 318-89 in terms of the cracking load (actually 1.2 times the cracking load). Minimum reinforcement requirements for reinforced concrete members may also be expressed in terms of the cracking load. In contrast to the legally specified loadings in the model building codes, the cracking loads are independent of the application, and are related only to the concrete strength and the section properties of the member under consideration. The cracking loads are also unrelated to the strength requirements of Section 9. For prestressed concrete members, the cracking loads are further complicated by the arbitrary value of service load tensile stress selected by the designer, by the relative span lengths, and by the selected tendon profile.

As an illustration of the difference between legally specified loads and cracking loads, Table 5 presents the multiples of a 50 psf live load required to produce cracking on the reinforced concrete T-beam design examples discussed in Section 7. These are assumed to be simple span beams with a cantilever at one end.

Concrete strength is taken as 4000 psi. The range of these multiples of the 50 psf live load for negative moment (17 to 31) raises a question as to whether it is reasonable to consider the cracking moment of T-beams in design when the flange is in tension. In any case, it is apparent from these multiples that the designer will opt for the provisions of Section 10.5.3 of ACI 318-95 in proportioning reinforcement for these T-beams rather than the amount required by the cracking load. This will be the case with most T-beam designs with the flange in tension. Accordingly, for T-beams the minimum reinforcement requirements of Section 10.5.3 of ACI 318-95 will, in a majority of cases, require that all reinforcement be increased to $1\frac{1}{3}$ the amount required by analysis. This comparison indicates that use of cracking load or cracking moment controls the design of routine members, and is not restricted to “members which for architectural or other reasons, are much larger in cross section than required by

Table 5—Multiples of 50 psf live load required to produce cracking in T-Beam example designs

Beam	Cantilever, ft	Span, ft	Depth, in.	Multiple of 50 psf LL to produce cracking	
				Neg. M	Pos. M
TB-1	10	50	50	29	None*
TB-2	10	50	50	31	None*
TB-3	10	50	36	17	None*
TB-4	6	25	24	23	None*
TB-5	6	25	24	24	0.5

* Section cracked under dead load

strength considerations” as indicated in the Commentary to Section 10.5.1 in ACI 318-89.

As discussed in the section of this report on the review of ACI 318-95 Section 10.5, Section 10.5.3 appears to require reinforcement at least one-third greater than required from analysis at every section (positive and negative) whenever the requirements of Section 10.5.1 and 10.5.2 are waived. Section 10.5.3 does not say specifically that only the flange reinforcement needs to be increased one-third when the requirements of Section 10.5.2 are waived.

Note that T-beams are cracked under positive dead load moment in most cases. For lightly reinforced T-beams under positive moment, visible cracking and deflection would necessarily result under small values of superimposed loading.

8—COMPARISON OF REINFORCEMENT REQUIREMENTS OF SECTION 10.5 OF ACI 318-95 WITH SECTION 18.9.2 OF ACI 318-89

Rectangular sections

The minimum bonded reinforcement for rectangular sections with unbonded tendons required by Section 18.9.2 can be expressed as $0.002 bh$. Note that this value exceeds the basic proposed minimum reinforcement equation in Section 6 ($0.0020 b_w d$) by the ratio of h/d . For member depths of 1 to 4 feet, width of 6 in., and a dimension of $1\frac{1}{2}$ inches from the bottom of the section to the centerline of the reinforcement, Section 18.9.2 can be compared with the minimum rein-

forcement requirements of as follows ($f'_c = 6000$ psi, d taken as $h - 1$ in.).

As illustrated by Table 6, the proposed ACI unified equation for minimum reinforcement ($0.0020 b_w d$), nearly identical to Section 18.9.2, provides sufficient reinforcement to resist the cracking moment of the plain concrete section (with a safety margin) in rectangular members prestressed with unbonded tendons. The difference between ACI 318-95 requirements for rectangular sections and Section 18.9.2 (or proposed ACI unified requirements) results primarily from the inaccuracy involved in the use of the single value of 3 as the multiplier in Equation (10-3) in ACI 318-95 for both rectangular sections and T-beams with the flange in compression. A multiplier of about 1.5 is sufficient to satisfy the cracking moment requirement for rectangular sections.

