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Shear Reinforcement For Slabs

Reported by ACI-ASCE Committee 421

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Tests have established that punching shear in slabs can be effectively resisted by reinforcement consisting of vertical members mechanically anchored at top and bottom of slabs. ACI 318 sets out the principles of design for slab shear reinforcement and makes specific reference to stirrups and shear heads. This report reviews other available devices and makes recommendations for their design. The application of these recommendations is illustrated through a numerical example.

Keywords: column-slab junction; concrete design; design; moment transfer; prestressed concrete; punching shear; shear stresses; stud shear; slabs; two-way floors.

Contents

Notation, pg. 421.1R-2

Chapter 1—Introduction, pg. 421.1R-2
1.1—Objectives
1.2—Scope
1.3—Evolution of the practice

Chapter 2—The role of shear reinforcement, pg. 421.1R-3

Chapter 3—Design procedure, pg. 421.1R-3
3.1—Strength requirement
3.2—Calculation of factored shear stress v_u
3.3—Calculation of shear strength v_n
3.4—Design procedure

Chapter 4—Prestressed slabs, pg. 421.1R-6
4.1—Nominal shear strength

Chapter 5—Suggested higher allowable values for v_n, s_o and s, pg. 421.1R-6
5.1—Justification
5.2—Upper limit for v_n
5.3—Upper limits for s_o and s

Chapter 6—Design example, pg. 421.1R-6

Chapter 7—References, pg. 421.1R-8
7.1—Recommended reference
7.2—Cited references

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Appendix A—Details of shear studs, pg. 421.1R-10
A.1—Geometry of stud shear reinforcement
A.2—Stud arrangements
A.3—Stud length

Appendix B—Properties of critical sections, pg. 421.1R-11

Conversion factors—Inch-pound to SI (metric), pg. 421.1R-11

NOTATION

A_c = area of concrete of assumed critical section
A_v = cross section area of the shear studs on one peripheral line parallel to the perimeter of the column section
b_o = perimeter of critical section
d = effective depth of slab
D = stud diameter
f_c' = specified compressive strength of concrete
f_{pc} = the average value of compressive stress in concrete in the two directions (after allowable of all prestress losses) at centroid of cross section
f_{yv} = specified yield strength of shear studs
J_x, J_y = property of assumed critical section analogous to polar moment of inertia about the axes x and y
ℓ_s = length of stud (including top anchor plate thickness; see Fig. 7
M_{Ox}, M_{Oy} = factored unbalanced moments transferred between the slab and the column about centroidal axes of the column
M_{ux}, M_{uy} = factored unbalanced moments transferred between the slab and the column about centroidal axes x and y of the assumed critical section
n_x, n_y = numbers of studs per line/strip running in x and y directions
s = spacing between peripheral lines of studs
s_o = spacing between first peripheral line of studs and column face
v_c = nominal shear strength provided by concrete in presence of shear studs
v_n = nominal shear strength at a critical section
v_s = nominal shear strength provided by studs
v_u = maximum shear stress due to factored forces
V_p = vertical component of all effective prestress forces crossing the critical section
V_u = factored shear force
x, y = coordinates of the point at which v_u is maximum with respect to the centroidal principal axes x and y of the assumed critical section
α = distance between column face and a critical section divided by d
α_s = dimensionless coefficient equal to 40, 30 and 20 for interior, edge and corner columns, respectively
β_c = ratio of long side to short side of column cross section
γ_{vx}, γ_{vy} = fraction of moment between slab and column that is considered transferred by eccentricity of the shear about the axes x and y of the assumed critical section
ϕ = strength reduction factor = 0.85

CHAPTER 1—INTRODUCTION

1.1—Objectives

In flat plate floors, slab-column connections are subjected to high shear stresses produced by the transfer of axial loads and bending moments between slab and columns. Section 11.12.3 of ACI 318 allows the use of shear reinforcement in the form of bars, as in the vertical legs of stirrups. The ACI 318R commentary emphasizes the importance of anchorage details of the shear reinforcement and accurate placement especially in thin slabs. The general procedure for evaluation of the punching shear strength of slab-column connections is given in Section 11.12 of ACI 318.

Shear reinforcement consisting of vertical rods (studs), or the equivalent, mechanically anchored at each end can be used. In this report, all types of mechanically anchored shear reinforcement are referred to as "shear stud" or "stud." To be fully effective, the anchorage must be capable of developing yield strength of the rods.

1.2—Scope

The present recommendations are for the design of shear reinforcement using shear studs in slabs. The design is in accordance with ACI 318, treating a stud as the equivalent of a vertical branch of a stirrup. A numerical design example is included.

1.3—Evolution of the practice

Extensive tests[1-6] have confirmed the effectiveness of mechanically anchored shear reinforcement (one example is shown in Fig. 1.1*) in increasing the shear strength and ductility of slab-column connections subjected to concentric punching or punching combined with moment. The Canadian Concrete Design Code (CAN3-A23.3) and the German Construction Supervising Authority, Berlin,[7] allow the use of shear studs (Fig. 1.1) for flat slabs. Design rules have been presented[8] for application of British Standard BS 8110 to stud design for slabs. Various forms of such devices were applied and tested by other investigators, as described in Appendix A.

* U.S. and Canada patents Nos. 4406103 and 1085642, respectively. Liscencee: Deha represented, by Decon, 105J Atsion Rd., P.O. Box 1575 Medford, N.J. 08055-6675 and 35 Devon Road, Bramton, Ontario L6T 5B6.

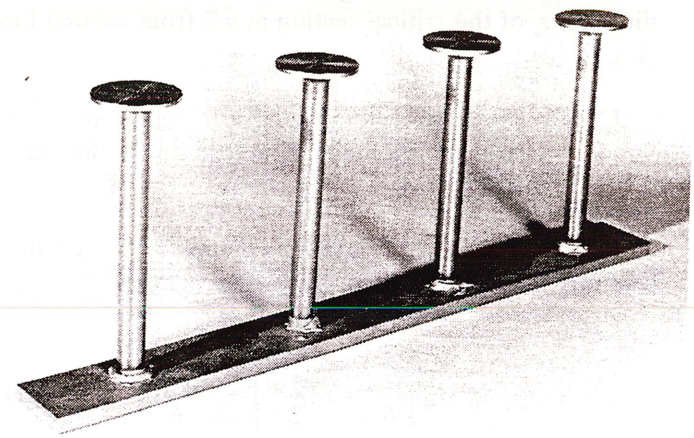


Fig. 1.1—Assembly of one type of shear studs

CHAPTER 2—THE ROLE OF SHEAR REINFORCEMENT

Shear reinforcement is required to intercept shear cracks and to prevent their widening. The intersection of shear reinforcement and cracks can be anywhere over the height of the shear reinforcement. The strain in the shear reinforcement is highest at that intersection.

Effective anchorage is essential and its location must be as close as possible to the structural member's outer surfaces. This means that the vertical part of the shear reinforcement must be as long as possible to avoid the possibility of cracks passing above or below it (i.e. cracks not intercepted by shear reinforcement).

