

# Design of Post-Tensioned Floor Slabs

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A consistently increasing proportion of concrete floor slab systems in the United States is being designed and constructed using unbonded post-tensioning. In California, post-tensioned floor slabs are the primary choice in concrete floor construction. An overview is presented on the application and design of such floor systems and the commonly selected cast-in-place floor system configurations are described. Limitations on slab dimensions and required material quantities are presented and discussed. Questions encountered in everyday design practice, such as structural modeling assumptions, placement of post-tensioning and reinforcement, one-way and two-way systems, and punching shear are introduced and clarified.

The effective introduction of unbonded post-tensioning, just over three decades ago, has added a new critical dimension to the design and application of concrete floor systems. Larger clear spans, thinner slabs, lighter structures, shorter overall building heights, and, more importantly, savings in overall costs have

made this type of construction a prime choice of flooring systems for concrete buildings in the United States, especially in California.

Extensive application of post-tensioning in building construction is relatively new. The first floor systems using unbonded tendons in the United States were erected in 1956-57 with the construction of post-tensioned flat plate roofs for some schools in Nevada.<sup>1</sup> Considerable developments in engineering and construction techniques were fuelled by the rapidly increased application of post-tensioning in slab construction.

Today, the initial period of probing, uncertainties, and problems associated with an emerging technology have been left behind. Design guidelines and construction practice have matured and are fully established within the building industry. These are backed by over 400 million ft<sup>2</sup> (37 million m<sup>2</sup>) of post-tensioned floor systems in satisfactory service throughout the United States.

The general recommendations governing the sizing and design of post-tensioned systems are given in ACI 318.<sup>2</sup> ACI 423<sup>3</sup> provides specific recommendations for slab floor systems post-tensioned with

unbonded tendons. The Post-Tensioning Institute has compiled several comprehensive publications regarding the design of post-tensioned floor systems<sup>4,5</sup> that include several design examples and practical construction details. Many consulting engineers and post-tensioning system suppliers in the United States have played a prominent role in the development of this industry.

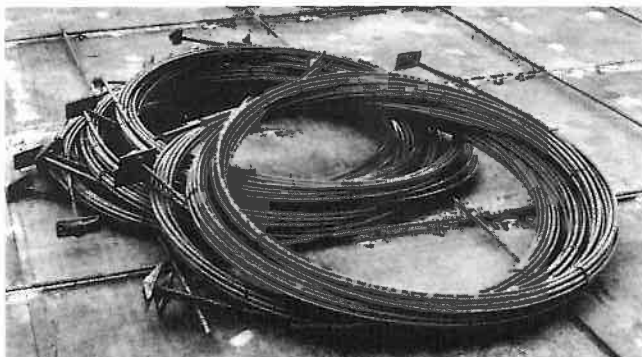
Following provisions introduced in the Canadian<sup>6</sup> and the British<sup>7</sup> codes, a notable move is underway to take advantage of post-tensioning in new construction in these two countries. France currently is preparing specific recommendations for this construction technology, and other countries are expected to follow suit.

This state-of-the-art report on the application and design of post-tensioned floor systems in the United States, with particular emphasis on California practice, describes commonly used cast-in-place floor system configurations and features that have evolved over the last two decades.

## Floor slab systems

The industry-preferred floor slab systems are rooted in the nonpre-

**Keywords:** concrete slabs; deflection; flat concrete plates; flat concrete slabs; floors; partial prestressing; post-tensioning; punching shear; stresses; structural design; shear properties; unbonded prestressing; waffle slabs.



Post-tensioning tendons are cut, bundled, and delivered to the job site ready to be unrolled and placed.



View of a two-way flat slab in the direction of distributed tendons prior to placement of concrete.

**Table 1—Common geometries of two-way slab systems**

1		<b>FLAT PLATE</b>
		Maximum Span: 26 ft (8 m) Limiting Criterion: Punching Shear Rebar: 0.22 psf (1.08 kg/m <sup>2</sup> ) PT: 0.58 psf (2.84 kg/m <sup>2</sup> )
2		<b>FLAT SLAB WITH SQUARE COLUMN CAPITALS</b>
		Maximum Span: 36 ft (11.8 m) Limiting Criterion: Rebar Congestion Rebar: 0.44 psf (2.15 kg/m <sup>2</sup> ) PT: 1.04 psf (5.09 kg/m <sup>2</sup> )
3		<b>FLAT SLAB WITH DROP PANEL</b>
		Maximum Span: 40 ft (12.2 m) Limiting Criterion: Deflection Rebar: 0.60 psf (2.94 kg/m <sup>2</sup> ) PT: 0.79 psf (3.87 kg/m <sup>2</sup> )
4		<b>FLAT SLAB WITH SQUARE CAPITAL AND DROP PANEL</b>
		Not advantageous for regular office or residential occupancies. Applicable to LL of 75 psf (3.60 kN/m <sup>2</sup> ) and higher.
5		<b>BANDED SLAB - UNIDIRECTIONAL</b>
		Maximum Span: 44 ft (13.4 m) Limiting Criterion: Rebar Congestion Rebar: 0.41 psf (2.01 kg/m <sup>2</sup> ) PT: 0.85 psf (4.16 kg/m <sup>2</sup> )
6		<b>WAFFLE SLAB</b>
		Maximum Span: 42 ft (12.8 m) Limiting Criterion: Rebar Congestion Rebar: 0.65 psf (3.18 kg/m <sup>2</sup> ) PT: 0.36 psf (1.76 kg/m <sup>2</sup> )
7		<b>WAFFLE SLAB WITH SOLID COLUMN LINES</b>
		Maximum Span: 42 ft (12.8 m) Limiting Criterion: Rebar Congestion Rebar: 0.65 psf (3.18 kg/m <sup>2</sup> ) PT: 0.36 psf (1.76 kg/m <sup>2</sup> )

Numbers relate to  $f'_c = 4000$  psi (28 N/mm<sup>2</sup>), 8 in. (200 mm) slab; LL = 50 psf (2.39 kN/m<sup>2</sup>), office superimposed DL = 30 psf (1.44 kN/m<sup>2</sup>); columns 20 x 20 in. (508 x 508 mm).

**Table 3 — Beam example material quantities**

	UBC — 36 in. (914 mm) beam		ACI — 30 in. (762 mm) beam	
	Post-tensioning	Reinforcement	Post-tensioning	Reinforcement
Beam	0.35*	0.65	0.35	0.51
Slab	0.20	0.40	0.20	0.40
Total	0.55	1.05	0.55	0.91

\*All values are in psf; multiply by 4.88 for kg/m<sup>2</sup>.

