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Structural Modeling and Analysis of Concrete Floor Slabs

Post-tensioning adds challenges

BY BIJAN O. AALAMI

The advent of computers and automation has had a major impact on the way engineers design buildings and prepare construction documents. Because they are much simpler to model, materials used in skeletal framing systems, especially steel, have seen the greatest benefit from automated design software. A major obstacle to similar automated design of concrete buildings is the structural modeling and treatment of floor slabs. This problem is exacerbated by the introduction of post-tensioning, especially when beamless floor systems (flat slabs) are used.

This article discusses the similarities and differences between traditional frame methods and finite element methods for the design of floor slab systems. Through this discussion, it will be illustrated that the finite element method has the greatest potential for being able to automate this design process due to the reduction in the amount of input required from the designer. Successful automation of repetitive tasks in the design process would greatly decrease design time and provide the designer with a greater opportunity for creativity and design optimization.

DISTINGUISHING FEATURES

Reinforced concrete has some unique features that distinguish its behavior, and consequently the design of concrete slabs, from other materials. At flexural failure, concrete slabs develop hinge lines. A hinge line mobilizes much of the reinforcement passing through it to resist the moment along its length, contributing to the safety of a concrete slab. The distribution of reinforcement along a design section generally isn't critical for strength. It's the total amount of reinforcement that governs the collapse load.

Once a slab has cracked, the reinforcement determines the manner in which the applied load is resisted. It's the orientation and the amount of reinforcement that govern the path that the load takes to the supports.

These features rely on the ability of the slab, once past the elastic limit, to redistribute forces. Hence, adequate ductility becomes a prerequisite. Stipulations contained in building codes safeguard this needed ductility.

LOAD PATH AND DETAILING

Prior to the calculation of the design moments and shears, the first step an engineer must take in the design of a two-way slab is to anticipate the load path, which sets the orientation and position of the reinforcement. As an example, consider the partial plan of a concrete slab supported on three walls shown in Fig. 1(a). If the slab is modeled as a strip spanning between Walls A and B, the load path requires bottom reinforcing bars placed as shown in Fig. 1(b). Alternatively, the slab could be modeled as a cantilever supported by Wall C. This load path requires top bars over Wall C as shown in Fig. 1(c). The two load paths are very different from one another and also different than the natural elastic response of the slab region, which would be that of a plate supported on three sides. However, each of the selected load paths leads to a safe design.

Apart from ductility, the designer must follow two other considerations. First, the load path selected must be capable of receiving the designated loads. For example, a concentrated load applied to the slab may require additional reinforcing to distribute its effect over the primary bars selected for the load path. Second, while the load path selected ensures adequate strength, additional crack control reinforcement may be necessary for satisfactory in-service performance. The top bars shown in Fig. 1(b) over Wall C and in Fig. 1(c) over Walls A and B enhance the in-service response of the slab.

In addition to designating the load path, post-tensioning requires that the amount of prestressing and the tendon profile be determined before a design is initiated. The final design will be significantly affected by the selections made at this point. While several different designs can meet the serviceability and safety requirements of the codes, some will be more economical than others. As a result, the engineer's experience and design tools can play a critical role in the successful design of a post-tensioned floor.