For members with unbonded tendons and a span-to-depth ratio of 35 or less, the ACI 318-89 stress in unbonded tendons at design loads is:

$$f_{ps} = f_{se} + 10000 + \frac{f'_c}{100\rho_p} \quad (18-4)$$

Based on $\rho_p = 0.0012$, the increase of stress from service load to design load for a concrete strength of 5000 psi is 51,667 psi. Estimating the stress in the unbonded tendons at service load of $0.60f = 162,000$ psi, the stress increase at design load is 32 percent. The combination of bonded reinforcement sufficient to provide a margin against the cracking load of the plain concrete section, and a 32 percent increase in the stress in the unbonded tendons after cracking obviously provides a member with a substantial margin of capacity relative to the cracking load.

T-beams

A comparison of bonded reinforcement requirements for various sizes of T-beams with unbonded tendons based on Section 18.9.2 and the requirements of ACI 318-95 is presented in Tables 7 and 8. These tabulations are based on the following parameters and criteria:

- $f'_c = 6000$ psi
- $h/d = 1.05$ or 1.20 (closest value)
- slab width = $b_w + 16t$, where t = slab thickness
- $A_s = 0.004A$, where A = area of that part of the cross section between the flexural tension face and center of gravity of gross section, sq. in.

Table 8 indicates that the bonded reinforcement provided by 18.9.2 for T-beams is approximately equal to the minimum amount required by ACI 318-95 for non-prestressed T-beams. In view of the stress increase in unbonded tendons from service to design load, as discussed in reference to rectangular sections above, T-beams reinforced with unbonded tendons, and bonded reinforcement in accordance with Section 18.9.2, provide substantial margins of safety against the cracking moment. In this regard, T-beams with unbonded tendons with the flange in tension provide the full M_{cr} capacity, whereas the minimum reinforcement for conventionally reinforced T-beams with the flange in tension is only re-

Table 6—Comparison of reinforcement provided by Section 18.9.2 and minimum reinforcement required by ACI 318-95 for rectangular sections

Member depth, in.	A_s , Section 18.9.2, in. ²	ρ , Section 18.9.2, percent	$A_{s,min}$, ACI 318-95, in. ²	$A_{s,min}$, $0.0020 b_w d$, in. ²
12	0.144	0.229	0.244	0.132
24	0.288	0.213	0.522	0.276
36	0.432	0.209	0.801	0.420
48	0.576	0.206	1.08	0.564

Table 7—T-beam section properties for comparative designs

Beam depth, in.	Flange, in.	Flange width, in.	b_w , in.	Depth to N.A., in.	A comp. flange, in. ²	A tension flange, in. ²
18	6	106	10	4.43	286.42	469.6
36	7	122	10	8.06	279.40	864.6
486	8	140	12	11.20	441.60	1159.0

Table 8—Comparison of Section 18.9.2 reinforcement requirements for T-beams with ACI 318-95

Beam depth, in.	A_s , (18.9.2)		A_s , (ACI 318-95)	
	Comp. flange	Tension flange	Comp. flange	Tension flange
	in. ²			
18	1.15	1.88	0.62	1.23
36	1.12	3.46	1.33	2.62
48	1.77	4.63	2.15	4.25

quired to provide $1\frac{1}{3} \times M_u$. As indicated by reinforced concrete T-beam examples with the flange in tension (Section 6), the $1\frac{1}{3} M_u$ reinforcement requirements are much less than the M_{cr} reinforcement requirements, and will therefore be used for most designs. Accordingly, T-beams with unbonded tendons will be the only T-beams designed in accordance with ACI 318 code provisions with both positive and negative moment capacity in excess of the cracking moment.

Conclusions

On the basis of the above discussion, the bonded reinforcement requirements of Section 18.9.2 for rectangular beams and T-beams with unbonded tendons equals or exceeds the bonded reinforcement required to resist the cracking moment of the plain concrete section with a margin of safety. In conjunction with the capacity provided by unbonded tendons, unbonded prestressed members designed in accordance with ACI 318 requirements inherently provide a satisfactory margin of flexural capacity beyond the cracking moment.