**Table 2—Common geometries of one-way slab systems**

1		<b>ONE-WAY BEAM AND SLAB</b>
		Beam Spans 60-65 ft (18-20 m) Slab Spans 18-20 ft (5.5-6.0 m) Slab Thickness 5-6 in. (125-150 mm) Beam Depth 30-36 in. (750-900 mm) Beam Width 16-18 in. (400-460 mm)
2		<b>JOIST SLAB</b>
		Example: Joist 14+3, 6+30* Beam Span 45 ft (13.7 m) Joist Span 42 ft (12.80 m) Rebar 1.10 psf (5.38 kg/m <sup>2</sup> ) PT 0.55 psf (2.69 kg/m <sup>2</sup> )
3		<b>SKIP JOIST SLAB</b>
		Beam spans comparable with joist slab. Joists may be spaced up to 4.5 ft (1400 mm) with two strands in each joist.

\*Note: Joist stem 14 in (356 mm) high, slab 3 in (76 mm) thick; joist tip 6 in (152 mm), clear distance of joists 30 in (762 mm); beam width 112 in (2845 mm); columns 20 x 20 in (508 x 508 mm)

### Supplementary information

**Table 1** — For all cases: Concrete — 4000 psi (28 N/mm<sup>2</sup>); slab thickness — 8 in. (203 mm); columns — 20 x 20 in., 10 ft high (508 x 508 mm, 3.05 m); superimposed dead load — 30 psf (1.44 kN/m<sup>2</sup>); live loading — 50 psf (2.39 kN/m<sup>2</sup>).

**Case 2:** Drop cap size — 45 x 45 in. (1143 x 1143 mm), extending 6 in. (152 mm) below slab soffit.

**Case 3:** Drop panel size — 14 x 14 ft (4.27 x 4.27 m), extending 6 in. (152 mm) below slab soffit.

**Case 5:** Band width — 45 in., extending 8 in. below slab (1143 mm, 203 mm); span in the transverse direction — 30 ft (9.14 m).  
**Case 6:** Total depth of system — 17 in., (432 mm); slab thickness — 3 in. (76 mm); modules — 36 in. (914 mm) square; stems — 6 in. (152 mm) at bottom; solid drops — 12 x 12 ft (3.66 x 3.66 m).

**Table 2** — **Case 1:** One-way beam frame and slab construction is used primarily in parking structures, with typical dimensions as given in the table. Slab thicknesses are 4.5 to 6.0 in. (114 to 152 mm). The thinner slabs do not boast a history of satisfactory long-term performance in aggressive environments, since for thin slabs it is more difficult to maintain the required minimum reinforcement cover and thinner slabs are more susceptible to poor workmanship. It is recommended to use a minimum of 5 in. (125 mm) slab thickness for exposed slabs in aggressive environments.

Beam dimensions are governed by the applicable building codes. On the west coast of the United States, where UBC<sup>2</sup> applies, beams typically are 36 in. (914 mm) in total depth and 18 in. (460 mm) wide. Elsewhere in the United States, where ACI<sup>2</sup> prevails, a 30 in. (762 mm) total depth and 14 in. (356 mm) stem width yield optimum designs. The primary difference between the two codes, in this particular case, is UBC's stringent requirement for one-way systems reinforced with unbonded tendons. UBC requires that the system be designed to sustain its dead loading plus 25 percent of unreduced live loading by means other than post-tensioning.

**Case 2, ACI design:** The joist dimensions are selected to be comparable with the waffle parameters of Case 6 in Table 1. For the dimensions selected, reinforcement congestion in the beam direction over the column limits the maximum span. The joists each are provided with two ½ in. (12.7 mm) post-tensioned strands.

stressed slab configurations that lent themselves advantageously to the application of post-tensioning. Reference 8 gives a thorough account of nonprestressed concrete slabs.

Tables 1 and 2 summarize the common geometries of two- and one-way post-tensioned slab systems, respectively. A first glance reveals that tapered column capitals no longer are among primary choices. Likewise, two-way slabs resting on a gridwork of beams supported by columns are no longer in common use. Slabs with square column capitals (Case 2), with drop panels (Case 3), and banded slabs (Case 5) are the favored selections.

Tables 1 and 2 offer maximum span lengths and other parameters typical to office/residential occupancies or parking structures using the current ACI and UBC<sup>9</sup> requirements. The tables provide a reference base for comparison with other alternatives. Nontypical particulars of a project may lead to values and quantities different from those given in the tables.

The quantities quoted in the tables are computed for code compliance in serviceability and strength. Support bars for the post-tensioning system, bars necessary to secure the reinforcement, and trim bars are not included.

The comparative designs for the grid layout shown in Fig. 1 have been performed with the ADAPT Post-Tensioning Software System.<sup>10</sup> The limiting criterion stated for each case is discussed in more detail in the section on principal limiting criteria.

Table 3 compares a parking structure beam designed according to both ACI and UBC. In this example the columns are spaced at 65 ft (20 m) and frames at 20 ft (6.0 m) on center. A 36 in. (914 mm) UBC beam design is compared to a 30 in. (762 mm) ACI alternative. Live loading is 50 psf. Observe that the ACI design reflects a 6 in. (152 mm) savings in total depth as well as slightly less reinforcement.

## Floor slab design Structural system

Once the geometry and loading are established, the design of a slab system consists of two principal steps, namely, determining actions (mo-

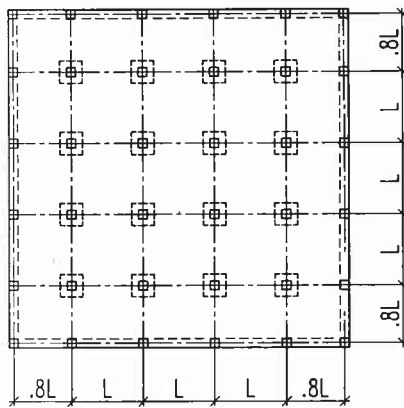


Fig. 1—Grid layout of two-way slab examples.

ments and shears) in the floor system, and designing reinforcement and detailing. The actions are computed using a structural system that models the actual slab. Slabs are modeled either as one-way or two-way systems.

One-way members carry the applied loading primarily in one direction. One-way systems, as shown in Table 2, are treated as one-dimensional beams or plane frames. The one-way slab is viewed as a beam strip.

The slab systems illustrated in Table 1 all are examples of two-way systems, which have the ability to sustain the applied loading in two directions. Several requirements establish this ability.