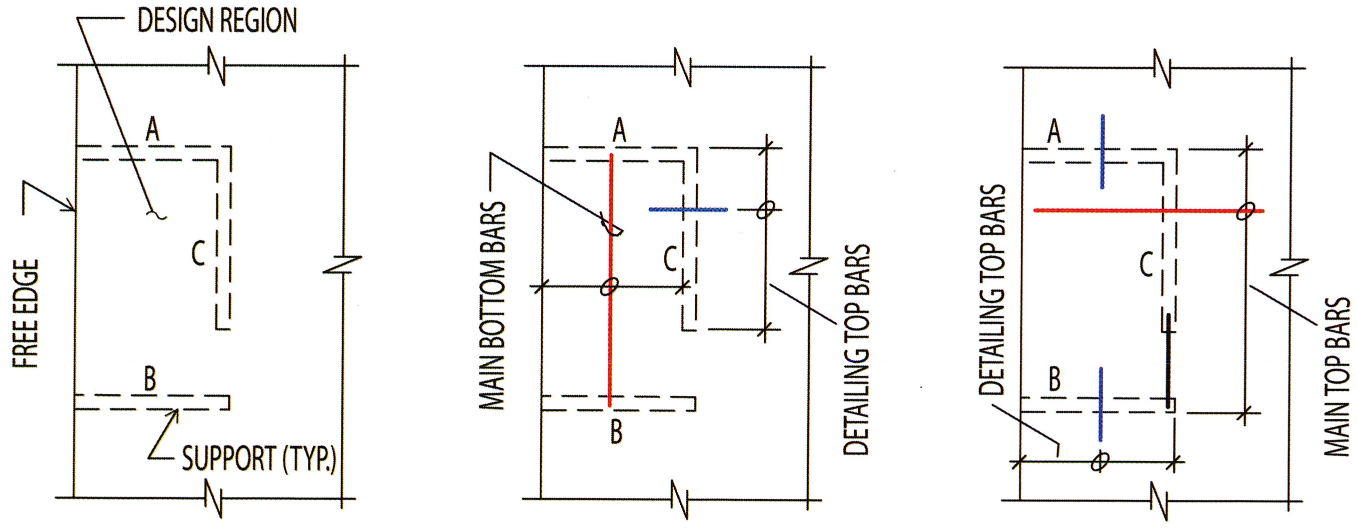
DESIGN PROCESS FOR CONCRETE SLABS

For concrete slabs, the design proceeds through the selection of support lines and design strips that lay out the load path. Following analysis, design values are calculated for the design sections, and finally, code checks can be performed for the determination of required reinforcement. The following example illustrates this process using the slab plan shown in Fig. 2.

First, lines joining adjacent column or wall supports are selected as *support lines* that identify the load path of the floor slab in each direction. Typically, these are the lines along which reinforcement will be placed. The support lines for the X-direction are indicated by the dashed lines in Fig. 2(a). Similar lines would be considered in the orthogonal direction.

The tributary width for each support line forms a *design strip*, as illustrated in Fig. 2. The tributary widths are typically delineated by lines mid-way between adjacent support lines.

Design sections are generally drawn normal to each support line and extend across the entire design strip. At a minimum, design sections are selected at critical locations for each span, as shown in Fig. 2(b). One section is located at the face of each support, and a third



(a) (b) (c)

Fig. 1: Modeling options for a slab region: (a) partial plan of slab region; (b) reinforcing layout for slab designed to span between Walls A and B; and (c) reinforcing layout for slab designed to cantilever over Wall C (after Reference 2)

design section is located at midspan. When design software is used, more design sections (typically 12 to 20) can be automatically generated for each span to arrive at optimum reinforcement lengths.

Design values are the result of the six actions (three forces and three moments) acting on the entire cross-sectional surface of a design section and expressed about its centroid. An enlargement of the first span of Design Strip B and the distribution of moment across the design strips at the first interior column and at midspan are shown in Fig. 3. The area within each of the two distributions is the total "integral" moment that constitutes the design value for the respective design section. The total area of reinforcement required for strength is determined by applying the design moment to the entire cross-sectional area of the design section.

The slab will perform properly under service conditions only if the appropriate distribution of reinforcement is provided. ACI 318-05[1] has recommendations for the distribution of primary reinforcement for both nonprestressed and prestressed floor systems. For nonprestressed slabs, ACI 318 recommends that the reinforcement be distributed based on column strips and middle strips. For prestressed slabs, ACI 318 allows the entire reinforcement to be banded over and adjacent to the column in one direction.

In addition to the primary reinforcement determined from the analysis and design, it's necessary to review the design with the objective of adding reinforcement for proper distribution of loads and crack control. For crack control of prestressed slabs, a representative tensile stress in the concrete is calculated for each design section assuming that the total design force is applied across the entire design section using simple beam theory. If this tensile stress exceeds the allowable code values, additional bonded reinforcement must be provided. For construction with unbonded tendons, bonded reinforcement is always required in negative moment regions at column supports and must be concentrated over the columns.