Anchorage of shear reinforcement in slabs is achieved by mechanical ends, bends and hooks. However, the following should be noted:

Tests[1] have shown that movement occurs at the bends of shear reinforcement, at point A of Fig. 2.1, before the yield strength can be reached in the shear reinforcement, causing a loss of tension. Furthermore, the concrete under the bend in the stirrups is subjected to stresses that could exceed 0.4 times the stirrup's yield stress f_{yv}, causing concrete crushing. These difficulties, including the consequences of improper stirrup details, have also been discussed by others.[9-12] The movement at the end of the vertical leg of a stirrup can be reduced by attachment to a flexural reinforcement bar as shown, at point B of Fig. 2.1. However, the flexural reinforcing bar cannot be placed any closer to the vertical leg of the stirrup, without reducing the effective slab depth, d. It should be noted that flexural reinforcing bars can provide such improvement to shear reinforcement anchorage only if direct contact exists at the intersection of the bars, at point B of Fig. 2.1. However, under normal construction conditions, it is very difficult to ensure such contact for all stirrups. Thus, such support is normally not fully effective, and the end of the vertical leg of the stirrup

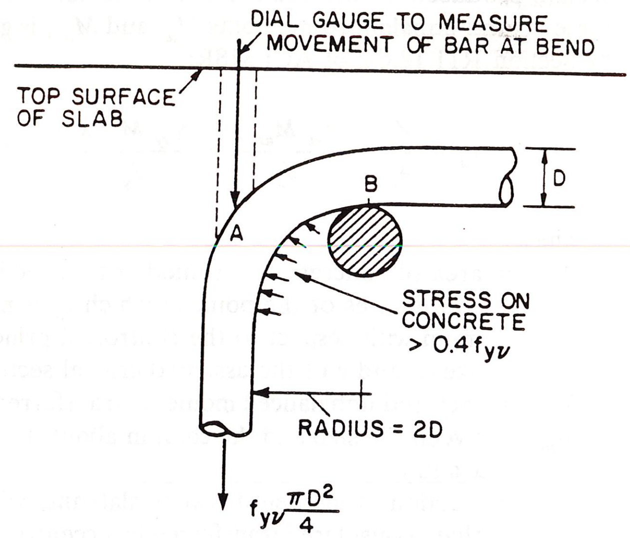


Fig. 2.1—Geometrical and stress conditions at a bend of a shear reinforcing bar

can move. The amount of movement is the same for a short or long shear reinforcing bar. Therefore, the loss in tension is very important and the stress is unlikely to reach yield in short shear reinforcement (in thin slabs). These problems are eliminated if shear reinforcement is provided with mechanical anchorage.

CHAPTER 3—DESIGN PROCEDURE

3.1—Strength requirement

This chapter presents the design procedure for mechanically anchored shear reinforcement required in the slab in the vicinity of a column transferring moment and shear. The requirements of ACI 318 are satisfied and a stud is treated as the equivalent of one vertical leg of a stirrup.

Design of critical slab sections perpendicular to the plane of the slab should be based upon

$$v_u \leq \phi\, v_n \tag{3.1}$$

in which v_u is the shear stress in the critical section caused by the transfer between the slab and the column of factored axial force or factored axial force combined with moment; v_n is the nominal shear strength (Eqs. 3.5 to 3.9).

Equation 3.1 should be satisfied at a critical section perpendicular to the plane of the slab at a distance $d/2$ from the column perimeter and at a critical section located so that its perimeter b_o, is minimum but need not approach closer than $d/2$ to the outermost peripheral line of shear studs.

3.2—Calculation of factored shear stress v_u

The maximum factored shear stress v_u at a critical section produced by the combination of factored shear force V_u and unbalanced moments M_{ux} and M_{uy}, is given by Section R11.12.6.2 of ACI 318R:

$$v_u = \frac{V_u}{A_c} + \frac{\gamma_{vx} M_{ux}\, y}{J_x} + \frac{\gamma_{vy} M_{uy}\, x}{J_y} \qquad (3.2)$$

in which

A_c = area of concrete of assumed critical section

x, y = coordinates of the point at which v_u is maximum with respect to the centroidal principal axes x and y of the assumed critical section

M_{ux}, M_{uy} = factored unbalanced moments transferred between the slab and the column about the axes x and y

γ_{vx}, γ_{vy} = fraction of moment between slab and column that i considered transferred by eccentricity of shear about the axes x and y of the assumed critical section. The coefficients γ_v are given by:

$$\gamma_{vx} = 1 - \frac{1}{1 + 2/3\sqrt{\ell_{y1}/\ell_{x1}}}\ ;\ \gamma_{vy} = 1 - \frac{1}{1 + 2/3\sqrt{\ell_{x1}/\ell_{y1}}} \qquad (3.3)$$

where ℓ_{x1} and ℓ_{y1} are lengths of the sides in the x and y directions, of the critical section at $d/2$ from column face (Fig. 3.1a).

J_x, J_y = property of assumed critical section, analogous to polar moment of inertia about the axes x and y

In the vicinity of an interior column, J_y for a critical section at $d/2$ from column face (Fig. 3.1a) is given by:

$$J_y = d\left[\frac{\ell_{x1}^3}{6} = \frac{\ell_{y1}\,\ell_{x1}^2}{2}\right] + \frac{\ell_{x1}\, d^3}{6} \qquad (3.4)$$

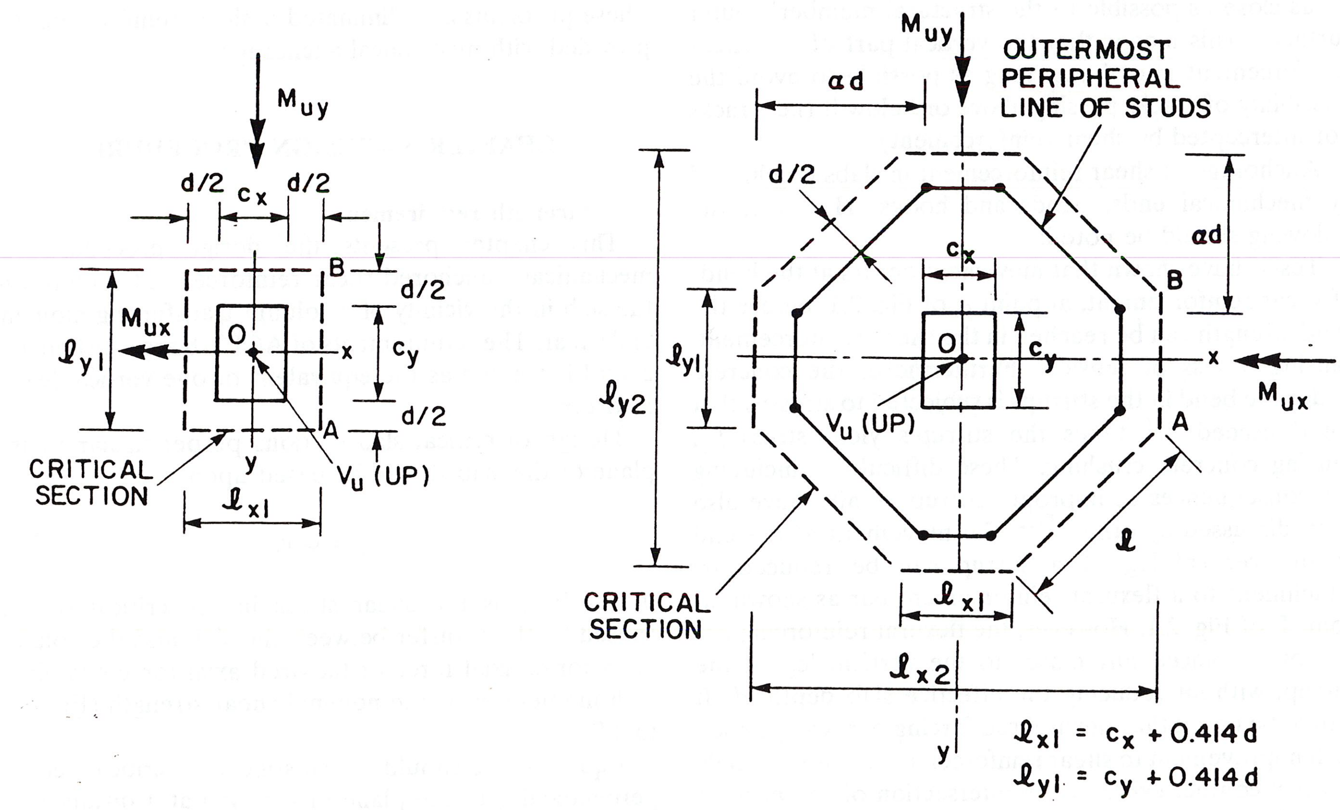
To determine J_x, interchange the subscripts x and y in Eq. 3.4.