### Qualifications for two-way action

**Biaxial precompression due to post-tensioning** — A biaxially precompressed slab with no, or moderate, crack formation performs as a homogeneous elastic plate with its inherent two-way behavior. The actual tendon location at a given point in a slab system is not critical to the slab's biaxial behavior since precompression, which is the decisive factor, commonly is deposited onto the slab at the slab's perimeter (Fig. 2).

Detailed elastic plane stress solutions indicate that the precompression spreads into the slab steeply, as displayed in the figure, resulting in a fairly uniform compression at about the 45 deg distribution assumed in simplified design. This is true for slabs of uniform thickness as well as slabs with beams in the direction of precompression. Slabs

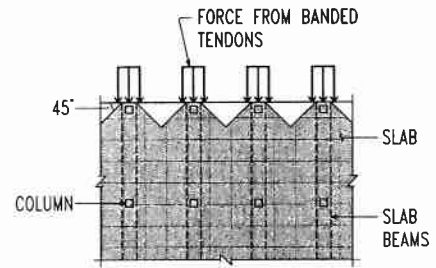


Fig. 2—Diffusion of precompression into slab system.

with banded post-tensioning and slabs with wide shallow beams also qualify for two-way action at regions away from the free slab edges where precompression is attained both directions.

**Aspect ratio** — Regardless of the precompression, for slabs supported on orthogonal rows of columns, the length-to-width ratio of a panel of uniform thickness (flat plate) bounded by four columns must be less than 2.5 to develop significant two-way behavior.

**Stiffness ratios in two directions** — For ribbed and other slabs of non-uniform cross section, the relative stiffnesses of the two directions affect the slab's biaxial load-sharing characteristics. In the absence of a rigorous evaluation, and for a square panel, a ratio between moments of inertia in two perpendicular directions greater than 10 is considered to render the otherwise two-way square panel into a primarily one-way system.

### Structural modeling of two-way slab systems

Two-way floor slabs resting on discrete supports form complex three-dimensional structural systems. Several methods are available for the analysis of such slabs: the classical elastic methods using plate theory, the yield line limit state method, finite differences using plate equations, the finite element method, and a host of approximate solutions.

In today's competitive consulting offices, where a design engineer is expected to design, document, and detail 50,000 to 100,000 ft<sup>2</sup> (5,000 to

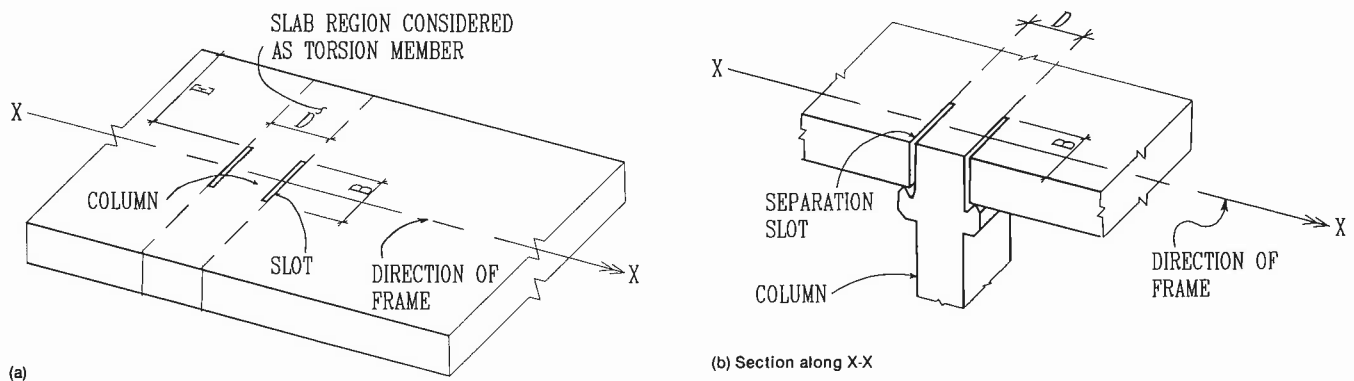


Fig. 3—Structural model of slab/column junction for transfer of bending moments using the equivalent frame method.

10,000 m<sup>2</sup>) per month of floor system while responding to ongoing job site queries and plan check comments, a fast, versatile, reliable, and economically proven design technique is the key. For this reason, in routine designs for gravity loading and post-tensioning, slab floor systems are modeled as rows of intersecting two-dimensional slab frames.

Floor levels together with the columns immediately below and above them each are treated separately. A line of column support bounded by its associated slab tributary forms a single-story slab frame. The influence of the dimension perpendicular to the assumed frame — not represented in the two-dimensional slab frame model — is accounted for by one of several options depending on the modeling scheme adopted.

Several methods have been proposed for the structural modeling of floor slabs<sup>11,12</sup> as two-dimensional slab frames, all of which are approximate. The most common scheme is the equivalent frame method given in ACI 318 and recommended in the Canadian and British codes.

One of the strengths of the equivalent frame method is that many concrete buildings, either nonprestressed or post-tensioned, have been designed and constructed over a period of more than two decades using the method, and such floor slabs that have been observed to exhibit the performance criteria intended in their designs. For this reason, among the approximate methods, the equivalent frame option is most credible.

The equivalent frame method is explained in detail in ACI 318 and its Commentary. The following analogies provide added insight toward the appreciation of its physical significance:

Columns in a three-dimensional slab system are subject to a lesser moment than a straight two-dimensional slab frame model would suggest. Hence, a correction to this effect is necessary in two-dimensional models that represent a three dimensional system.

In the equivalent frame method the moment transfer between the slab and column is modeled to take place through strips of slab on each side of the column and normal to the frame direction as shown in Fig. 3. The torsional stiffnesses of the two strips at each joint are combined into a single spring, which is considered in series between the slab and the column at the joint.

Fig. 4 illustrates this model for a simple two-span slab. The consequence of this modeling is an increase in the effective length of the columns at each joint [Fig. 4(c)], and thereby a reduction in the moments that the columns would be subjected to from the frame analysis.

One other strength of the equivalent frame method is its ability to model such moderately complex geometries as those shown in Table 1. Each span is modeled as a nonprismatic beam, which in the general case consists of up of seven different sections, each section with a different moment of inertia (Fig. 5). The first and last sections represent the regions over the supports; the remainder are commonly, but not necessarily, for column drops and

drop panels. This feature provides the flexibility necessary for modeling other changes in slab properties, such as openings, reductions in slab tributary, or changes in slab thickness between supports.

Of equal importance, the equivalent frame method is simple to model and apply.