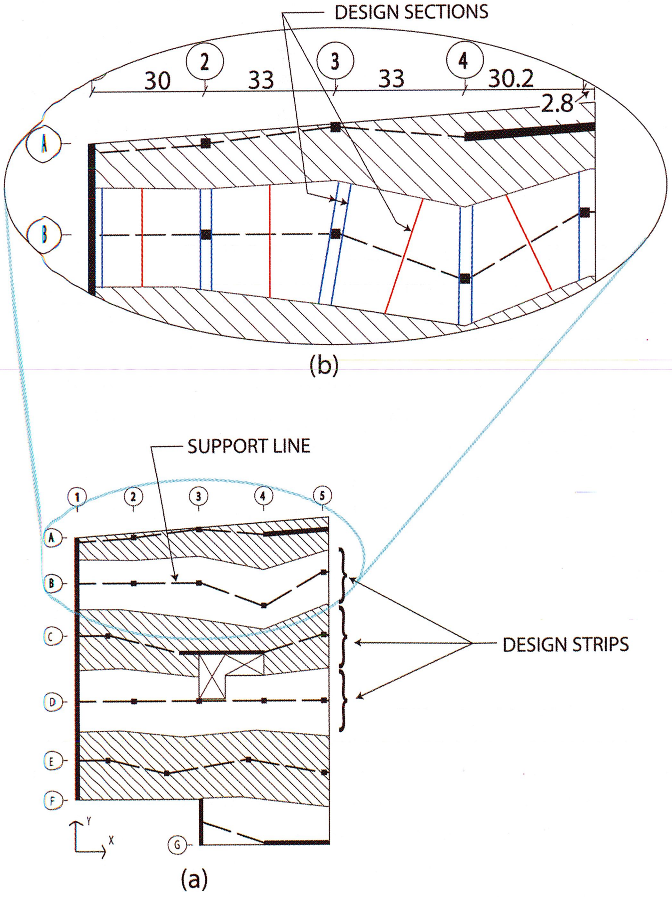


Fig. 2: Design strips in the X-direction for an irregular floor slab: (a) support lines and design strips for the entire floor; and (b) design sections (indicated by red and blue lines) for design strip along Column Row B (after Reference 2). All dimensions in ft

ANALYSIS OPTIONS

Three methods are commonly used to analyze the structure and determine design values for the slab. In all three methods, the floor slab and a story of supports immediately above and below it are isolated for analysis

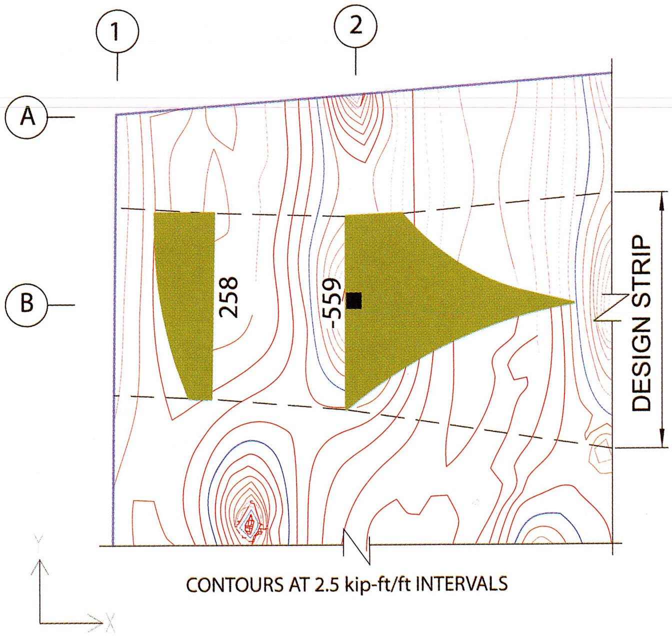


Fig. 3: Service load moment distribution for design sections in first bay of Column Row B. The total design moment at each location is equal to the area under the moment curve for the entire section width (after Reference 2)

of gravity loads (Fig. 4). The far ends of the supports in the structural model are assumed rotationally fixed.

Simple frame method

In the simple frame method, each design strip is analyzed independently from the rest of the slab system using an approximate plane frame model (Fig. 4). The stiffnesses of the slab and columns are based strictly on their cross-sectional geometry.