9—REVIEW OF MINIMUM REINFORCEMENT REQUIREMENTS FOR PRECAST, PRESTRESSED CONCRETE ELEMENTS

Development of general equation for minimum reinforcement for pretensioned members and bonded post-tensioned members

Comparative studies of minimum reinforcement requirements for nonprestressed members as well as for pretensioned members and bonded post-tensioned members in this report are based on the following equation:

$$A_{s, \min} = 0.0020b_w d$$

As for nonprestressed members, the formula may be generalized to include the concrete strength by dividing by the square root of 4000 and adding $\sqrt{f'_c}$ as follows:

$$A_{s, \min} = 0.000032\sqrt{f'_c} b_w d$$

These comparative studies illustrate that this equation provides satisfactory minimum reinforcement values for members reinforced with 270 ksi strand. However, 250 ksi strand may still be used in some cases for pretensioned members, and bars with tensile strength of 150 ksi to 160 ksi are sometimes used for bonded post-tensioned members. Strand with tensile strength of 300 ksi is also available. To make the proposed minimum reinforcement equation for members with bonded prestressing steel applicable to the range of reinforcement strengths available, the equation is multiplied by 270,000, and f_{pu} is added to the denominator. This provides:

$$A_{s, \min} = 8.64 \frac{\sqrt{f'_c}}{f_{pu}} b_w d$$

As in the case of the formula for nonprestressed members, the formula proposed for consideration for members with bonded prestressed reinforcement in the ACI Code is rounded up to:

$$A_{s, \min} = 9.0 \frac{\sqrt{f'_c}}{f_{pu}} b_w d$$

The change of the multiplier from 8.64 to 9.0 increases the minimum reinforcement 4 percent above the values tabulated for pretensioned and bonded post-tensioned members, increasing the tabulated safety margins with respect to the cracking moment by 4 percent.

Minimum reinforcement requirements for pretensioned double tees

Application of the proposed $0.000032\sqrt{f'_c}$ minimum reinforcement criteria to the standard double tee sections in the fourth edition of the PCI Handbook is illustrated in Table 9. For positive moment, the range of $\phi M_n/M_{cr}$ is from 1.15 to 1.33. If the ϕ factor is considered as an additional safety factor, the range of M_n/M_{cr} is 1.28 to 1.48. In general, the area of prestressing steel resulting from the above equation is approximately equal to, or is less than, the smallest strand area shown for the double tee sections in the PCI Handbook. Accordingly, double tees will rarely be used with an amount of reinforcement as small as that obtained from the proposed ACI equation. However, it is apparent that the proposed equation provides an adequate amount of reinforcement to preclude failure at the time of cracking. As shown in Fig. 3, the margin of safety between M_{cr} and M_n for the 10DT26 section increases as the value of ρ increases above the proposed minimum value. The same qualitative results are ob-