For other conditions any rational method may be used.

3.3—Calculation of shear strength v_n

Whenever the specified compressive strength of concrete f_c' is used in Eqs. 3.5 to 3.10 to follow, its value must be in pounds per square inch.

3.3.1 *Shear strength without shear reinforcement*—For nonprestressed slabs, the shear strength of concrete at a critical section at $d/2$ from column face where shear reinforcement is not provided should be the smallest of:



a) AT d/2 FROM COLUMN FACE   b) AT d/2 FROM THE OUTERMOST PERIPHERAL LINE OF SHEAR STUDS

Fig. 3.1—Critical sections for shear in slab in the vicinity of an interior column

a) $$v_n = \left(2 + \frac{4}{\beta_c}\right)\sqrt{f_c'} \qquad (3.5)$$

where β_c is the ratio of long side to short side of the column cross-section.

b) $$v_n = \left(\frac{\alpha_s d}{b_o} + 2\right)\sqrt{f_c'} \qquad (3.6)$$

where α_s is 40 for interior columns, 30 for edge columns, 20 for corner columns and

c) $$v_n = 4\sqrt{f_c'} \qquad (3.7)$$

At a critical section outside the shear-reinforced zone,

$$v_n = 2\sqrt{f_c'} \qquad (3.8)$$

Equation 3.1 should be checked first at a critical section at $d/2$ from the column face (Fig. 3.1a). If Eq. 3.1 is not satisfied, shear reinforcement is required.

3.3.2 *Shear strength with studs*—The shear strength v_n at a critical section at $d/2$ from the column face should not be taken greater than 6 $\sqrt{f_c'}$ when stud shear reinforcement is provided. The shear strength at a critical section within the shear-reinforced zone should be computed by:

$$v_n = v_c + v_s \qquad (3.9)$$

in which

$$v_c = 2\sqrt{f_c'} \qquad (3.10)$$

and

$$v_s = \frac{A_v f_{yv}}{b_o s} \qquad (3.11)$$

where A_v is the cross-sectional area of the shear studs on one peripheral line parallel to the perimeter of the column section; s is the spacing between peripheral lines of studs.

The distance s_o between the first peripheral line of shear studs and the column should not be smaller than $d/4$ (Fig. 3.2). The upper limits for s_o and for the spacing s between the peripheral lines should be:

$$s_o \le 0.5d \qquad (3.12)$$

$$s \le 0.5d \qquad (3.13)$$

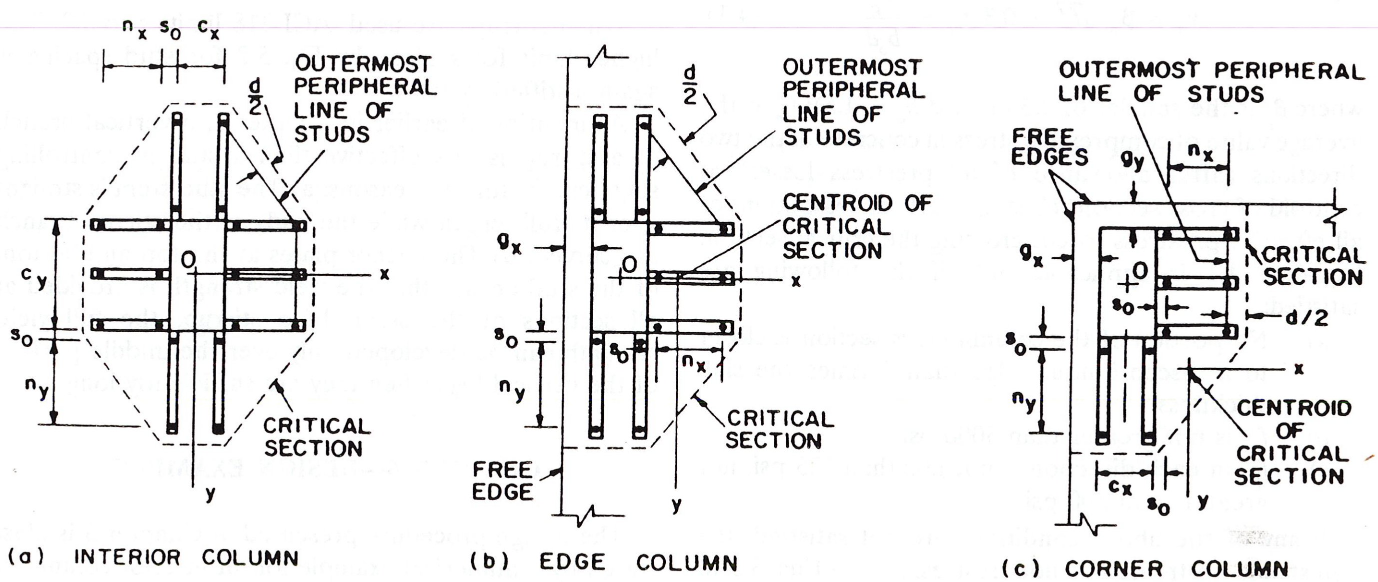
The upper limit of s_o is intended to eliminate the possibility of shear failure between the column face and the innermost peripheral line of shear studs. Similarly, the upper limit of s is to avoid failure between consecutive peripheral lines of studs.

The shear studs should extend away from the column face so that the shear stress v_u at a critical section at $d/2$ from outermost peripheral line of shear studs (Fig. 3.1b and Fig. 3.2) does not exceed ϕv_n, where v_n is calculated using Eq. 3.8.

3.4—Design procedure

The values of f_c', f_{yv}, M_u, V_u, h and d are given. The design of stud shear reinforcement can be performed by the following steps:

1. At a critical section at $d/2$ from column face, calcu-



Note: n_x and n_y are number of equally spaced studs in x and y direction, respectively. Only the innermost and outermost studs are shown.

Fig. 3.2—Typical arrangement of shear studs and critical sections outside the shear reinforced zone

late v_u and v_n by Eqs. 3.2 and 3.5 to 3.7. If $(v_u/\phi) \leq v_n$, no shear reinforcement is required.

2. If $(v_u/\phi) > v_n$, calculate the contribution of concrete, v_c to the shear strength (Eq. 3.10) at the same critical section. The difference $[(v_u/\phi) - v_c]$ gives the shear stress v_s to be resisted by studs.

3. Select s_o and stud spacing s within the limitations of Eqs. 3.12 and 3.13, and calculate the required area of stud for one peripheral line, A_v by solution of Eq. 3.11. Find the minimum number of studs per peripheral line.