## Post-tensioning

The general application of post-tensioning in buildings is described in the Post-Tensioning Institute's (PTI) Design Manual.<sup>5</sup> Floor slabs are described in a special PTI publication.<sup>4</sup> Unbonded post-tensioning used in floor slab systems consists predominantly of single ½ in., 270 ksi (12.7 mm, 1850 MPa) seven-wire low-relaxation strands greased and encased in a plastic protective sheathing.<sup>13</sup> The sheathing is formed over the strand using a heat-induced extrusion process. For added protection against weathering in aggressive environments, the exposed strand tails at the anchorages may be encapsulated in sealed plastic covers.

The tendons are placed with a profile in the vertical plane for the purpose of exerting a net uplift onto the slab system along the tendon path in addition to the precompression that the tendons impart to the slab at the slab edges. Tendons are stressed and anchored after the slab is cast.

## Tendon layout

Several schemes were proposed initially and were in use for the layout of tendons in two-way floor slabs: tendons distributed uniformly in both directions; tendons banded in

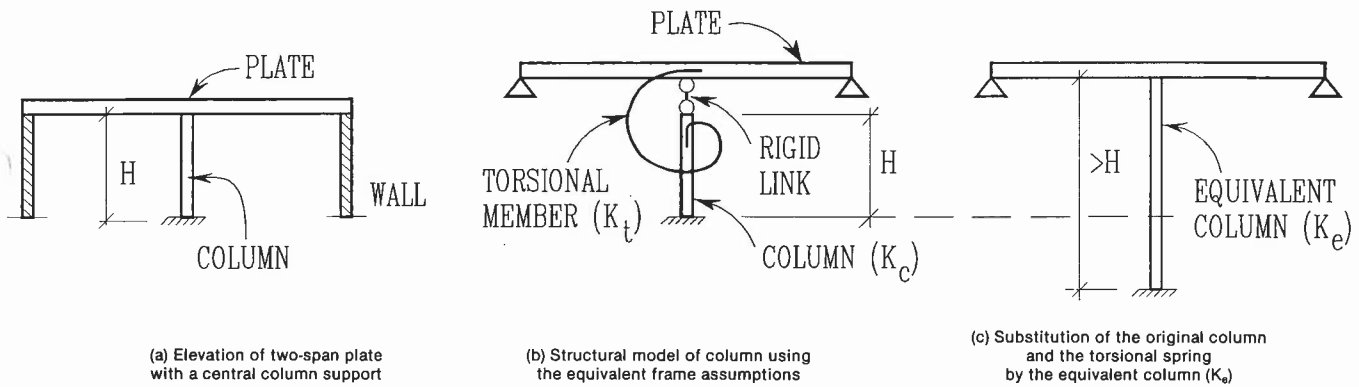


Fig. 4—Illustration of the equivalent frame assumption for column stiffness.

one direction and distributed in the other direction; and tendons banded in both directions. Test results concluded that the arrangement of tendons is not critical to the performance of slab;<sup>4</sup> it is the entire tendon force in a bay and the tendon drape that govern the slab behavior.

From the tendon arrangements probed, the second alternative, as shown in Fig. 6 and commonly referred to as a "banded system," prevails in industry. The banded system is simpler to place, and provides the same performance as the other alternatives.

### Load balancing

In computing moments and shears in slab systems, the post-tensioning is viewed as an applied loading following the load-balancing approach presented by Lin.<sup>14</sup> In the load-balancing method, the tendons are considered removed from their ducts. The influence of post-tensioning on the structure is represented by the forces tendons exert on the slab system when in place. These forces are referred to as the balanced loading. Balanced loading is handled similarly to dead and live loading for the determination of actions in the slab system.

### General design considerations

#### Principal limiting criteria

In the selection of the limiting slab and column dimensions, the following parameters pose as the principal design constraints:

**Punching shear** — Limitations on punching shear capacity of slab regions around columns is the first

restriction to the selection of large spans if no special shear reinforcement is to be provided (Case 1, Table 1). Provision of a drop cap of sufficient strength around the column allows an increase in span length until the punching shear capacity of the slab immediately beyond the drop becomes critical or other limitations are encountered (Case 2, Table 1).

**Reinforcement congestion** — Reinforcement congestion over the column is an important construction consideration in the selection of large spans. Reinforcement may be required over a support to supplement post-tensioning in achieving strength demands. At the same time, for adequate crack control and sufficient ductility, ACI stipulates a minimum amount of reinforcement to be placed over each support. The minimum reinforcement is a function of the geometry of the slab and is required to be placed over the column within a narrow band of slab.

Observations on construction suggest that the total area of reinforcement over a column in each direction should not exceed  $6.75 \text{ in.}^2$  ( $4200 \text{ mm}^2$ ) to achieve effective consolidation of concrete and avoid the necessity of special procedures in placement. The condition is aggravated by the selection of small bar diameters (#4 and #5 [12 to 16 mm]) favored by most engineers for maximizing the moment arm of thin slabs and also to avoid reducing the effective depth of tendons that cross in the perpendicular direction below the top reinforcement.

For an optimum design, the engi-

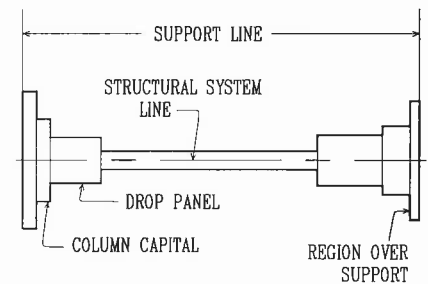


Fig. 5—Idealization of a slab span as a nonprismatic beam.

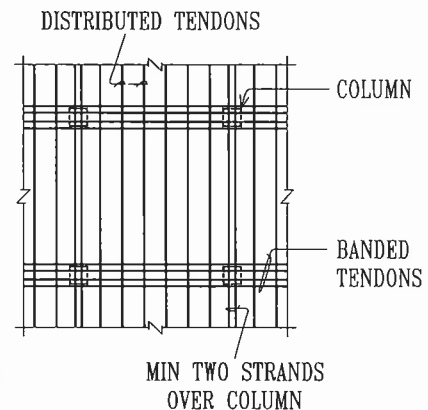


Fig. 6—Two-way system with banded tendon layout.

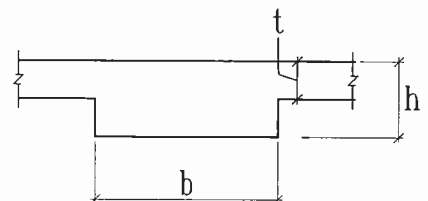


Fig. 7—Cross section of a slab-beam.

