

DEFLECTION OF CONCRETE FLOOR SYSTEMS FOR SERVICEABILITY¹

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Deflection control is a central consideration in serviceability of floor systems. This Technical Note reviews the range of acceptable deflections and the currently available methods for their estimate. The work concludes with a review of the merits and applicability of each method.

1 - OVERVIEW

The primary reasons for deflection control are:

- ❖ A concrete floor should possess adequate stiffness to mitigate damage to non-structural elements due to floor's deflection.
- ❖ The deflection of a floor should not be large enough to be noticeable by occupants, and convey a sense of inadequacy, safety concerns or discomfort.
- ❖ Deflection shall not be large enough to impair the function of a floor, such as adequate drainage or to avoid ponding of water.

2 - LIMITS FOR ACCEPTABLE DEFLECTION

2.1 Deflection Index (d/L)

With respect to allowable values, the deflection of a member is generally expressed in terms of deflection "d" to span "L" ratio, as illustrated in Fig. 2.1-1 for a simply supported member.

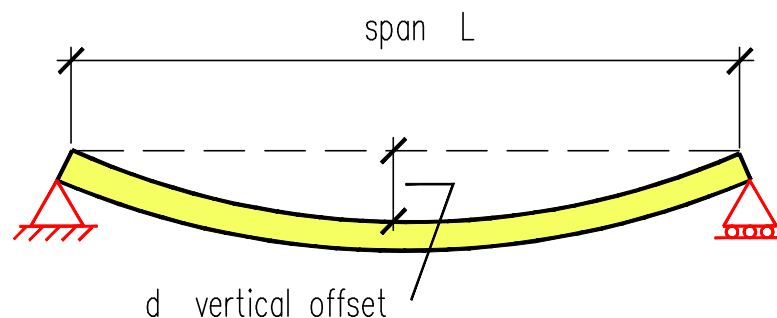


FIGURE 2.1-1 Deflection Index Expressed in Terms of d/L

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2.2 Aesthetics and Sense of Comfort

In considering aesthetics and sense of comfort for occupants, the important criterion is the out-of-level condition of a floor, as opposed to its stiffness. Sensitive individuals, when walking over or viewing a floor in elevation, are claimed to perceive a floor’s sag when the vertical out-of-level to span ratio is in excess of 1/250, and for cantilevers in excess of 1/125. The out-of-level condition of a floor system can be controlled through camber at time of construction.

2.3 Deflection Limits to Mitigate Damage to Non-structural Construction

Incremental displacement of a floor from the time a non-structural element likely to be damaged is installed shall not cause damage to the installed element.

The recommended limits in most major building codes are similar. ACI-318-11 recommends 1/240 and 1/480 for comfort and potential damage. EC2³ Recommends 1.250 and 1/500 for the same. Specifically, ACI-318-11’s recommendation along with its comments is reproduced in Table 1. ACI-318-11 permits the recommended values to be exceeded, if the for the specific condition the computed deflection is deemed to be acceptable.

TABLE 2.3-1 MAXIMUM PERMISSIBLE COMPUTED DEFLECTIONS (ACI-318-11 Table 9.5(b))

Type of member	Deflection to be considered	Deflection limitation
Flat roofs not supporting or attached to nonstructural elements likely to be damaged by large deflection	Immediate deflection due to live load	L/180 *
Floors not supporting or attached to nonstructural elements likely to be damaged by large deflection	Immediate deflection due to live load	L/360
Roof or floor construction supporting or attached to nonstructural elements likely to be damaged by large deflection	That part of the total deflection occurring after attachment of nonstructural elements(sum of the long-time deflection due to all sustained loads and the immediate deflection due to any additional live load)****	L/480 **
Roof or floor construction supporting or attached to nonstructural elements not likely to be damaged by large deflection		L/240 ***

Notes:

- * Limit not intended to safeguard against ponding. Ponding should be checked by suitable calculations of deflection, including added deflections due to ponding of water, and considering long-term effects of all sustained loads, camber, construction tolerances, and reliability of provisions for drainage.
- ** Limit may be exceeded if adequate measures are taken to prevent damage to supported or attached elements.

³ EC2 (EN 1992 -1-1:2004)

- *** But not greater than tolerance provided for nonstructural elements. Limit may be exceeded if camber is provided so that total deflection minus camber does not exceed limit.
- **** Long-time deflection shall be determined using established procedures, but may be reduced by amount of deflection calculated to occur before attachment of nonstructural elements. This amount shall be determined on basis of accepted engineering data relating to time-deflection characteristics of members similar to those being considered.

3 - DEFLECTION CONTROL THROUGH LIMITATION ON SPAN TO DEPTH RATIO

For common residential and commercial buildings, designers can forego deflection calculation, if the stiffness of the member selected is large enough. Selection of adequate member thickness can provide the necessary stiffness for deflection control. The minimum span to thickness ratio for different types of floor members are listed in the following:

3.1 One-Way Conventionally Reinforced Slabs and Beams

TABLE 3.1-1 MINIMUM THICKNESS OF CONVENTIONALLY REINFORCED BEAMS OR ONE-WAY SLABS⁴

Member	Simply supported	One end continuous	Both ends continuous	Cantilever
Solid one-way slabs	L/20	L/24	L/28	L/10
Beams or ribbed one-way slabs	L/16	L/18.5	L/21	L/8

Notes:

L =span length

Values given shall be used directly for members with normal weight concrete and Grade 60 ksi (400 MPa) reinforcement. For other conditions, the values shall be modified as follows:

- a) For lightweight concrete having equilibrium density, w_c , in the range of 90 to 115 lb/ft³ (1440-1840 kg/m³), the values shall be multiplied by $(1.65-0.005 w_c)$ but not less than 1.09 [in SI units, $(1.65-0.003 \gamma_c)$ but not less than 1, where γ_c is the density in kg/m³].
- b) For f_y other than 60,000 psi(400 MPa), the values shall be multiplied by $(0.4+f_y/100,000)$ [in SI units $(0.4+f_y/670)$].