4. Repeat step 1 at a trial critical section at αd from column face to find the section where $(v_u/\phi) \leq 2\sqrt{f_c'}$. No other section need to be checked and s is to be maintained constant. Select the distance between the column face and the outermost peripheral line of studs to be $\geq (\alpha d - d/2)$.

The position of the critical section can be determined by selection of n_x and n_y (Fig. 3.2); in which n_x and n_y are numbers of studs per line running in x and y directions, respectively. For example, the distance in the x direction between the column face and the critical section is equal to $s_o + (n_x - 1)\,s + d/2$. The two numbers n_x and n_y need not be equal; but each must be ≥ 2.

5. Arrange studs to satisfy the detailing requirements described in Appendix A.

CHAPTER 4—PRESTRESSED SLABS

4.1—Nominal shear strength

When a slab is prestressed in two directions, the shear strength of concrete at a critical section at $d/2$ from the column face where stud shear reinforcement is not provided is given by ACI 318:

$$v_n = \beta_p \sqrt{f_c'} + 0.3 f_{pc} + \frac{V_p}{b_o d} \qquad (4.1)$$

where β_p is the smaller of 3.5 or $(\alpha_s d/b_o + 1.5)$; f_{pc} is the average value of compressive stress in concrete in the two directions (after allowance of all prestress losses) at centroid of cross section; V_p is the vertical component of all effective prestress forces crossing the critical section. Equation 4.1 is applicable only if the following are satisfied:

a) No portion of the column cross section is closer to a discontinuous edge than 4 times the slab thickness

b) f_c' is not greater than 5000 psi

c) f_{pc} in each direction is not less than 125 psi, nor greater than 500 psi

If any of the above conditions are not satisfied, the slab should be treated as nonprestressed and Eqs. 3.5 to 3.8 apply. Within the shear-reinforced zone, v_n is to be calculated by Eq. 3.9.

In thin slabs, the slope of the tendon profile is hard to control. Special care should be exercised in computing V_p in Eq. 4.1 due to the sensitivity of its value to the as built tendon profile.

CHAPTER 5—SUGGESTED HIGHER ALLOWABLE VALUES FOR v_n, s_o and s

5.1—Justification

Section 11.5.3 of ACI 318 requires that "stirrups and other bars or wires used as shear reinforcement shall extend to a distance d from extreme compression fiber and shall be anchored at both ends according to Section 12.13 to develop the design yield strength of reinforcement." Test results[1-6] using studs with anchor heads of area equal to 10 times the cross section area of stem clearly satisfied that requirement. Further, use of the shear device shown in Fig. 1.1 demonstrated a higher shear capacity. Other researchers, as briefly mentioned in Appendix A, applied successfully other configurations. This justifies the following deviations[1] from the Code:

5.2—Upper limit for v_n

The nominal shear strength, v_n resisted by concrete and steel in Eq. 3.9 can be taken as high as $8\sqrt{f_c'}$ instead of $6\sqrt{f_c'}$. This enables use of thinner slabs.

5.3—Upper limits for s_o and s

The upper limits for s_o and s can be based on the value of v_u at the critical section at $d/2$ from column face:

$$s_o \leq 0.5d \text{ and } s \leq 0.75d \quad \text{when } \frac{v_u}{\phi} \leq 6\sqrt{f_c'} \qquad (5.1)$$

$$s_o \leq 0.35d \text{ and } s \leq 0.5d \quad \text{when } \frac{v_u}{\phi} > 6\sqrt{f_c'} \qquad (5.2)$$

When stirrups are used, ACI 318 limits s to $d/2$. The higher limit for s given by Eq. 5.2 for stud spacing is again justified by tests.

As mentioned earlier in Chapter 2, a vertical branch of a stirrup is less effective than a stud in controlling shear cracks for two reasons: a) The stud stem is straight over its full length while the ends of the stirrup branch are curved; b) The anchor plates at the top and bottom of the stud ensure that the yield strength is provided at all sections of the stem. In a stirrup, the full yield strength can be developed only over the middle portion of the vertical legs when they are sufficiently long.

CHAPTER 6—DESIGN EXAMPLE

The design procedure presented in Chapter 3 is illustrated by a numerical example for an interior column of a nonprestressed slab. Design example for studs at edge column is presented in Ref. 13. There is divergence of opinions with respect to the treatment of corner and irregular columns; see Refs. 13, 14 and 15.

The design of studs is required at an interior column based on the following data: column size c_x by c_y = 12.0 in. x 20.0 in.; slab thickness = 7.00 in.; concrete cover = 0.75 in.; f_c' = 4000 psi; yield strength of studs f_{yv} = 60 ksi; flexural reinforcement diameter = ⅝ in. The factored forces transferred from the column to the slab are: V_u = 110 kip and M_{uy} = 50 ft-kip. The five steps of design outlined in Chapter 3 are followed:

Step 1: The effective depth of slab

$$d = 7.00 - 0.75 - 0.63 = 5.62 \text{ in.}$$

Properties of a critical section at $d/2$ from column face shown in Fig. 6.1 (Eqs B.1 and B.2; see Appendix B):

$$b_o = 86.5 \text{ in.}; A_c = 486 \text{ in.}^2$$

$$J_y = 28.0 \times 10^3 \text{ in.}^4;\ \ell_{x1} = 17.62 \text{ in.}$$

$$\ell_{y1} = 25.62 \text{ in.}$$

The fraction of moment transferred by shear (Eq. 3.3):

$$\gamma_{vy} = 1 - \frac{1}{1 + \frac{2}{3}\sqrt{\frac{17.62}{25.62}}} = 0.36$$

The maximum shear stress occurs at x = 17.62/2 = 8.81 in. and its value is (Eq. 3.2):

$$v_u = \frac{110 \times 1000}{486} + \frac{0.36\ (50 \times 12000)\ 8.81}{28.0 \times 10^3} = 294\ psi$$

$$\frac{v_u}{\phi} = \frac{294}{0.85} = 346\ psi = 5.5\sqrt{f_c'}$$

The nominal shear stress that can be resisted without shear reinforcement at the critical section considered (Eqs. 3.5-3.7):

$$v_n = \left(2 + \frac{4}{1.67}\right)\sqrt{f_c'} = 4.4\sqrt{f_c'}$$

$$v_n = \left[\frac{40(5.62)}{86.5} + 2\right]\sqrt{f_c'} = 4.6\sqrt{f_c'}$$

$$v_n = 4\sqrt{f_c'}$$

use the smallest value: $v_n = 4\sqrt{f_c'}$ = 253 psi

Step 2: The quantity v_u/ϕ is greater than v_n indicating that shear reinforcement is required; the same quantity is less than the upper limit $v_n = 6\sqrt{f_c'}$, which means that the slab thickness is adequate.

The shear stress resisted by concrete in presence of the shear reinforcement (Eq. 3.10) at the same critical section:

$$v_c = 2\sqrt{f_c'} = 126 \text{ psi}$$

Use of Eqs. 3.1, 3.9 and 3.11 gives:

$$v_s \geq \frac{v_u}{\phi} - v_c = 346 - 126 = 220\ psi$$

$$\frac{A_v}{s} \geq \frac{v_s\, b_o}{f_{yv}} = \frac{220(86.5)}{60000} = 0.32\ in.$$

Step 3:

$$s_o \leq 0.5d = 2.8 \text{ in.};\ s \leq 0.5d = 2.8 \text{ in.}$$

This example has been provided for one specific type of shear stud reinforcement, but the approach can be adapted and used also for other types mentioned in Appendix A.