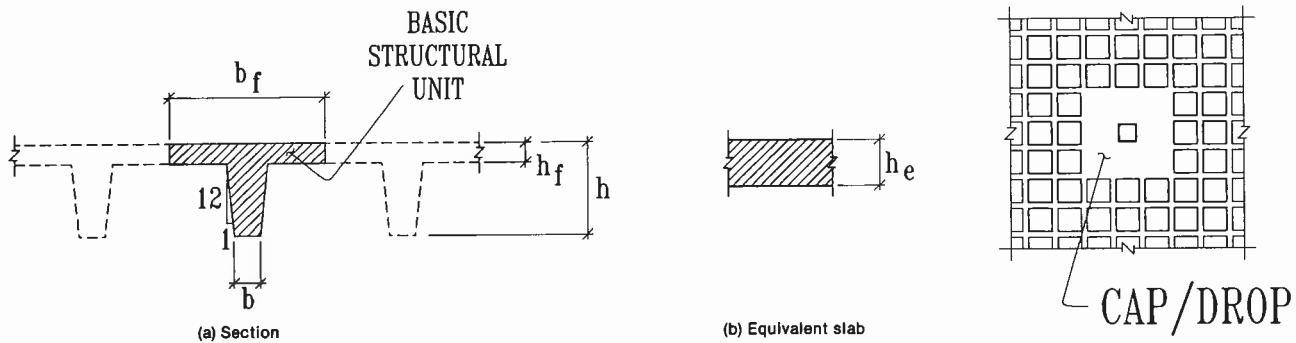


Fig. 8—Ribbed or waffle slab cross-sectional geometry.

neer aims at slab thickness and span dimensions that use the entire code-required minimum reinforcement to supplement prestressing in strength requirements.

**Deflection** — Deflection deficiencies are very rare in post-tensioned slab systems built according to ACI recommendations. Deflections seldom pose as a limiting criterion. As a guideline, a maximum long-term deflection equal to span/250 is used as a limiting design criterion. This is the sum of long-term deflections obtained from the frame analyses in two perpendicular directions. In calculating the long-term deflection, the gross moment of inertia is used and a creep factor of 2 is applied to sustained loading (dead load plus post-tensioning). Table 9.5(b) of ACI 318 gives accounts for special limits on deflections.

**Average precompression** — ACI 423 recommends a minimum precompression of 150 psi (1.0 MPa) and that for precompressions over 500 psi (3.4 MPa) attention should be focused on the consequences of shortening. The amount of post-tensioning selected affects the reinforcement requirements. The more post-tensioning is used, the less reinforcement is likely to be required. Unlike nonprestressed slab systems, a multitude of acceptable designs are possible for a given slab geometry and loading. The best selection is governed by the relative in-place unit costs of reinforcement and prestressing.

Experienced post-tensioning engineers fine tune the drapes and amounts of prestressing for each span to achieve an optimum design. However, for common dimensions and residential/office occupancies, an average precompression between

150 and 250 psi (1.0 and 1.75 MPa) yields a satisfactory solution. In floor slab systems, it rarely becomes necessary to exceed 300 psi (2.0 MPa) average precompression. At higher precompressions, constraint of supports and slab shortening become important considerations in design, particularly for low rise buildings and buildings with unfavorable arrangement of supports.<sup>15</sup>

### Stresses

A serviceability check required for post-tensioned members is the control of concrete stresses under working conditions. The objective of the stress check is to determine whether or not visible cracks develop in service and, if they do, to provide reinforcement to control crack width.

It is recognized that, in elastic plates supported over discrete columns, moments concentrate at supports. Using the equivalent frame method, a single moment for the entire bay is obtained. For the purposes of design, the stress at the line of support is determined by applying this total moment to the entire bay section. ACI 318 does not recommend that the bay moment be distributed nonuniformly over the support line, with a higher concentration over the column. The column-strip/middle-strip concept is prescribed for the direct design method and nonprestressed concrete. ACI 318 explicitly excludes their application to post-tensioned floor systems.

Detailed finite element plate analyses carried out for post-tensioned floor slabs<sup>16</sup> indicate that the computed stresses differ by as much as 100 percent from the simplified

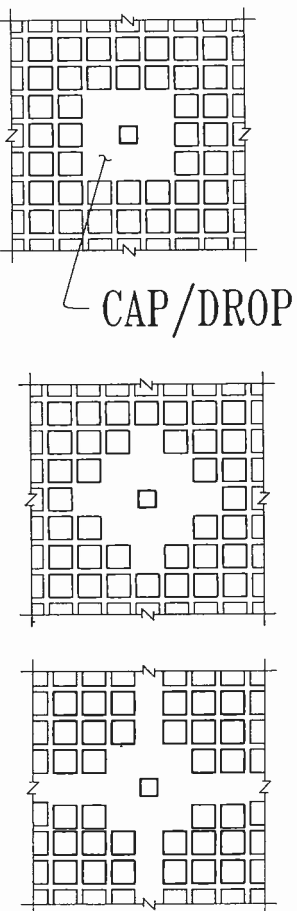


Fig. 9—Examples of solid regions in waffle slab construction.

beam formulas used in regular calculations. However, for the purposes of member sizing and design, the simplified stress formulas are acceptable.

The simplified stresses should be viewed as a check on parameters that represent the applied loading, floor plan, and cross-sectional features of a slab, duly fine tuned by code-specified permissible stress values to yield predictable and satisfactory floor slab performance. In other words, the code-specified permissible stresses are compatible with the code-recommended methodology for their estimate and deliver the types of structures that are intended by the design concept.

### Nonprestressed reinforcement Arrangement of nonprestressed reinforcement

For post-tensioned slab systems designed using the equivalent frame method, the top reinforcement is concentrated over the supports.