⁴ ACI318-8, Table 9.5(a)

3.2 Two-Way Conventionally Reinforced Slabs and Beams

TABLE 3.2-1 MINIMUM THICKNESS OF SLABS WITHOUT INTERIOR BEAMS* ⁵

fy psi**	Without drop panels***			With drop panels***		
	Exterior panels		Interior panels	Exterior panels		Interior panels
	Without edge beams	With edge beams****		Without edge beams	With edge beams****	
40,000	Ln/33	Ln/36	Ln/36	Ln/36	Ln/40	Ln/40
60,000	Ln/30	Ln/33	Ln/33	Ln/33	Ln/36	Ln/36
75,000	Ln/28	Ln/31	Ln/31	Ln/31	Ln/34	Ln/34

Notes:

* For two-way construction, Ln is the length of clear span in the long direction, measured face-to-face of supports in slabs without beams and face-to-face of beams or other supports in other cases.

** For fy between the values given in the table, minimum thickness shall be determined by linear interpolation.

*** Drop panels are defined as extension of slab thickening into span not less than span/6, and extension of thickening below slab not less than slab thickness/4.

**** Slabs with beams between columns along exterior edges. The ratio of edge beam stiffness to the stiffness of the edge beam’s design strip shall not be less than 0.44

3.3 - Post-Tensioned Members

For post-tensioned beams and slabs, the recommended values by the Post-Tensioning Institute [PTI, 1990] are as follows:

TABLE 3.3-1 RECOMMENDED SPAN TO DEPTH RATIOS FOR POST-TENSIONED MEMBERS

	Continuous Spans		Simple Spans	
	Roof	Floor	Roof	Floor
One-way solid slabs	50	45	45	40
Two-way solid slabs (supported on columns only)	45-48	40-45		
Two-way waffle slabs (1m pans)	40	35	35	30
Beams	35	30	30	26
One-way joists	42	38	38	35

Note: The above ratios may be increased if calculations verify that deflection, camber, and vibrations are not objectionable.

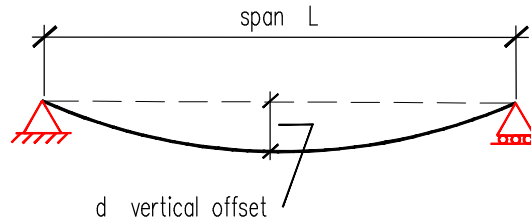
⁵ ACI-318-8, Table 9.5(c)

4 – MEASUREMENT OF DEFLECTION

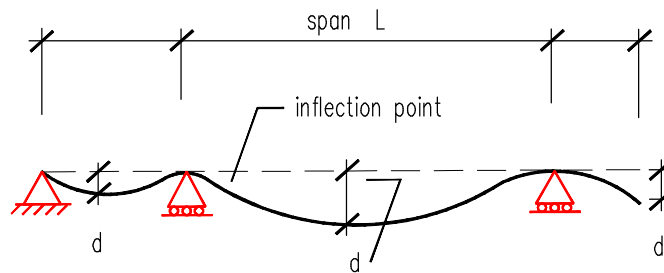
4.1 Definition of notional span (L) and deflection (d)

Evaluation of a floor’s deflection is based on a notional span (L) and the vertical offset (d) from a datum line. Figure 4.1-1 illustrates the traditional definition of (L) and (d).

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(a) Simply supported span

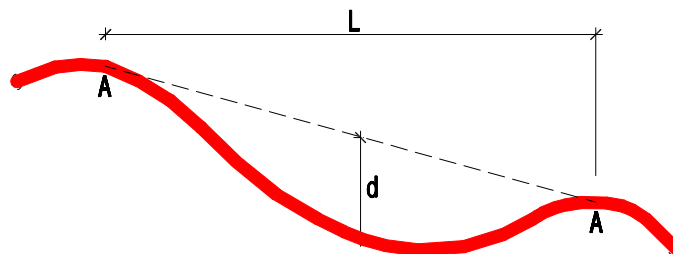


(b) Continuous member with cantilever

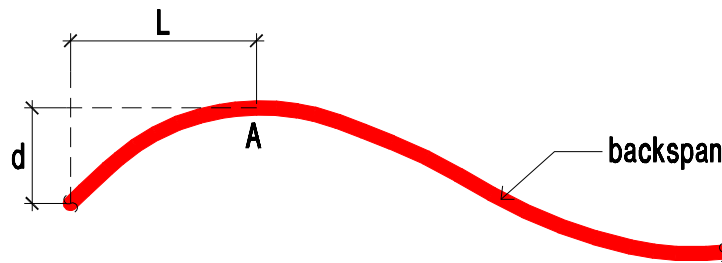
FIGURE 4.1-1 Traditional Parameters for Evaluation of Deflections

The traditional approach requires the recognition of “span” as is common in the structural engineering practice. The identification of “span,” however is not always a straight forward operation, when using automated designs, such as those based on the Finite Element Method. Moreover, the impact of a floor’s displacement on the elements that it supports is governed by the deflected contour of the floor surface, as opposed to distance between its supports below the floor. For this reason, for automated design a generalized definition as illustrated in Fig. 4-1.2 is used.