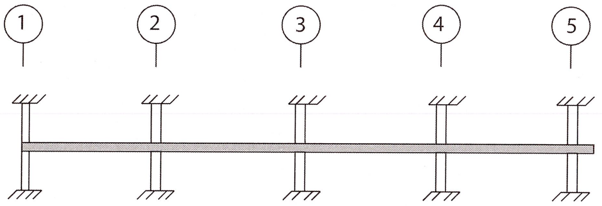


Fig. 4: Elevation of design strip at Column Row B (after Reference 2)

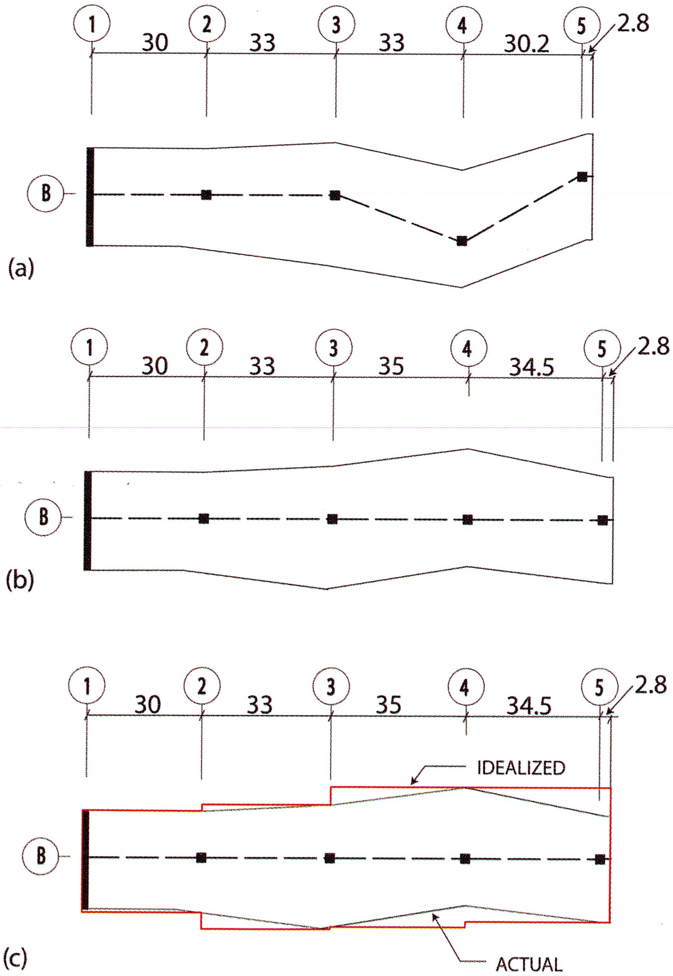


Fig. 5: Plan views of design strip at Column Row B: (a) actual design strip; (b) straightened design strip; and (c) idealized design strip (after Reference 2). All dimensions in ft

To simplify the plane frame analysis, two approximations can be used if the actual supports are not colinear. First, the actual frame shown in Fig. 5(a) can be modeled as a plane frame with span lengths equal to the centerline distance between supports as shown in Fig. 5(b). Second, tributary widths that vary within the span can be conservatively approximated with constant widths as shown in Fig. 5(c).

Equivalent frame method

The equivalent frame method is based on essentially the same concept as the simple frame method, with one significant difference. Using an ACI 318 recommended procedure, the relative stiffness between the column and the slab is modified to simulate the two-way action of a column supported slab that is otherwise lost in the plane frame modeling. The equivalent frame method generally results in reduced support moments, at the expense of an increase in midspan values, when compared to the simple frame method. This is a great advantage in post-tensioned slabs, where equal moment capacities are generally available over the supports and at midspan as a result of the continuous tendons. This can lead to more economical designs.

Finite element method

In the finite element method, the floor system is subdivided into discrete elements (an example is shown in Fig. 6). Unlike the frame methods, the entire floor is analyzed at one time to obtain an overall solution. However, just like in the frame methods, the total forces must be obtained for the design strips and design sections so that code requirements for strength and serviceability can be checked and the reinforcement can be properly distributed.[3] However, the design strips do not need to be selected until after the analysis is completed. Figure 7 shows the distribution of bending moments that is processed from the finite element results for each of the design strips.