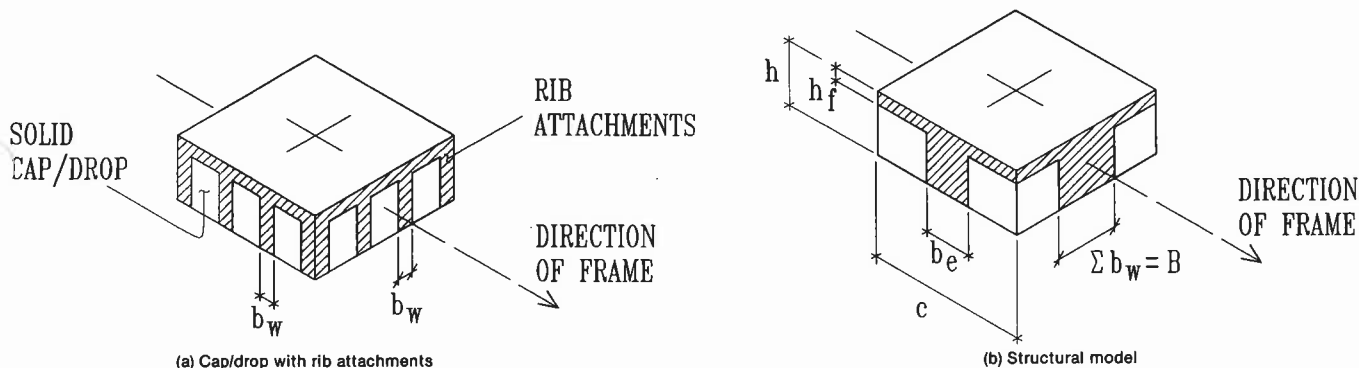


Fig. 10—Pictorial view of a solid region at support of a waffle slab system.

There are no requirements regarding the breakdown of reinforcement into column and middle strips. In the banded direction, the bottom reinforcement commonly is placed within the width of banded tendons. In the direction of distributed tendons, the bottom bars are distributed uniformly among the tendons across the entire bay.

#### Amount of nonprestressed reinforcement

Much information has become available since the first requirements on minimum reinforcement were formulated in ACI 318. Consultants have extended the application of the two-way flat plate requirement of minimum support reinforcement ( $0.00075hl$ ; ACI 318, Section 18.9.3.3) to two-way slab systems having drop caps or drop panels and to slab systems with wide shallow beams. The observed serviceability from surveys made by the author on floor slabs containing over one thousand such columns has been excellent. No cracking distress was observed over the supports reinforced using the requirement stipulated in ACI 318, Section 18.9.3.3.

Another consideration regarding the provision of minimum nonprestressed reinforcement is the sufficiency in ductility. The ductility requirement is safeguarded provided the computed moments are applied to the real geometry of the column/slab region. That is to say, the drop cap or panel is considered as part of the compression zone of the actual geometry for which the reinforcing index stipulated in ACI 318, Section 18.8.1, is maintained. It is emphasized that the two-way flat plate minimum reinforcement re-

quirement (ACI 318, Section 18.9.3.3) may be extended to two-way slab systems only if it is accompanied by a ductility check based on ACI 318, Section 18.8.1, using the actual geometry of the nonprismatic slab.

### Special design considerations

#### Wide shallow beams

When the support arrangement of a uniform floor slab is such that the spans in one direction are substantially longer than the perpendicular direction, such as Case 5 of Table 1, the longer span governs the slab thickness. However, in post-tensioned slab construction, the adverse effects of the longer span can be reduced if tendons are banded in the long direction and placed with an increased drape to provide significant upward forces.

Beam slabs, (Fig. 7), also known as wide shallow beams, were conceived as a thickening of a slab along the line of columns for the purpose of providing cover to the banded tendons, which were designed to protrude below the soffit of slab proper. The dimensions of the wide shallow beam are selected so as to retain the two-way action of the floor system. Referring to Fig. 7, the recommended parameters are:

$$h \leq 2t$$

$$h \leq b/3$$

The parameters are selected using ACI 318, Section 11.5.5.1, as a guide and are substantiated through observed performance. No absolute maximum value for slab beam depth  $h$  is proposed. The selected

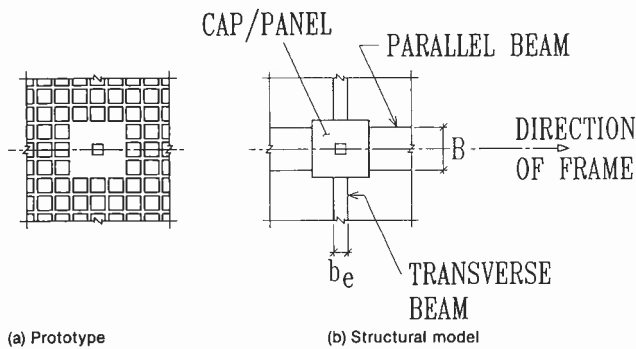
parameters do not result in a localized stiffening of the slab to an extent that would inhibit the slab deformation significantly. Hence, they are not considered as beam supports.

One-way shear is resisted by the entire cross section and is not critical; rather, the punching shear around supports is checked. For improved serviceability, two continuous bottom bars, one at each corner of the stem, are placed. The bars are held by open stirrups spaced 24 in. (600 mm) on center. The geometry of the wide shallow beam is accounted for in modeling the span in calculation of stresses and reinforcement. This inclusion is particularly critical in consideration of the shallow beam as a compression zone over the supports.

#### Waffle slabs

Fig. 8 shows the cross section of a typical waffle slab unit. The waffles are made solid around the columns or along the lines of support, as illustrated in Fig. 9. In applying the equivalent frame method for modeling of waffle slabs, the ribs in the direction of the frame and perpendicular to it are combined into representative solid members. Consider the solid cap/drop shown in Fig. 10. Part (a) of the figure displays the contact surface of the ribs to the solid support region.

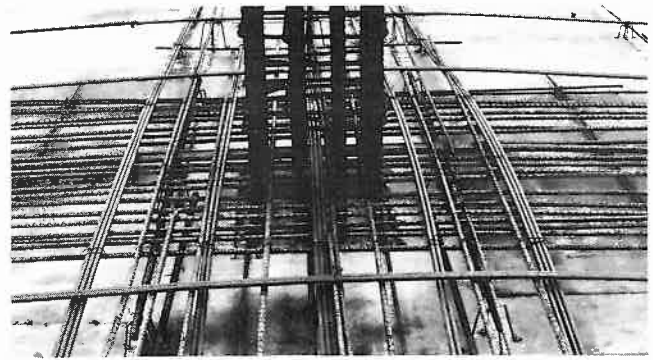
In the direction of the frame, the stems reaching the cap are combined into a single T-section [Fig. 10(b)] with stem width equal to the sum of the individual stems of the associated ribs. Thus, the moment of inertia of the region joining the solid sections of two adjacent columns is retained unchanged. In the transverse direction, the stems in



(a) Prototype

(b) Structural model

Fig. 11—Structural modeling of a waffle slab.



Two-way construction: View of banded tendons at column location.

contact with the solid region are combined into a torsional member having the same cross-sectional constant  $J$  as the sum of individual stems reaching the solid block.