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(a) Elevation of an interior span



(b) Elevation of a cantilever span

FIGURE 4.1-2 Deflected Floor Slab Profiles

For the interior region of a floor system (Fig. 4.1-2a), for the purposes of deflection evaluation, the notional span is defined as distance between two adjacent crests (points of zero slope) on the floor. The points do not necessarily coincide with the location of supports below the floor slab. For slab edges, the notional span is defined as the distance from a point on the slab edge to a crest at interior of the floor (Fig. 4.1-2b).

4.2 – Numerical Example

Figure 4.2-1 shows the plan of a column supported floor with overhangs. It is required to evaluate the deflection of the overhang identified for code compliance.

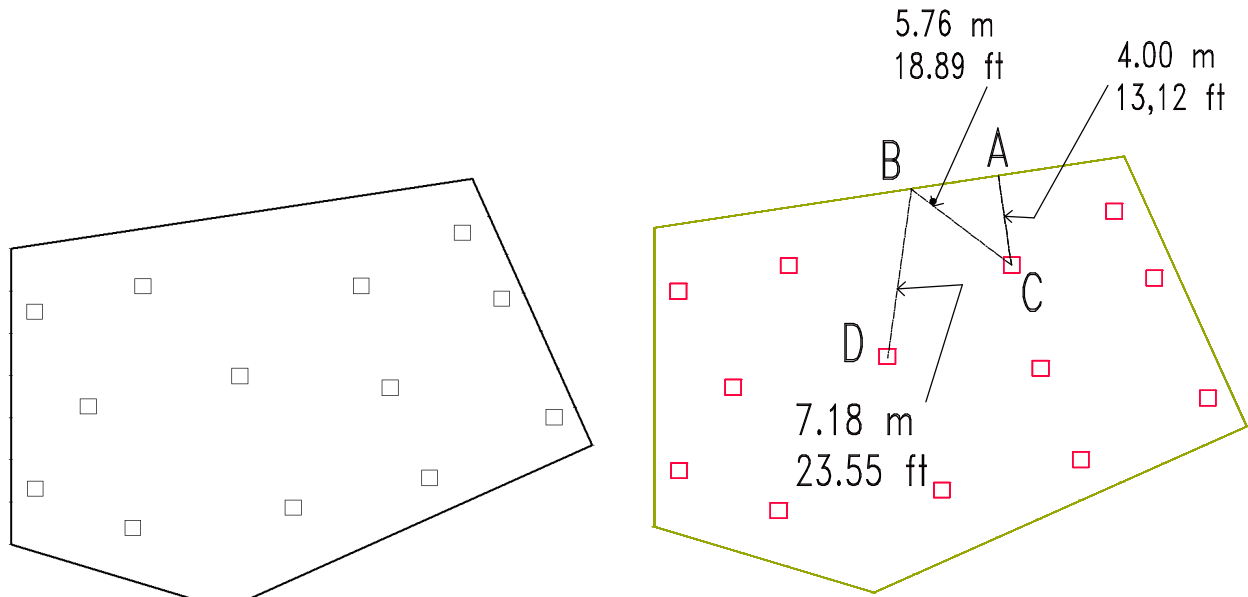
The deflected contour of the floor is shown in Fig. 4.2-2a. Maximum deflection is at point B near the center of the overhang. The deflection values for point A the shortest distance from the support to the tip of the cantilever and point B are:

Point A	$d = 12.05 \text{ mm (0.47 in)}$
Point B	$d = 15.5 \text{ mm (0.61 in)}$

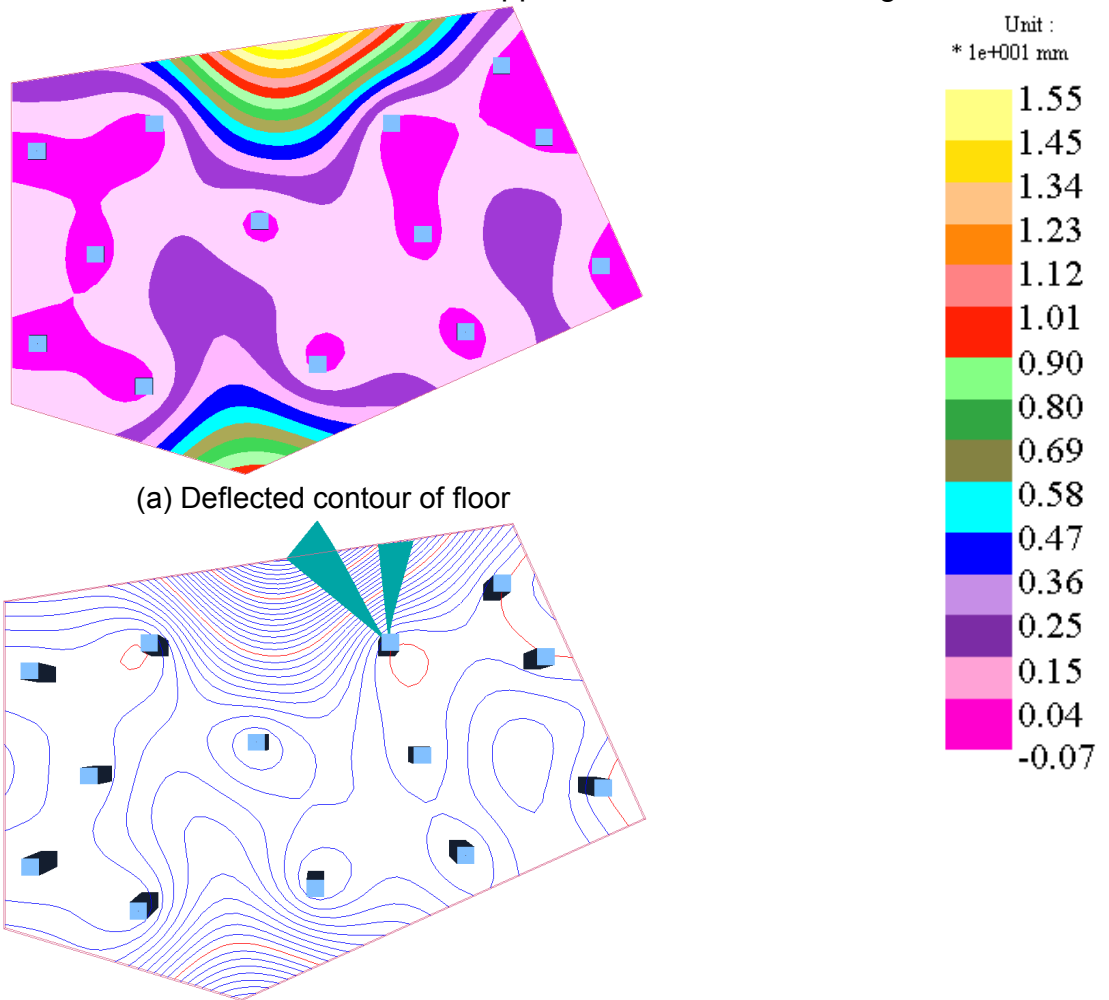
The deflection to span ratios are:

Point A	$d/L = 12.0/4,000 = 1/333$
Point B	$d/L = 15.5/5760 = 1/385$

Note that the point of maximum deflection (B) with a smaller deflection index has a lesser adverse impact than point (A) with smaller absolute deflection.



(a) Plan view
 (b) Distance of overhang points to supports
 FIGURE 4.2-1 Plan of a Column-Supported Slab with Overhangs



(a) Deflected contour of floor
 (b) Two deflected profiles from support C to overhang edge

FIGURE 4.2-2 Details of Floor Deflection

5 – CALCULATION OF INSTANTANEOUS DEFLECTION

Under otherwise unchanged conditions, the deformation of an exposed and loaded concrete member continues to increase, albeit at a reduced rate with time. The increase is due to creep under applied load, and shrinkage from loss of moisture and chemical reactions. While rigorous methods of computation for time-dependent long-term deflections are available⁶, the common engineering practice is to determine the instantaneous response of a structure under the applied load, and modify the instantaneous displacement accounting for the time-dependent factors of creep and shrinkage. The focus of this Technical Note is to follow the common engineering approach.

5.1 Calculation Methods for Instantaneous Deflection

Instantaneous deflection is mostly calculated using concrete's modulus of elasticity at 28 days, linear elastic theory, and in most cases the gross-cross sectional area. Detailed calculations can account for cracking of member and loss of stiffness. The common methods are;

1. Closed form formulas or tables, available primarily for uncracked sections;
2. Strip method, using (i) linear elastic response; and (ii) no cracking;
3. Strip method, using (i) linear elastic response, (ii) allowing for cracking, and (iii) an average equivalent moment of inertia (I_e) due to cracking;
4. Strip method, using (i) linear elastic response; (ii) allowance for cracking, and (iii) local equivalent moment of inertia (I_e) combined with numerical integration;
5. Modeling of (i) the entire floor system, using Finite Element Method (FEM) based program, (ii) linear elastic response; and (iii) no cracking; and
6. Modeling of the (i) entire floor system, using FEM; (ii) linear elastic response, and (iii) allowance for cracking.

Each of the above procedures is briefly discussed in the following;

5.2 Method 1 - Closed Form Formulas

Closed form formulas are readily available for beams and one-way slabs. The variables that describe the geometry of a two-way panel within a floor system, however, are so extensive that it becomes impractical to compile a meaningful set of tables or relationships without some approximation. For non-cracked sections, compilations such as the one listed in Table 5.2-1 are available in the literature [Bares, 1971].

In the application of tables, such as the one provided herein, engineering judgment is required in selection of support conditions. .