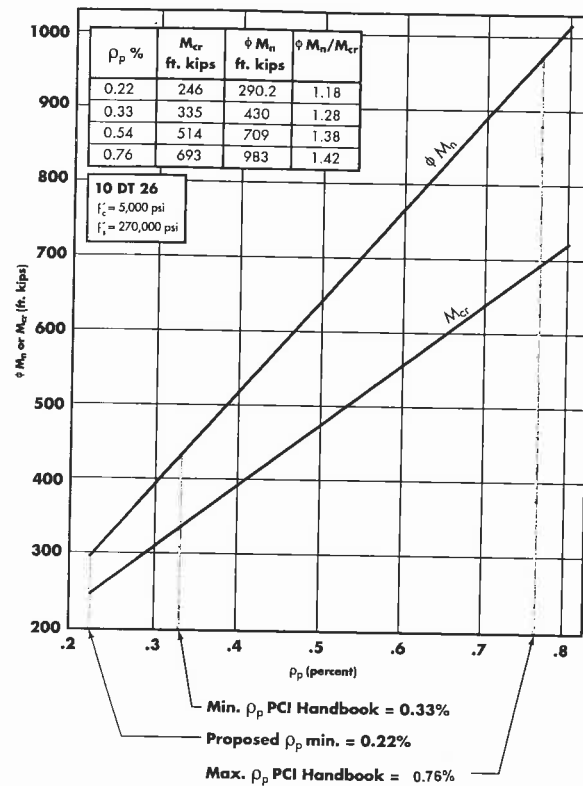


Fig. 3—Application of proposed minimum reinforcement criteria to precast 10DT26

Table 9—Minimum reinforcement requirements for precast double tees

Member	Positive moment			Negative moment		
	Proposed ACI min. A_s ,* in. ²	Min. area strand PCI handbook, in. ²	$\frac{\phi M_n}{M_{cr}}$ (Proposed ACI criteria)	Cantilever, ft.	$1.33 M_u$ A_s ,† in. ²	$\frac{\phi M_n}{1.33 M_u}$
8DT12	0.23	0.306	1.18	6	0.62	1.11
8DT14	0.306	0.306	1.33	6	0.62	1.28
8DT16	0.34	0.306	1.19	10	1.20	1.04
8DT18	0.34	0.306	1.15	10	1.20	1.09
8DT20	0.46	0.612	1.21	10	1.20	1.20
8DT24	0.612	0.918	1.26	10	1.20	0.97
8DT32	0.918	1.84	1.27	12	1.20	1.13
10DT24	0.612	0.918	1.23	12	1.50	1.04
10DT32	0.918	1.836	1.22	12	1.20	0.98
10DT26	0.612	0.918	1.18	10	1.20	1.04
10DT34	0.918	1.84	1.17	10	1.20	1.24

Note: $f = 5000$ psi
 * $f_{pu} = 270,000$ psi
 † $f_s = 60,000$ psi

tained when the amount of prestressing is increased above the minimum value for other standard precast sections.

As discussed in previous sections of this report, it is not economically feasible or reasonable to provide reinforcement for the cracking moment for T-sections with the flange in tension. For these reasons, reinforcement for cantilevered

T-sections is based on the amount required to provide capacity of $1.33M_u$. Since T-sections and other cantilevered precast sections are more often reinforced for negative moments by use of deformed reinforcement, rather than prestressing steel, it is proposed that all precast cantilevers have capacity of $1.33M_u$. Review of Table 10 reveals that very small amounts of nonprestressed reinforcement satisfy the $1.33M_u$ requirement. Accordingly, there is little economic penalty involved in providing $1.33M_u$ capacity for precast sections.

Minimum reinforcement requirements for other standard precast sections

The results of application of the $0.000032\sqrt{f'_c} b_w d$ minimum reinforcement criteria to standard precast sections in the PCI Handbook, other than double tees, is illustrated in Table 10. The proposed minimum amount of prestressed reinforcement is less than the minimum amounts shown in the PCI Handbook in all but one case (FS8 in Table 10). The values of $\phi M_n/M_{cr}$ are all larger than 1.0. Due to the shallow depth and small value of b_w for hollow core products, a value of b_w equal to one-half of the section width is arbitrarily proposed for use in Equation (18-6) for hollow core products. Also, for L-shaped and inverted T-shaped ledger beams, the flange L or T width was used in calculating the minimum reinforcement values in Table 10.

Conclusions

In summary, the minimum reinforcement of standard precast sections as reflected in Tables 9 and 10 is considered to be satisfactory. The final form of the equation proposed for the ACI Code for pretensioned members and post-tensioned members with bonded reinforcement (as discussed in previous sections) provides 4 percent more capacity than the values in Tables 9 and 10. Additional safety is provided in cantilevers above the $1.33 M_u$ values due to strain hardening of bonded deformed reinforcement.

10—REVIEW OF MINIMUM REINFORCEMENT REQUIREMENTS FOR POST-TENSIONED MEMBERS WITH BONDED TENDONS

In Section 9, an equation is developed for the minimum area of prestressing steel in pretensioned members and post-tensioned members with bonded tendons. As in the case of pretensioned members, comparison of minimum bonded reinforcement for bonded post-tensioned members in this section are based on the equation:

$$A_{s, \min} = 0.000032\sqrt{f'_c} b_w d$$

For 5000 psi concrete, this formula provides a reinforcement ratio of 0.226 percent.