Try ⅜ in. diameter studs welded to a bottom anchor strip 3/16 in. x 1 in. Taking cover of ¾ in. at top and bottom, the length of stud ℓ_s (Fig. 6.1) should not exceed:

$$\ell_{smax} = 7 - 2\left(\frac{3}{4}\right) - \frac{3}{16} - 5\frac{5}{16}\ in.$$

Also, ℓ_s should not be smaller than:

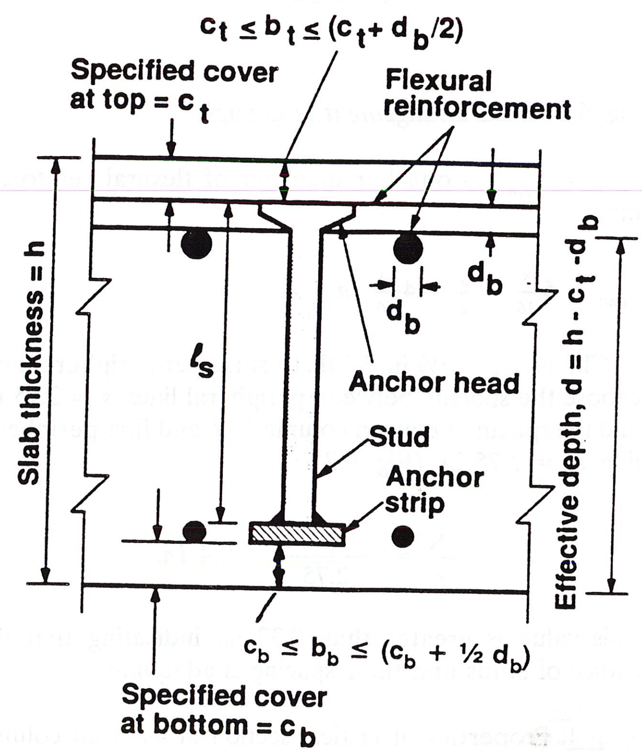


Fig. 6.1—Section in slab perpendicular to a shear stud line

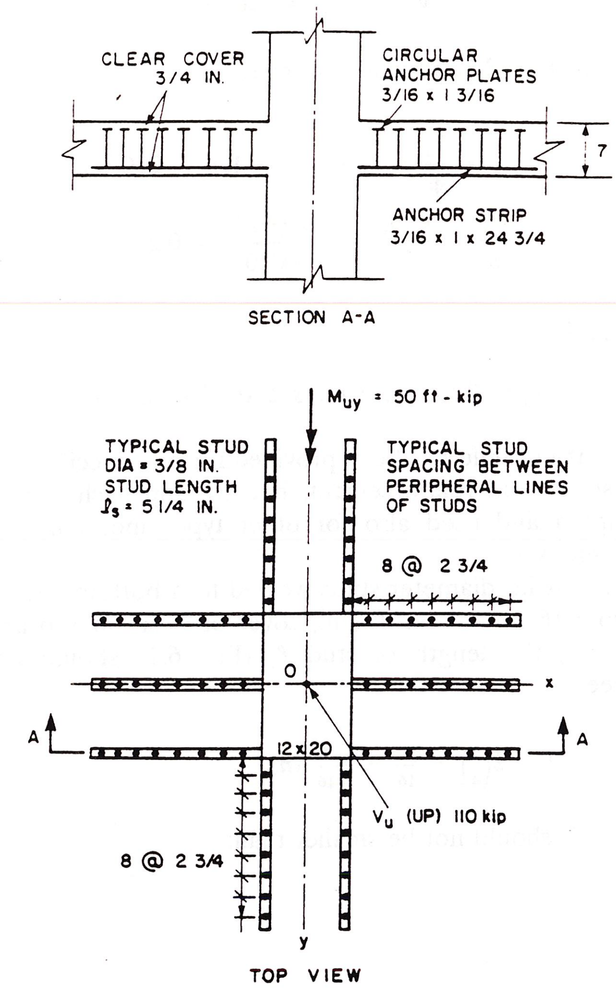


Fig. 6.2—Stud arrangement in example

$\ell_{smin} = \ell_{smax}$ - one bar diameter of flexural reinforcement

$$\ell_{smin} = 5\frac{5}{16} - \frac{5}{8} = 4\frac{11}{16}\ in.$$

Choose $\ell_s = 5\frac{1}{4}$ in. With 10 studs per peripheral line, choose the spacing between peripheral lines, s = 2.75 in. and the spacing between column face and first peripheral line, s_o = 2.75 in. (Fig. 6.2).

$$\frac{A_v}{s} = \frac{10\ (0.11)}{2.75} = 0.4\ in.$$

This value is greater than 0.32 in. indicating that the choice of studs and their spacing is adequate.

Step 4: Properties of critical section at $4d$ from column face (Fig. 3.1b, Eqs. B.1 to B.3):

$\alpha = 4.0;\ \alpha_d = 4(5.62) = 22.5$ in.;

$\ell_{x1} = 14.3$ in.; $\ell_{y1} = 22.3$ in.;

$\ell_{x2} = 57.0$ in.; $\ell_{y2} = 65.0$ in.;

$\ell = 30.2$ in.; $b_o = 194.0$ in.;

$A_c = 1090$ in.^{2}; $J_y = 449.5 \times 10^3$ in.^{4}

The maximum shear stress in the critical section occurs on line AB at:

$x = 57/2 = 28.5$ in. Eq. 3.2 gives:

$$v_u = \frac{110000}{1090} + \frac{0.36\ (50 \times 12000)\ 28.5}{449.5 \times 10^3} = 115\ psi$$

$$\frac{v_u}{\phi} = \frac{115}{0.85} = 135\ psi$$

$$v_n = 2\sqrt{f_c'} = 126\ psi$$

The value v_u/ϕ = 135 psi is greater than v_n = 126 psi, which indicates that shear stress should be checked at $\alpha > 4$. Try 8 peripheral lines of studs; distance between column face and outermost peripheral line of studs:

$$= s_o + 7s = 2.75 + 7(2.75) = 22.0\ in.$$

Check shear stress at a critical section at a distance from column face

$$= 22 + d/2 = 22.0 + 5.62/2 = 24.8\ in.$$

$$\alpha = \frac{24.8}{d} = \frac{24.8}{5.62} = 4.4$$

$$\frac{V_u}{\phi} = 125\ psi$$

$$v_n = 2\sqrt{f_c'} = 126\ psi$$

Step 5: The value v_u/ϕ is less than v_n, which indicates that details of stud arrangement as shown in Fig. 6.2 are adequate.

The value of V_u used to calculate the maximum shear stress could have been reduced by the counteracting factored load on the slab area enclosed by the critical section.

CHAPTER 7—REFERENCES

7.1—Recommended references

The documents of the various standards-producing organizations referred to in this document are listed below with their serial designation.

American Concrete Institute

ACI 318/318R Building Code Requirements for

Reinforced Concrete and Commentary

British Standards Institution

BS 8110 Structural Use of Concrete

Canadian Standards Association

CAN3-A23.3 Design of Concrete Structures for Buildings

The above publications may be obtained from the following organizations:

American Concrete Institute
P.O. Box 19150
Detroit, MI 48219-0150

British Standards Institution
2 Park Street
London W1A 2BS
England

Canadian Standards Association
178 Rexdale Blvd.
Rexdale, Ontario M9W 1R3
Canada

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APPENDIX A—DETAILS OF SHEAR STUDS

A.1—Geometry of stud shear reinforcement

Several types and configurations of shear studs have been reported in the literature. Shear studs mounted on a continuous steel strip, as discussed in the main text, have been developed and investigated.[1-6] T-headed reinforcing bars were developed and applied in Norway[16] for high strength concrete structures, and it was reported that such applications improved significantly the structural performance.[17] Another type of T-headed shear reinforcement was implemented for increasing the punching shear strength of lightweight concrete slabs and shells.[18] Several other approaches for mechanical anchorage in shear reinforcement can be used.[9, 19-21] Several types are depicted in Fig. A1; the figure also shows the required details of stirrups when used in slabs according to ACI 318R.