⁶ Computer program ADAPT-ABI; www.adaptsoft.com

TABLE 5.2-1 DEFLECTION COEFFICIENTS k FOR TWO-WAY SLABS

γ				
	1	2	3	4
1	0.0457	0.0143	0.0653	0.0491
1.1	0.0373	0.0116	0.0548	0.0446
1.2	0.0306	0.0094	0.0481	0.0422
1.3	0.0251	0.0075	0.0436	0.0403
1.4	0.0206	0.0061	0.0403	0.0387
1.5	0.0171	0.0049	0.0379	0.0369
2.0	0.0071	0.0018	0.0328	0.0326

Notes and Legend

Poisson's ratio assumed 0.25

$\gamma = a/b$ (aspect ratio)

Boundary conditions

1 = rigid supports; rotationally free;

2 = rigid supports; rotationally fixed;

3 = central panel from an array of identical panels supported on columns; deflection at center; and

4 = similar to case 3, but deflection at center of long span at support line

$$w = k (a^4 \cdot q / E \cdot h^3)$$

Where,

w = deflection normal to slab;

a = span along X-direction;

E = Modulus of elasticity; and

h = slab thickness.

EXAMPLE EX5.2-1

Consider the floor system shown in Fig. EX5.2-1-1. Estimate the deflection of the slab panel identified in part b of the figure for the following condition. Other particulars of the floor system are noted in Appendix A.

Given:

Span length along X-X direction = 30' (9.14 m)

Span length along Y-Y direction	= 26.25' (8.0 m)
Slab thickness	= 8 in (203 mm)
Ec (modulus of elasticity)	= 4.287 * 10 ⁶ psi (29,558 MPa)
Superimposed dead load	= 25 psf (1.2 kN/m ²)
Including allowance for partitions	
Live load	= 40 psf (1.9 kN/m ²)

Required

Immediate deflection of the panel at midspan for the following load combination

$$1*DL + 1*LL$$

$$\text{Aspect ratio } \gamma = 30/26.25 = 1.14$$

$$\text{Total service load } q = [(25 + 40) + 150*8/12]/144 = 1.146 \text{ lb/in}^2 (7.9*10^{-3} \text{ N/mm}^2)$$

Using closed form formulas (Table 5)

$$(a^4*q / E*h^3) = [(30*12)^4 * 1.146 / (4.287*10^6 * 8^3) = 8.77 \text{ in (222.76 mm)}$$

For mid-panel deflection, consider case 3 from Table 5

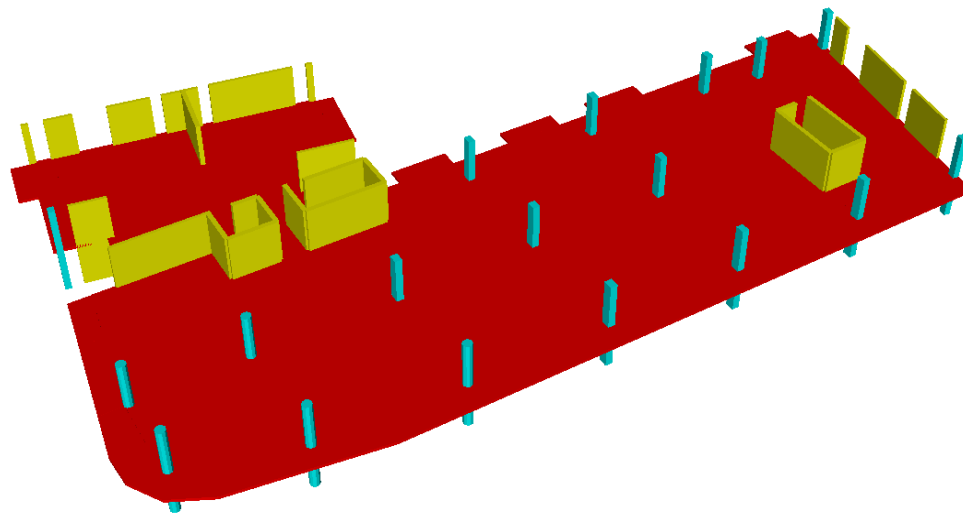
$$k = 0.0548$$

$$\text{Deflection, } \Delta = k (a^4*q / E*h^3) = 0.0548*8.77 = 0.48 \text{ in (12.20 mm)}$$

For deflection at midpoint of column lines in X-direction, from Table 5

$$k = 0.0446$$

$$\text{Deflection, } \Delta = k (a^4*q / E*h^3) = 0.0446*8.77 = 0.39 \text{ in (9.91 mm)}$$