The research on post-tensioned beams at the University of Illinois reviewed in Section 3 demonstrated that post-tensioned beams with bonded tendons and reinforcement ratios as low as 0.101 and 0.107 percent increased after cracking. There was no loss of load capacity at the time of cracking for any of the 67 beams with bonded prestressing steel or bonded reinforcement in the University of Illinois tests. There was also a substantial amount of deflection after cracking for all of these beams. On

Table 10—Minimum reinforcement requirements for standard precast sections

Member	Positive moment			Negative moment		
	Proposed ACI min. A_s ,* in. ²	Min. area strand PCI handbook, in. ²	$\frac{\phi M_n}{M_{cr}}$ (Proposed ACI criteria)	Cantilever, ft.	$1.33 M_u$ A_s , [†] in. ²	$\frac{\phi M_n}{1.33 M_u}$
4HC6	0.24	0.51	1.24	6	0.80	1.08
4HC8	0.35	0.51	1.07	6	0.55	1.02
4HC10	0.46	0.61	1.08	6	0.55	1.21
4HC12	0.56	0.595	1.08	6	0.44	1.19
FS4	0.255	0.51	1.20	4	0.66	1.00
FS6	0.51	0.51	1.31	4	0.44	1.08
FS8	0.75 [‡]	0.51	1.37	4	0.33	1.00
12RB16	0.345	0.765	1.33	4	0.11	1.01
18LB20	0.765	1.377	1.41	6	1.32	1.17
24IT20	1.071	1.377	1.42	6	1.24	1.06
30IT20	1.22	2.142	1.42	6	1.20	1.01
36IT20	1.53	2.754	1.42	6	1.24	1.00

Note: $f'_c = 5000$ psi

* $f_{pu} = 270,000$ psi

† $f_y = 60,000$ psi

‡ Only case where proposed ACI minimum reinforcement requirements are greater than minimum area of strand in PCI Handbook

Table 11—University of Illinois tests of bonded post-tensioned members with low reinforcement ratios

Member	Reinforcement ratio ρ , percent	M_n/M_{cr}	$\Delta_{fail}/\Delta_{cr}$
OB14.030	0.107	1.95	37.62
OB24.061	0.177	1.64	37.21
OB34.027	0.101	1.12	24.65
OB34.037	0.198	1.51	19.27
OB44.032	0.108	1.13	24.04
RB34.031*	0.166	1.64	32.44

* Pretensioned

the basis of this research, the proposed ACI equation is satisfactory for rectangular beams. Table 11 provides a summary of M_n/M_{cr} results from the University of Illinois tests of bonded rectangular post-tensioned members with low reinforcement ratios (all much lower than the proposed 0.226 percent).

The results of application of the $0.000032\sqrt{f'_c} b_w d$ criteria to four rectangular bonded post-tensioned beams, and two bonded post-tensioned T-beams (all under positive moment) are presented in Table 12. The ratio of $\phi M_n/M_{cr}$ for the rectangular beams range from 1.37 to 1.47, and the ratios of M_n/M_{cr} for the T-beams are 1.12 and 1.15. Concrete strength of 5000 psi, and 270,000 psi seven wire, low relaxation strand were used for these examples.

The actual post-tensioning forces required in the T-beam examples in Table 12 were much larger than the minimum reinforcement provided by the proposed ACI equation. For Example 5.5.1, the design post-tensioning force was 728,473 lb as compared to 242,190 lb provided by the proposed ACI equation for minimum reinforcement. For Example 5.5.2,

Table 12—Comparison of proposed ACI minimum reinforcement and cracking moment for post-tensioned beams with bonded reinforcement

Member	Proposed ACI A_s , in. ²	A_s used in calculation	$\phi M_n/M_{cr}$	ρ , percent
RB 12 × 12	0.276	0.306	1.38	0.24
RB 12 × 24	0.598	0.612	1.47	0.23
RB 12 × 36	0.92	0.918	1.38	0.22
RB 12 × 48	1.24	1.22	1.37	0.22
T-beam Example 5.5.1 PTI P/T Mnl	1.37	1.38	1.15	0.067
T-beam Example 5.5.2 PTI P/T Mnl	1.066	1.071	1.12	0.061

the design post-tensioning force was 348,780 lb as compared to 187,960 lb provided by the proposed ACI equation. Example 5.5.2 was for a T-beam with unbonded tendons, so bonded rebar was provided in accordance with Section 18.9.2. As in the case of precast, pretensioned sections, the amount of post-tensioning steel used in practical designs will generally be much greater than the minimum amount of steel determined by the proposed ACI equation.