The anchors should be in the form of circular or rectangular plates, and their area must be sufficient to develop the yield strength of studs f_{yv}. It is recommended that the performance of the shear stud reinforcement be verified before their use. The user can find such information in the references cited above.

A.2—Stud arrangements

Shear studs in the vicinity of rectangular columns should be arranged on peripheral lines. The term peripheral line is used in this report to mean a line running parallel to and at constant distance from the sides of the column cross section. Fig. 3.2 shows a typical arrangement of stud shear reinforcement in the vicinity of a rectangular interior, edge and corner columns. Tests[1] showed that studs are most effective near column corners. For this reason shear studs in Figs. 3.2a, b and c are aligned with column faces. In the direction parallel to a column face, the distance g between lines of shear studs should not exceed $2d$, where d is the effective depth of the slab.

The stud arrangements for circular columns are shown in Fig. A2. The minimum number of peripheral lines of shear studs, in the vicinity of rectangular and circular columns, is two.

A.3—Stud length

The studs are most effective when their anchors are as close as possible to the top and the bottom surfaces of the slab. Unless otherwise protected, the minimum concrete cover of the anchors should be the same as the

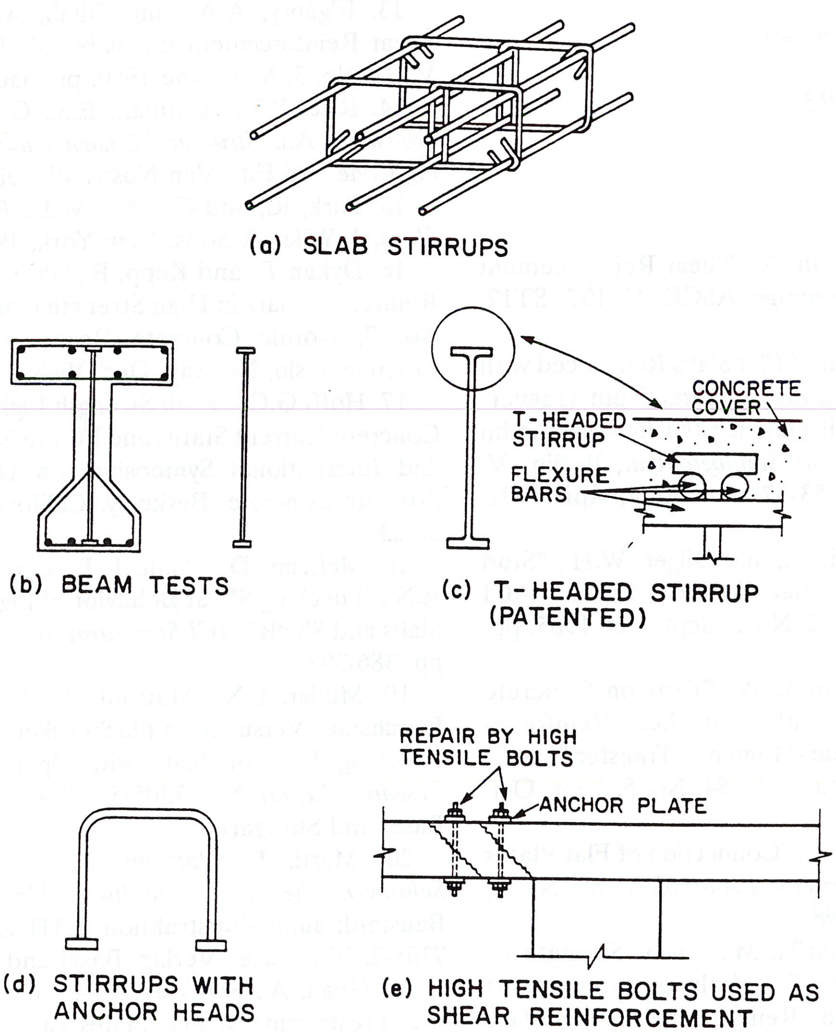

Fig. A1—Shear reinforcement types (a) to (e) are from reference ACI 318, and cited references 16, 18, 19 and 21, respectively.

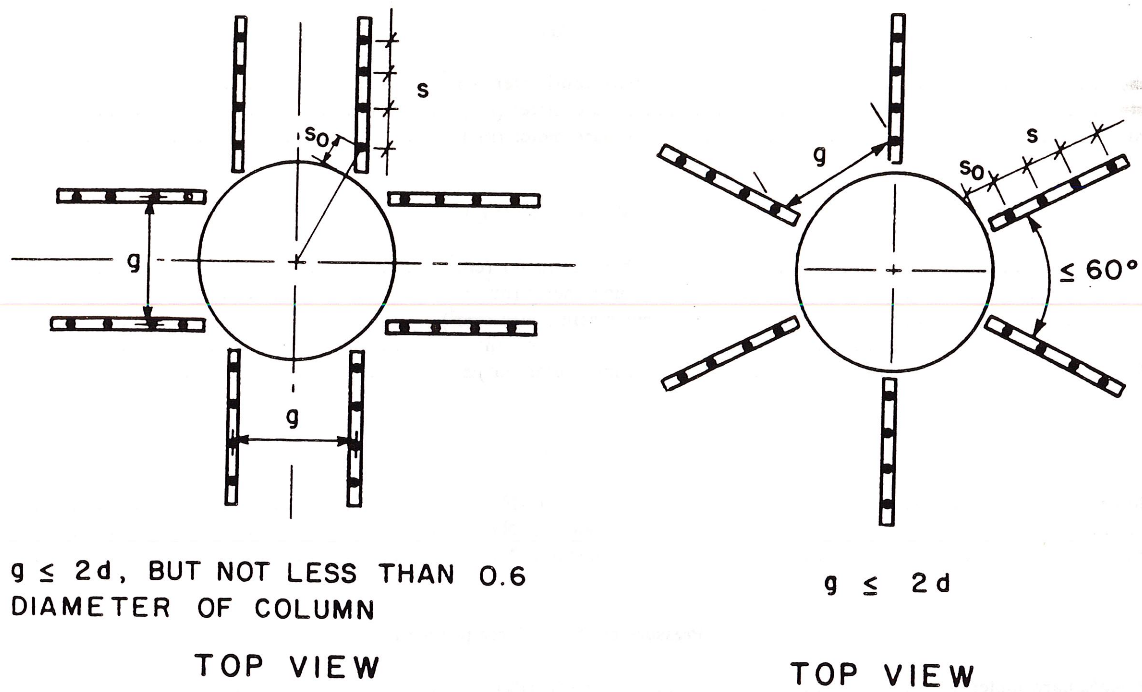

Fig. A.2—Stud shear reinforcement arrangement for circular columns

minimum cover for the flexural reinforcement following Section 7.7 of ACI 318. The cover of the anchors should not exceed the minimum cover plus one half bar diameter of flexural reinforcement (Fig. 6.1). The mechanical anchors should be placed in the forms above reinforcement supports which ensure the specified concrete cover.