(a) 3D View of typical floor system

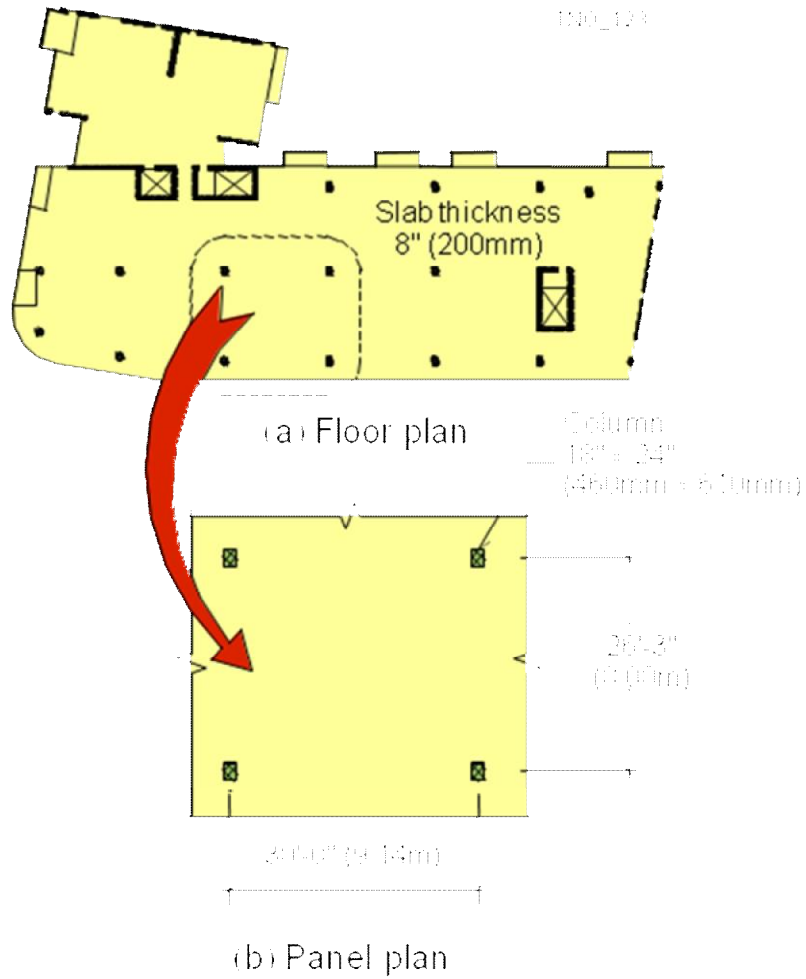


FIGURE EX5.2-1-1 Typical Floor Highlighting the Span under Consideration

5.3 Method 2 - Strip method, using (i) linear elastic response; and (ii) no cracking

This method has been widely in use, since the advent of personal computers., and the availability of software based on strip methods. The background to this method is detailed in reference [Aalami, 2005]. In this method the floor system is subdivided into strips along the line of columns, each covering in width the tributary of a line of column supports. Each strip is extracted and analyzed in isolation . Figure EX5.3-1 illustrates the strips that are applicable for the panel under consideration.

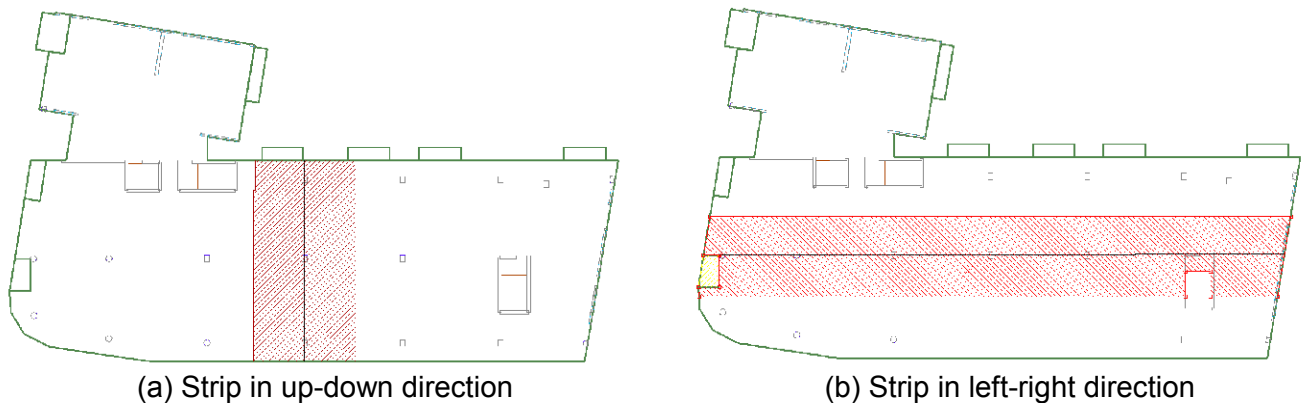


FIGURE 5.3-1 Subdivision of the Floor into Design Strips

When calculating the deflections from the strips in orthogonal directions, the deflection at center of panel (d) is taken to be the sum of the deflections obtained from each of the orthogonal strips (d1, and d2) (Fig. 5.3-2a. When using the displacement at center of panel (d), the evaluation of deflection with respect to its code compliance is related to the notional span (L) being the diagonal length of the panel (Fig. 5.3-2b).

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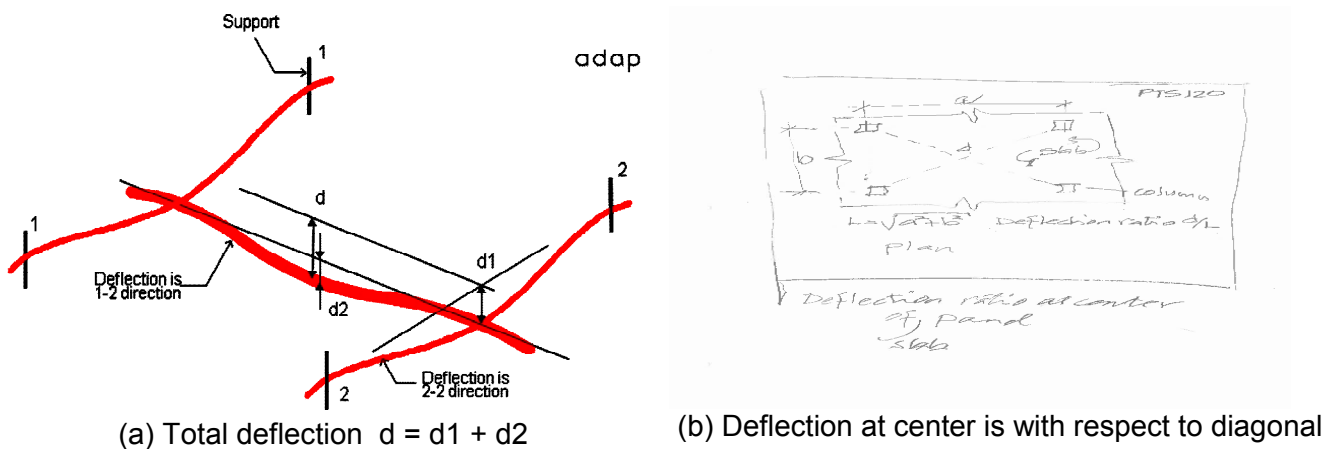