As discussed in previous sections of this report, it is not considered economically feasible to provide reinforcement for the cracking moment of statically determinate post-tensioned T-beams with bonded tendons when the flange is in tension. As in the case of precast, prestressed T-beams, it is proposed that reinforcement for statically determinant post-tensioned, bonded T-beams with the flange in tension be designed for $1.33 M_{cr}$.

11—COMPARISON OF MINIMUM REINFORCEMENT REQUIREMENTS FOR REINFORCED CONCRETE FLAT PLATES AND PRESTRESSED FLAT PLATES

Existing Section 10.5.3 (ACI 318-89) and the revision of this section in Section 10.5.4 of ACI 318-89 state: "For structural slabs of uniform thickness, minimum area and maximum spacing of reinforcement in the direction of the span shall be as required for shrinkage and temperature according to 7.12." The commentary to Section 10.5.3 states: "The minimum reinforcement required for slabs is somewhat less than that required for beams, since an overload would be distributed laterally and a sudden failure would be less likely."

Joint ACI-ASCE Committee 423's report, *Recommendations for Concrete Members Prestressed with Unbonded Tendons*, 1983 and 1989 editions,^{6,7} Section 3.3.2—Limits for Reinforcement include the following recommendation and discussion concerning application of Section 18.8.3 to two-way post-tensioned systems with unbonded tendons:

"3.3.2—Limits for reinforcement: It is recommended that Committee 318 waive the requirement of Section 18.8.3 of ACI 318 for a total amount of prestressed and nonprestressed reinforcement sufficient to develop 1.2 times the cracking load for two-way post-tensioned systems with unbonded tendons. Due to the very limited

amount and extent of the initial cracking in the negative moment region near columns of two-way flat plates, load-deflection patterns do not reflect any abrupt change in stiffness at this point in the loading history.

Only at load levels beyond the design (factored) loads is the additional cracking extensive enough to cause an abrupt change in the load-deflection pattern. Tests have also shown that it is not possible to rupture (or even yield) unbonded post-tensioning tendons in two-way slabs prior to a punching shear failure. The use of unbonded tendons in combination with the minimum bonded reinforcement requirements of Sections 18.9.3 and 18.9.4 of ACI 318 has been shown to assure post-cracking ductility and that a brittle failure mode will not develop at first cracking."

Section 18.12.4 of ACI 318-89 requires a minimum P/A for slab systems of 125 psi, or 125 percent of the shrinkage and temperature reinforcement requirement (the basis for minimum reinforcement in conventional flat plates). Sections 18.9.3.2 and 18.9.3.3 include minimum bonded reinforcement requirements in positive and negative moment areas, respectively, of two-way flat plates.

In consideration of the recommendation of joint ACI-ASCE Committee 423 quoted above, the exception granted to conventionally reinforced slabs in Section 10.5.3 of ACI 318-89 and in ACI 318-95, Section 10.5.4, as well as the minimum reinforcement requirements for prestressed slabs in Sections 18.12.4, 18.9.3.2, and 18.9.3.3, a waiver of any other minimum reinforcement requirements is considered to be warranted for prestressed flat plates.

12—RECOMMENDATIONS FOR UNIFIED MINIMUM FLEXURAL REINFORCEMENT REQUIREMENTS

General

The discussions in previous sections of this study indicate that unified minimum flexural reinforcement requirements for non-prestressed and prestressed concrete members are generally feasible on the basis of the following equation:

$$A_{s, \min} = 0.0020b_w d$$

Exceptions to this equation are necessary for statically determinant T-beams with the flange in tension, for structural slabs of uniform thickness, and for members prestressed with unbonded tendons. Application of discussions in previous sections of this study result in the following, essentially unified, minimum flexural reinforcement provisions for reinforced and prestressed concrete members.