APPENDIX B—PROPERTIES OF CRITICAL SECTIONS

Figure 3.1 shows the top view of critical sections for shear in slab in the vicinity of interior column. The centroidal x and y axes of the critical sections, V_u, M_{ux} and M_{uy} are shown in their positive directions. The shear force V_u is acting at the column centroid; V_u, M_{ux} and M_{uy} represent the effects of the column on the slab.

The equations given below give the area properties of a general critical section for interior column. When the critical section is at $d/2$ from the column face (Fig. 3.1a), the same equations apply by setting $\ell_{x1} = \ell_{x2}$, $\ell_{y1} = \ell_{y2}$, and $\ell = 0$.

The properties of the critical section shown in Fig. 3.1b are:

$$b_o = 2(\ell_{x1} + \ell_{y1}) + 4_{\ell} \quad \text{(B-1)}$$

$$J_y = d\left\{\frac{\ell_{x1}^3}{6} + \frac{\ell_{y1}\ell_{x2}^2}{2} + \frac{\ell}{4}\left[(\ell_{x2} + \ell_{x1})^2 + 1/3\,(\ell_{x2} - \ell_{x1})^2\right]\right\} + \frac{\ell_{x2}d^3}{6} \quad \text{(B-2)}$$

where

$$\ell = \sqrt{1/2(\ell_{x2} - \ell_{x1})^2} \quad \text{(B-3)}$$

To determine J_x, interchange the subscripts x and y in Eq. B-2.

CONVERSION FACTORS—INCH-POUND TO SI (METRIC)*

To convert from	to	multiply by
Length		
inch	millimeter (mm)	25.4E+
foot	meter (m)	0.3048E
yard	meter (m)	0.9144E
mile (statute)	kilometer (km)	1.609

to convert from	to	multiply by
Area		
square inch	square centimeter (cm^2)	6.451
square foot	square meter (m^2)	0.0929
square yard	square meter (m^2)	0.8361
Volume (capacity)		
ounce	cubic centimeter (cm^3)	29.57
gallon	cubic meter (m^3)±	0.003785
cubic inch	cubic centimeter (cm^3)	16.4
cubic foot	cubic meter (m^3)	0.02832
cubic yard	cubic meter (m^3)±	0.7646
Force		
kilogram-force	newton (N)	9.807
kip-force	newton (N)	4448
pound-force	newton (N)	4.448
Pressure or stress (force per area)		
kilogram-force/square meter	pascal (Pa)	9.807
kip-force/square inch (ksi)	megapascal (MPa)	6.895
newton/square meter (N/m^2)	pascal (Pa)	1.000E
pound-force/square foot	pascal (Pa)	47.88
pound-force/square inch (psi)	kilopascal (kPa)	6.895
Bending moment or torque		
inch-pound-force	newton-meter (Nm)	0.1130
foot-pound-force	newton-meter (Nm)	1.356
meter-kilogram-force	newton-meter (Nm)	9.807
Mass		
ounce-mass (avoirdupois)	gram (g)	28.34
pound-mass (avoirdupois)	kilogram (kg)	0.4536
ton (metric)	megagram (mg)	1.000E
ton (short, 2000 lbm)	megagram (Mg)	0.9072
Mass per volume		
pound-mass/cubic foot	kilogram/cubic meter (kg/m^3)	16.02
pound-mass/cubic yard	kilogram/cubic meter (kg/m^3)	0.5933
pound-mass/gallon	kilogram/cubic meter (kg/m^3)	119.8
Temperature§		
degrees Fahrenheit (F)	degrees Celsius (C)	$t_C = (t_F - 32)/1.8$
degrees Celsius (C)	degrees Fahrenheit (F)	$t_F = 1.8t_C + 32$

* This selected list gives practical conversion factors of units found in concrete technology. The reference source for information on SI units and more exact conversion factors is "Standard for Metric Practice" ASTM E 380. Symbols of metric units are given in parenthesis.

+E Indicates that the factor given is exact.

± One liter (cubic decimeter) equals 0.001 m^3 or 1000 cm^3.

§ These equations convert one temperature reading to another and include the necessary scale corrections. To convert a difference in temperature from Fahrenheit degrees to Celsius degrees, divide by 1.8 only, i.e., a change from 70 to 88 F represents a change of 18 F or 18/1.8 = 10 C deg.

ACI STRUCTURAL JOURNAL

COMMITTEE REPORT

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ACI 421.1R

Abstract of: Shear Reinforcement for Slabs

reported by ACI Committee 421

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Tests have established that punching shear in slabs can be effectively resisted by reinforcement consisting of vertical members mechanically anchored at the top and bottom of slabs. ACI 318 sets out the principles of design for slab shear reinforcement and makes specific reference to stirrups and shear heads. This report reviews other available devices and makes recommendations for their design. The application of these recommendations is illustrated with a numerical example.

Keywords: column-slab junction; moment transfer; prestressed concrete; punching shear; **shear stresses**; slabs; structural design; stud shear; two-way floors.

CONTENTS OF FULL REPORT

Chapter 1—Introduction
1.1—Objectives
1.2—Scope
1.3—Evolution of the practice

Chapter 2—Role of shear reinforcement

Chapter 3—Design procedure
3.1—Strength requirement
3.2—Calculation of factored shear stress v_u
3.3—Calculation of shear strength v_n
3.3.1—Shear strength without shear reinforcement
3.2.2—Shear strength with studs
3.4—Design procedure

Chapter 4—Prestressed slabs
4.1—Nominal shear strength

Chapter 5—Suggested higher allowable values for v_n, s_o, and s
5.1—Justification
5.2—Upper limit for v_n
5.3—Upper limits for s_o and s

Chapter 6—Design example

Chapter 7—References
7.1—Recommended reference
7.2—Cited references

Appendix A—Details of shear studs
A.1—Geometry of stud shear reinforcement
A.2—Stud arrangements
A.3—Stud length

Appendix B—Properties of critical sections

INTRODUCTION

In flat plate floors, slab-column connections are sub-

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jected to high shear stresses produced by the transfer of axial loads and bending moments between slab and columns. Section 11.12.3 of ACI 318 allows the use of shear reinforcement in the form of bars, as in the vertical legs of stirrups. The ACI 318R commentary emphasizes the importance of anchorage details of the shear reinforcement and accurate placement, especially in thin slabs. The general procedure for evaluation of the punching shear strength of slab-column connections is given in Section 11.12 of ACI 318.

Shear reinforcement consisting of vertical rods (studs), or the equivalent, mechanically anchored at each end can be used. In this report, all types of mechanically anchored shear reinforcement are referred to as "shear stud" or "stud." To be fully effective, the anchorage must be capable of developing yield strength of the rods.

Extensive tests have confirmed the effectiveness of mechanically anchored shear reinforcement (one example is shown in Fig. 1*) in increasing the shear strength and ductility of slab-column connections subjected to concentric punching or punching combined with moment.

ROLE OF SHEAR REINFORCEMENT

Shear reinforcement is required to intercept shear cracks and to prevent their widening. The intersection of shear reinforcement and cracks can be anywhere over the height of the shear reinforcement. The strain in the shear reinforcement is highest at that intersection.

Effective anchorage is essential and its location must be as close as possible to the structural member's outer surfaces [Fig. 1(b)]. This means that the vertical part of the shear reinforcement must be as long as possible to avoid the possibility of cracks passing above or below it (i.e., cracks not intercepted by shear reinforcement). Upper and lower limits of the cover b_t and b_b to the top and bottom anchor plates are specified in Fig. 1(b).