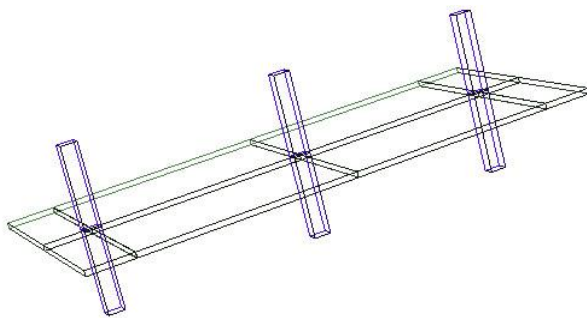
FIGURE 5.3-2 Deflections Determined from Sum of Values of Orthogonal Strips

Using the parameters described in the preceding and Appendix A, along with a computer

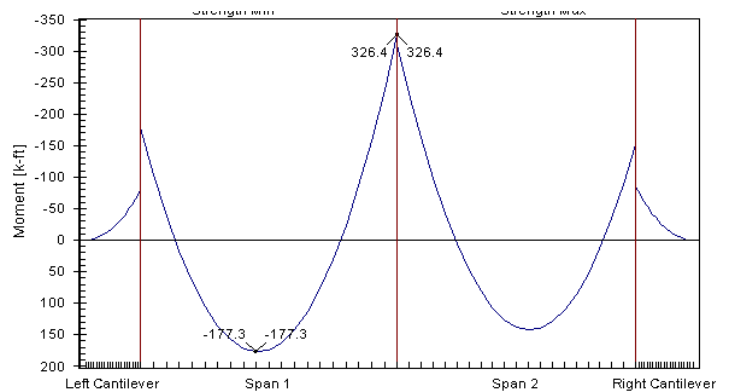
program⁷ the strip in up-down direction was modeled (Fig. 5.3-3a) and its deflection profile determined (part b of the same figure), using elastic linear theory with no allowance for cracking. The maximum deflection for the span under consideration is 0.231 in. (5.9 mm).

Strictly speaking, a similar analysis has to be performed for the strip in the orthogonal direction to determine the associated deflection. However, for expediency, and recognizing that the panel is almost square, the deflection at the panel center is estimated to be twice the value calculated from one of the strips.

Deflection at center of panel = $0.231 * 2 = 0.462$ in. (11.7 mm)



(a) View of design strip in up-down direction



(b) deflection profile of the design strip

FIGURE 5.3-3 Design Strip in Up-down Direction and its Deflected Profile

Change the above with deflection diagram

5.4 METHOD 3 Strip method, (i) linear elastic response,(ii) allowing for cracking, and (iii) an average equivalent moment of inertia (Ie);

This method is also referred to as simplified ACI-318 method based on equivalent moment of inertia (Ie). We start with a brief review of underlying principles, and conclude with a numerical example for its application.

5.4.1 Background

Where the applied moment (Ma) at a section along a member exceeds the cracking moment (Mcr) of the same section, a crack forms. The consequence of crack is a reduction in the second moment of area of the section at the location of crack. Fig. 5.4.1-1 illustrates the formation of crack in a member and the associated reduction in its second moment of area, referred to in common practice as “moment of inertia I.” The reduced moment of inertia is called “equivalent moment of inertia, Ie.”

⁷ ADAPT-RC

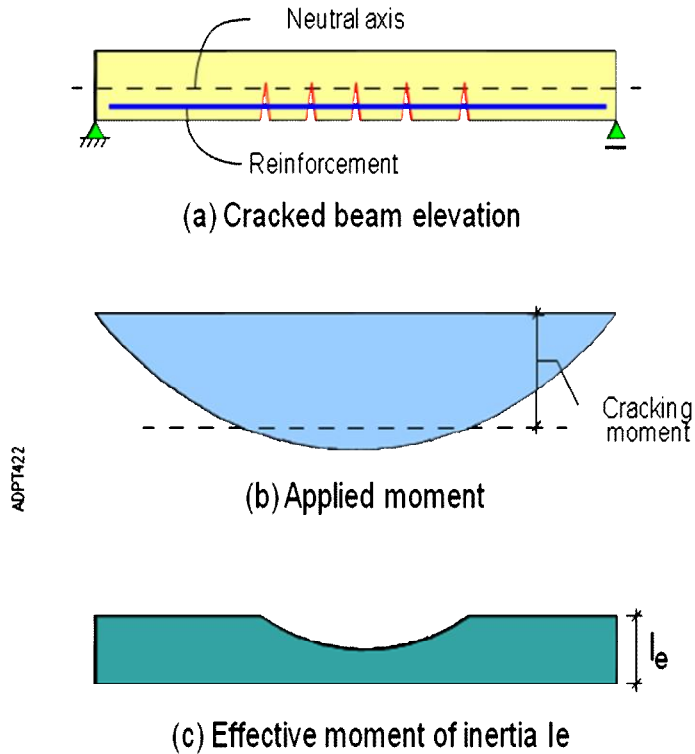


FIGURE 5.4.1-1 Illustration of Effective Moment of Inertia in Partially Cracked Slab