Minimum flexural reinforcement for reinforced concrete members (revision of Section 10.5 of ACI 318-95)

Minimum flexural reinforcement provisions for reinforced concrete members resulting from this study are proposed as follows:

Section 10.5 (ACI 318-95)—Notation

- $A_{s, \min}$ = minimum area of nonprestressed reinforcement, sq. in.
 b_w = web width, in. The average web width shall

f_{su} = be used for webs with sloped sides.
= minimum tensile strength of nonprestressed reinforcement, psi.

Section 10.5.1 (ACI 318-95)

At every section of a flexural member where tensile reinforcement is required by analysis, except as provided 10.5.2 and 10.5.3, the area of nonprestressed reinforcement provided shall not be less than required by 9.2 and 9.3, nor that given by Equation (10-3).

$$A_{s, \min} = 3 \frac{\sqrt{f'_c}}{f_{su}} b_w d \quad (10-3)$$

Section 10.5.2 (ACI 318-95)

For a statically determinate T-section with the flange in tension, the requirements of 10.5.1 shall be waived, and reinforcement provided shall not be less than required to provide capacity of $1.33 M_u$.

Section 10.5.3 (ACI 318-95)

For structural slabs and footings of uniform thickness, the minimum area of tensile reinforcement in the direction of the span shall be the same as required by 7.12. Maximum spacing of the reinforcement shall not exceed three times the thickness or 18 in.

Minimum flexural reinforcement requirements for prestressed concrete members (revision of Section 18.8.3 of ACI 318-95)

Minimum flexural reinforcement provisions for prestressed concrete members resulting from this study are proposed as follows:

Section 18.8.3 (ACI 318-95)

The total amount of prestressed and non-prestressed reinforcement shall be not less than required by 9.2 and 9.3, nor the amounts specified in 18.8.3.1, 18.8.3.2, 18.8.3.3, and 18.8.3.4. The yield strength of deformed bar reinforcement used to satisfy the requirements of this section shall not be less than 60,000 psi.

Section 18.8.3.1 (ACI 318-95)

Except as provided in 18.8.3.2, the area of longitudinal tensile prestressing steel in pretensioned members, and in post-tensioned members with bonded tendons, shall not be less than that given by Equation (18-6):

$$A_{s, \min} = 9 \frac{\sqrt{f'_c}}{f_{pu}} b_w d \quad (18-6)$$

where b_w = web width. For webs with sloped sides, the average width shall be used. (Note: Subsequent equations in Chapter 18 to be renumbered). For pretensioned L-shaped and inverted T-shaped ledger beams, the flange width shall be used for b_w , and for pretensioned hollow core products, b_w shall be taken as one-half the width of the section.

Section 18.8.3.2 (ACI 318-95)

For statically determinant pretensioned or bonded post-tensioned T-beams with the flange in tension, the requirements of 18.8.3.1 shall be waived, and reinforcement shall not be less than required to provide capacity of $1.33 M_u$. Minimum reinforcement required by Equation (18-6) for other pretensioned, cantilevered cross sections shall be waived if at every section of the cantilever, prestressed and/or nonprestressed reinforcement is not less than required to provide capacity of $1.33 M_u$.

Section 18.8.3.3 (ACI 318-95)

For post-tensioned beams, T-beams, or one-way slabs with unbonded tendons the area of prestressing steel and deformed bar reinforcement shall be not less than required by 18.4.2, and 18.9.2, respectively. (Note: Section 18.9.2 also to be revised to require use of Grade 60 reinforcement).

Section 18.8.3.4 (ACI 318-95)

For post-tensioned flat plates with unbonded tendons, the area of longitudinal prestressing steel and deformed bar reinforcement in each direction shall not be less than required by 18.12.4, 18.9.3.2, and 18.9.3.3. (Note: 18.9.3.2 and 18.9.3.3 to be revised to require use of Grade 60 reinforcement).

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* Ref. 2 and 3 are combined in University of Illinois Engineering Experiment Station *Bulletin* No. 464, by J. Warwaruk, M. Sozen, and C. P. Siess, 1962. *Bulletin* No. 464 is more generally available in technical libraries