Anchorage of shear reinforcement in slabs is achieved by mechanical ends—bends and hooks. Tests have shown that movement occurs at the bends of shear reinforcement, before the yield strength can be reached in the shear reinforcement, causing a loss of tension. The amount of movement is the same for a short or long shear reinforcing bar. Therefore, the loss in tension is very important and the stress is unlikely to reach yield in short shear reinforcement (in thin slabs). These problems are eliminated if shear reinforcement is provided with mechanical anchorage.

DESIGN PROCEDURE

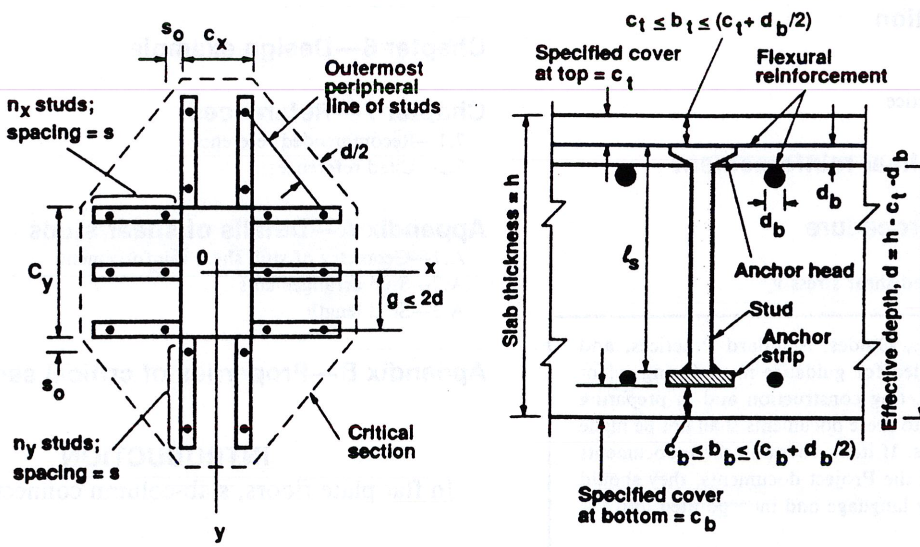
This chapter presents the design procedure for mechanically anchored shear reinforcement required in the slab in the vicinity of a column transferring moment and shear. The requirements of ACI 318 are satisfied and a stud is treated as the equivalent of one vertical leg of a stirrup.

Design of critical slab sections perpendicular to the plane of the slab should be based upon

$$v_n \leqslant \phi v_u \qquad (1)$$

in which v_u is the shear stress in the critical section caused by the transfer between the slab and the column of factored axial force or factored axial force combined with moment, v_n is the nominal shear strength, and ϕ is the strength reduction factor = 0.85.

*U.S. and Canada patents No. 4406103 and 1085642, respectively. Licensee: Deha represented by Decon, 105J Atsion Rd., P.O. Box 1575, Medford, N.J. 08055-6575, and 35 Devon Road, Brampton, Ontario L6T 5B6.



(a) Top View, Interior Column **(b) Section In Slab Perpendicular to a Shear Stud Line**

Fig. 1— Critical section outside the shear-reinforced zone and typical arrangement of stud strips

Eq. (1) should be satisfied at a critical section perpendicular to the plane of the slab at a distance $d/2$ from the column perimeter and at a critical section located so that its perimeter b_o is minimum, but need not approach closer than $d/2$ to the outermost peripheral line of shear studs [Fig. 1(a)].

Calculation of the maximum factored shear stress v_u at a critical section produced by the combination of factored shear force V_u and unbalanced moments M_{ux} and M_{uy} is given by Section 11.12.6.2 of ACI 318R.

For nonprestressed slabs, the shear strength of concrete v_n at a critical section at $d/2$ from column face where shear reinforcement is not provided is given by Section 11.12 of ACI 318.

Whenever the specified compressive strength of concrete f'_c is used in the following, its value must be in lb/in.².

At a critical section outside the shear-reinforced zone

$$v_n = 2\sqrt{f'_c} \qquad (2)$$

Eq. (1) should be checked first at a critical section at $d/2$ from the column face. If Eq. (1) is not satisfied, shear reinforcement is required.

The shear strength v_n at a critical section at $d/2$ from the column face should not be taken greater than $6\sqrt{f'_c}$ when stud shear reinforcement is provided. The shear strength at a critical section within the shear-reinforced zone should be computed by

$$v_n = v_c + v_s \qquad (3)$$

in which

$$v_c = 2\sqrt{f'_c} \qquad (4)$$

and

$$v_s = \frac{A_v f_{yv}}{b_o s} \qquad (5)$$

where A_v is the cross-sectional area of the shear studs on one peripheral line parallel to the perimeter of the column section and s is the spacing between peripheral lines of studs.

The distance s_o between the first peripheral line of shear studs and the column should not be smaller than $d/4$ [Fig. 1(a)]. The upper limits for s_o and for the spacing s between the peripheral lines should be

$$s_o \leqslant 0.5d \qquad (6)$$

$$s \leqslant 0.5d \qquad (7)$$

The upper limit of s_o is intended to eliminate the possibility of shear failure between the column face and the innermost peripheral line of shear studs. Similarly, the upper limit of s is to avoid failure between consecutive peripheral lines of studs.

The shear studs should extend away from the column face so that the shear stress v_u at a critical section at $d/2$ from the outermost peripheral line of shear studs [Fig. 1(a)] does not exceed ϕv_n, where v_n is calculated using Eq. (2).

SUGGESTED HIGHER ALLOWABLE VALUES FOR v_n, s_o, AND s

Justification

Section 11.5.3 of ACI 318 requires that "stirrups and other bars or wires used as shear reinforcement shall extend to a distance d from extreme compression fiber and shall be anchored at both ends according to Section 12.13 to develop the design yield strength of reinforcement." Test results using studs with anchor heads of area equal to 10 times the cross-sectional area of stem clearly satisfied that requirement. Further, use of the shear device shown in Fig. 1 demonstrated a higher shear capacity. Other researchers, as briefly mentioned in Appendix A of the full report, applied successfully other configurations. This justifies the following deviations from the ACI Building Code:

Upper limit for v_n—The nominal shear strength v_n resisted by concrete and steel in Eq. (3) can be taken as high as $8\sqrt{f'_c}$ instead of $6\sqrt{f'_c}$. This enables use of thinner slabs.

Upper limits for s_o *and* s—The upper limits for s_o and s can be based on the value of v_u at the critical section at $d/2$ from column face

$$s_o \leqslant 0.5d \text{ and } s \leqslant 0.75d \quad \text{when } \frac{v_u}{\phi} \leqslant 6\sqrt{f'_c}$$
$$s_o \leqslant 0.35d \text{ and } s \leqslant 0.5d \quad \text{when } \frac{v_u}{\phi} > 6\sqrt{f'_c} \qquad (8)$$

When stirrups are used, ACI 318 limits s to $d/2$. The higher limit for s given by Eq. (8) for stud spacing is again justified by tests. A vertical branch of a stirrup is less effective than a stud in controlling shear cracks for two reasons: (a) the stud stem is straight over its full length while the ends of the stirrup branch are curved, and (b) the anchor plates at the top and bottom of the stud insure that the yield strength is provided at all sections of the stem. In a stirrup, the full yield strength can be developed only over the middle portion of the vertical legs when they are sufficiently long.