METHOD COMPARISONS

Much of the design procedure is the same regardless of whether finite element or frame methods are used. The final goal of determining the reinforcement necessary depends on the determination of the proper design values for each design section along the load path. In the frame methods, these values are obtained directly. In the finite element method, there is generally much more information produced than is strictly necessary for design. The processing of this large amount of information

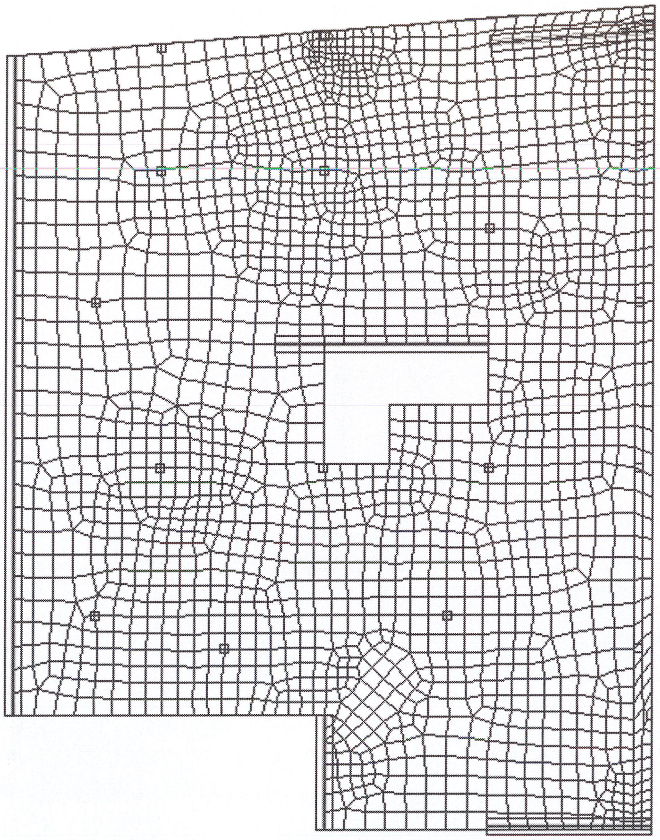


Fig. 6: Plan of entire floor slab showing finite element distribution (after Reference 2)

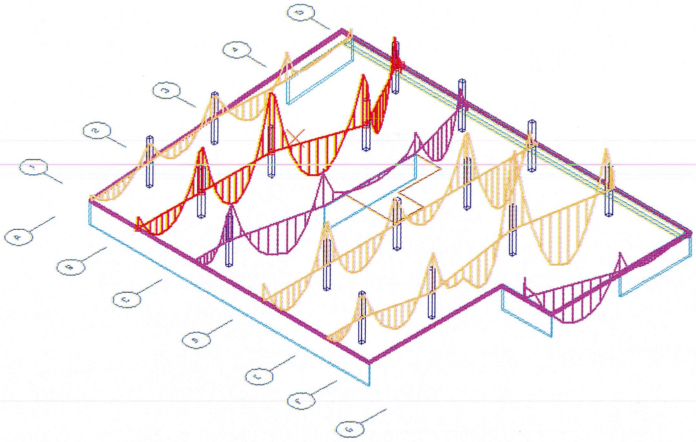


Fig. 7: Integrated floor slab service load moments for X-direction design strips

to obtain the design forces is one of the challenges of using this method. However, it does have the advantage that greater economy in design is achieved when the calculated design values are closer to the elastic response of the floor slab. When the support layout of a floor system is irregular, finite element analysis has the potential of leading to a more economical design and more appropriate structural detailing because it allows the load paths to be selected after the flow of forces can be visualized.

The greatest, but not yet fully tapped, potential for the finite element method is to fully automate the design of concrete floor slabs. For conventionally reinforced floors, current state-of-the-art software is close to full automation. For post-tensioned floor systems, however, full automation is further away. As noted previously, the reason lies in the need for additional input information from the designer prior to the initiation of the analysis. Because the equivalent frame method has the ability to assist the post-tensioning designer in arriving at an optimum design—by helping the engineer determine an economical force and profile—it's anticipated that it will retain its significance and application as the cornerstone tool for post-tensioned design for the foreseeable future. For complex structures, a combination of the equivalent frame method for the determination of initial post-tensioning values, followed by a detailed finite element analysis, is likely to yield the best design.

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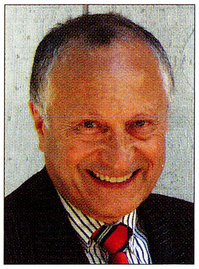

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