For the prototype:

$$J = (c h_c^3/3) + \Sigma\{(h - h_f)/3\} b_w^3$$

For the structural model:

If  $b_e < (h - h_f)$

$$b_e^3 = [3/(h - h_f)] \times \Sigma\{(h - h_f)/3\} b_w^3$$

If  $b_e > (h - h_f)$

$$b_e = [3/(h - h_f)^3] \times \Sigma\{(h - h_f)/3\} b_w^3$$

This modeling results in the representation shown in Fig. 11 for a typical interior joint. The modeled structure is treated as a two-way system using the equivalent frame approximation. The substitute beam in the direction of the frame is not subject to the same dimensional restrictions as stated for slab beams in the preceding, since the biaxial performance of the prototype slab is achieved through closely intersecting ribs.

Strictly speaking, the flexure of a waffle slab construction is more faithfully modeled through the orthotropic plate approximation in which the slab is substituted by one of uniform thickness having biaxial rigidities equivalent to the waffle's geometry [Fig. 8(b)]. The orthotropic plate approximation is not used in common practice since the equivalent slab thickness does not provide for a direct stress check and reinforcement calculation.

### Partial prestressing

Partial prestressing is a terminology codified in Canada.<sup>6</sup> It is also practiced to various degrees in Great Britain<sup>7</sup> and the United States.<sup>2</sup> It covers prestressed members that are designed on the premise that, under service conditions, the hypothetical maximum tension stresses computed from the gross section properties exceed the cracking strength of concrete used. Crack-width control and deflection checks become important considerations.

In the Canadian code, crack-width control is dealt with directly through crack-width computations (crack control parameter  $z$ ). Hence, no limit is imposed on hypothetical tensile stresses. ACI and the British codes deal with crack control by specifying minimum reinforcement quantities in regions where stresses exceed the cracking limit. Both codes stipulate an upper limit on calculated hypothetical tensile stresses. This limitation ties the application of partial prestressing to a range for which satisfactory test data were available to ACI committees.

In partially prestressed members, the strength requirements generally are achieved by a combination of prestressing and reinforcement. The maximum stress limitation of  $6f'_c/2$  ( $0.50 f'_c/2$ ) for two-way systems given in ACI 318 does extend the slab to the cracking range since the stress computation is based on moments and section properties spread uniformly over the entire bay. It is recognized that over the support these may peak to much higher values, exceeding the cracking limit of concrete. That is one explanation for code-specified<sup>2</sup> maximum permissible stresses for

two-way systems being one-half of those for one-way systems, for which the stress computations are more authentic.

ACI Committee 423 currently is working on a specific document for partial prestressing providing detailed recommendations for its application.

### Punching shear

It is noted in Case 1 of Table 1 that punching shear is the primary restriction in the selection of large spans. Cases 2 through 7 of the same table illustrate alternatives for surmounting the punching shear problem. All the proposed options rest on the enlargement of the critical sections over the support. The preferred alternative is to retain a flat soffit and provide added reinforcement in the slab to build up its punching shear resistance.

ACI 318 has provisions for punching shear reinforcement and makes specific references to stirrups and shear heads. However, stirrups are not practical for thin post-tensioned slabs since it is difficult to anchor the stirrup legs effectively so that they may develop their full yield strength. They also interfere with the placement of tendons as well as concentrated reinforcement over supports. Likewise, the shear heads cannot be used satisfactorily in slabs less than 8 in. (200 mm) deep. This excludes the application of shear heads in office/residential slabs that are mostly 8 in. (200 mm) or less.

A promising alternative is the use of shear studs, as shown in Fig. 12. Shear studs are vertical rods, or the equivalent, that are mechanically anchored at each end. To be fully effective, the anchorage must be ca-



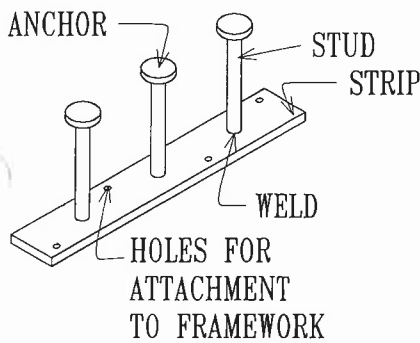


Fig. 12—View of stud shear reinforcement.

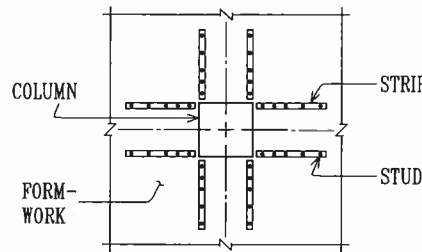


Fig. 13—Plan view of typical arrangement of stud shear reinforcement around column.

pable of developing the yield strength of the rods. Shear studs, properly proportioned, can develop their yield strength and also have minimum interference with the horizontal reinforcement. A stud shear reinforcement arrangement is shown in Fig. 13. Shear studs are designed as stirrups, with the difference that the total area of vertical stirrup legs is substituted by an equal area of shear stud.

Ghali<sup>17,18</sup> and his colleagues have conducted extensive tests on stud shear reinforcement. Their findings are being compiled into a recommendation by the Joint ACI-ASCE Committee 421, Concrete Slab Design.

### Design for lateral loading

The equivalent frame method, in which the entire slab tributary is considered effective in modeling the beam frame, is used primarily in design for gravity loading. Studies on the moment rotation relationship between column and slab subjected to lateral loading or side sway<sup>11,12,19,20</sup> suggest different stiffness values than those obtained from the direct application of the equivalent frame modeling.

At this time there is no simple widely acceptable procedure for modeling of slab frames under lateral loading. The common practice is to use between one-half to one-third of the slab tributary to be effective for the determination of moments due to lateral loading.

### Remarks

The ACI-recommended equivalent frame method is viewed as a strong and satisfactory method for the structural modeling of common two-way post-tensioned floor slab

systems. However, for complex geometries large openings, significant changes in geometry, largely irregular support conditions, and unusual loadings a closer examination in structural modeling is required. Or, when the equivalent frame method is used, the solutions obtained must be followed by a review of the actual slab to insure that local serviceability and strength are provided at irregularities.

Currently, the Joint ACI-ASCE Committee 421 is working on structural modeling of floor slab systems with the objective to recommend simplified guidelines for the structural modeling and treatment of irregular conditions.

### References

1. Freyermuth, Clifford L., "Practice of Partial Prestressing for Continuous Post-Tensioned Structures in North America," *Journal, Prestressed Concrete Institute*, V. 30, No. 1, Jan.-Feb. 1985, pp. 154-172.
2. ACI Committee 318, "Building Code Requirements for Reinforced Concrete (ACI 318-83)," American Concrete Institute, Detroit, 1983, 111 pp. Also, "Commentary on Building Code Requirements for Reinforced Concrete (ACI 318-83)," American Concrete Institute, Detroit, 1983, 155 pp.
3. ACI Committee 423, "Recommendations for Concrete Members Prestressed with Unbonded Tendons," (ACI 423.3R-83), American Concrete Institute, Detroit, 1983, 16 pp.
4. "Design of Post-Tensioned Slabs," Post-Tensioning Institute, Phoenix, 1977, 52 pp.
5. *Post-Tensioning Manual*, 4th Edition, Post-Tensioning Institute, Phoenix, 1985, 406 pp.
6. "Code for the Design of Concrete Structures for Buildings," (CAN3-A23.3-M84), Canadian Standards Association, Rexdale, 1984, 281 pp.
7. "Structural Use of Concrete," (BS8110:1985), British Standards Institution, London, 1985.
8. Fling, Russell S., *Practical Design of Reinforced Concrete*, John Wiley & Sons, New York, 1987, 516 pp.

9. *Uniform Building Code*, International Conference of Building Officials, Whittier, 1988, 926 pp.

10. ADAPT Post-Tensioning Software System, 1733 Woodside Rd., Redwood City, CA 94061.

11. Vanderbilt, M. Daniel, and Corley, W. Gene, "Frame Analysis of Concrete Building," *Concrete International: Design & Construction*, V. 5, No. 12, Dec. 1983, pp. 33-43.

12. Cano, Mary Theresa, and Klingner, Richard E., "Comparison of Analysis Procedures for Two-Way Slabs," *ACI Structural Journal*, V. 85, No. 6, Nov.-Dec. 1988, pp. 597-608.

13. Post-Tensioning Institute, "Specification for Unbonded Single Strand Tendons," *Journal, Prestressed Concrete Institute*, V. 30, No. 2, Mar.-Apr., 1985, pp. 22-39.

14. Lin, T'ung-yen, and Burns, Ned H., *Design of Prestressed Concrete Structures*, John Wiley & Sons, New York, 1981, 646 pp.

15. Aalami, Bijan O., and Barth, Florian G., "Restraint Cracks and Their Mitigation in Unbonded Post-Tensioned Building Structures," *Cracking in Prestressed Concrete Structures*, SP-113, American Concrete Institute, Detroit, 1989, pp. 157-202. Also, Post-Tensioning Institute, Phoenix, 49 pp.

16. Scordelis, A. C.; Lin, T. Y.; and Itaya, R., "Behavior of a Continuous Slab Prestressed in Two-directions," *ACI JOURNAL, Proceedings* V. 56, No. 6, Dec. 1959, pp. 441-459.

17. Dilger, Walter H., and Ghali, Amin, "Shear Reinforcement for Concrete Slabs," *Proceedings, ASCE*, V. 107, ST12, Dec. 1981, pp. 2403-2420.

18. Mokhtar, Abdel-Salam; Ghali, Amin; and Dilger, Walter, "Stud Shear Reinforcement for Flat Concrete Plates," *ACI JOURNAL, Proceedings* V. 82, No. 5, Sept.-Oct. 1985, pp. 676-683.

19. Aalami, Bijan, "Moment-Rotation Relation Between Column and Slab," *ACI JOURNAL, Proceedings* V. 69, No. 5, May 1972, pp. 263-269.

20. Mehrain, Mehrdad, and Aalami, Bijan, "Rotational Stiffness of Concrete Slabs," *ACI JOURNAL, Proceedings* V. 71, No. 9, Sept. 1974, pp. 429-435.

Selected for reader interest by the editors.

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# Appendix

## Torsional Member Modeling in Waffle Slab Construction

In direction perpendicular to the frame, the ribs joining the solid column block are lumped into a solid rectangle of width,  $b_e$ . The effective width,  $b_e$ , is determined such as to offer a torsional stiffness equal to the sum of the individual stiffnesses of the stems meeting the solid block. A pictorial modeling of the column support region is shown in Fig. 10, with a plan view in Fig. 11. The equivalent width,  $b_e$ , of the torsional member model is determined using the assumption that the substitute torsional member section consists of a relatively flange ( $c$ ,  $h_f$ ), and a thin stem ( $b_e$ ,  $h - h_f$ ), for each of which the torsional constant is one-third of length times the respective thickness cubed. The actual torsional constant will be higher if the constituent rectangles are bulkier.

Torsional stiffness,  $J$ , of the prototype is given by:

$$J = (ch^3/3) + \sum[(h - h_f)b_w^3]/3 \quad (1)$$

For the structural model, depending on the aspect ratio of the model, one of the two relationships apply:

(i) If  $b_e < (h - h_f)$

$$J_e = (ch^3/3) + b_e^3 (h - h_f)/3 \quad (2)$$

Substituting for,  $J_e$ , its equivalent,  $J$ , from Eqn (1), and solving for  $b_e$

$$b_e = (\sum b_w^3)^{1/3}$$

(ii) If  $b_e > (h - h_f)$

$$J_e = (ch^3/3) + b_e (h - h_f)^3 / 3 \quad (4)$$

Substituting for,  $J_e$ , its equivalent,  $J$ , from Eqn (1), and solving for  $b_e$

$$b_e = \sum b_w^3 / (h - h_f)^2 \quad (5)$$

Where,  $b_e$ , is the width of the equivalent torsional stiffness member. Other parameters are as defined in Figs.10 and 11.

**Example :**

Calculate the effective width,  $b_e$ , of the stem for the torsional model member of the waffle construction shown in the figure.

$$c = 1200 \text{ mm} ; \quad h = 600 \text{ mm.}; \quad b_w = 250 \text{ mm (idealized)} ; \quad h_r = 125 \text{ mm}$$

Three ribs are in contact with the solid column block (part, a, of the figure)

Alternative (i):  $b_e = (\sum b_w^3)^{1/3}$

$$b_e = (3 \times 250^3)^{1/3} = 360.5 \text{ mm}$$

$$b_e = 360.5 \text{ mm} < (h - h_r) = 600 - 125 = 475 \text{ mm} \quad \text{OK}$$

Alternative (ii):  $b_e = \sum b_w^3 / (h - h_r)^2$

$$b_e = 3 \times 250^3 / 475^2 = 207.7 \text{ mm}$$

$$b_e = 207.7 \text{ mm} < (h - h_r) = 600 - 125 = 475 \text{ mm} \quad \text{NG}$$

Alternative (i) applies

Use  $b_e = 475 \text{ mm}$ .