In this method, in computation of a member’s deflection, the original moment of inertia, referred to as “gross moment of inertia I_g ” is substituted by a reduced value I_e to account for the loss of local stiffness. . Among the available expressions put forward to estimate the reduced value of moment of inertia due to cracking, ACI-318-11 recommends the following:

$$I_e = (M_{cr} / M_a)^3 * I_g + [1 - (M_{cr} / M_a)^3] * I_{cr} \leq I_g \tag{5.4.1-1}$$

Where,

- I_g = Gross moment of inertia;
- I_{cr} = Moment of inertia of cracked section;
- I_e = Effective moment of inertia;
- M_a = Maximum moment in member at stage deflection is computed; and,
- M_{cr} = Cracking moment.

The applied moment, M_a , is calculated using elastic theory and the gross moment of inertia (I_g) for the uncracked section. The change in distribution of moment in indeterminate structures resulting from cracking in concrete is generally small, and is already accounted for in the empirical formula (1) for equivalent moment of inertia. The cracking moment is given by:

$$M_{cr} = f_r I_g / y_t \tag{5.4.1-2}$$

Where,

f_r = Modulus of rupture, flexural stress causing cracking. It is given by:

$$f_r = 7.5 f'_c{}^{1/2} \quad (\text{lb, in, units}) \quad (5.4.1-3)$$

$$f_r = 0.625 f'_c{}^{1/2} \quad (\text{N, mm}) \quad (\text{lb, in, units}) \quad (5.4.1-3 \text{ SI})$$

Using the European Code EC2⁸ the cracking stress is given by:

$$f_r = f_{ctm} = 0.3 * f_c{}^{2/3} \quad (\text{N, mm}) \quad (5.4.1-3 \text{ EC2})$$

y_t = distance of section centroid to farthest tension fiber

For all-lightweight concrete, f_r is modified as follows:

$$f_r = 0.75 * 7.5 f'_c{}^{1/2} \quad (5.4.1-4)$$

The value of cracking moment of inertial I_{cr} and the geometry of the section depend on the location and amount of reinforcement. For rectangular sections with single reinforcement layer (Fig. 5.4.1-2) the value is given by:

$$I_{cr} = (bk^3d^3)/3 + nA_s(d-kd)^2 \quad (5.4.1-5)$$

Where,

$$kd = [(2dB+1)^{1/2} - 1]/B \quad (5.4.1-6)$$

d = distance from compression fiber to center of tension reinforcement

$$B = b/(nA_s)$$

$$n = E_s/E_c$$

E_s = modulus of elasticity of steel

E_c = modulus of elasticity of concrete

For more details and treatment of other cross-sections refer to reference [ADAPT Technical Note TN293, 2008].

⁸ EC2 (EN 1992 – 1-1:2004)

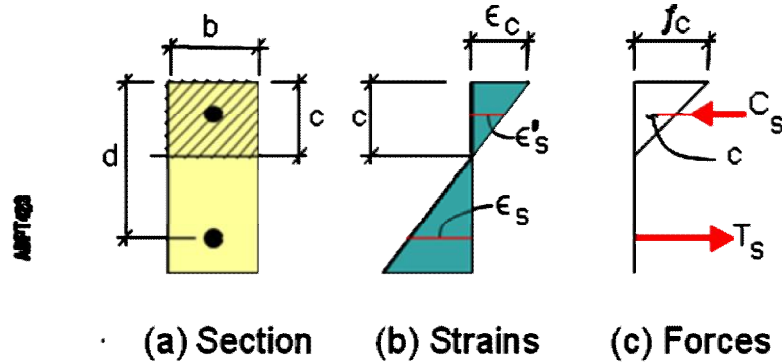


FIGURE 5.4.1-2 Post-Cracking Stress and Force Distribution

In the simplified method an average value of l_e is used for the entire span. For each span, the average value is calculated

$$l_{e, av} = 0.5 [(l_{e, \text{left support}} + l_{e, \text{right support}}) / 2 + l_{e, \text{midspan}}] \quad (5.3.1-7)$$

For cantilevers, the equivalent moment of inertia at the support is used for the entire span

EXAMPLE 5.4.1-1

Consider the floor system shown in Fig. EX5.2-1-1. Estimate the deflection of the slab panel identified under the same loading and conditions expressed in Example 5.2-1, using the averaging option of ACI-318 for equivalent moment of inertia l_e

Given:

- Span length along X-X direction = 30' (9.14 m)
- Span length along Y-Y direction = 26.25' (8.0 m)
- Slab thickness = 8 in (203 mm)
- E_c (modulus of elasticity) = $4.287 \cdot 10^6$ psi (29558 MPa)
- Other details of the slab are given in Example 1 and the Appendix A.

Required

Determine the deflection at the center of the panel identified in Example EX5.2-1 due to the sum of dead and live loads.

Consider the strip in up-down (Y-Y) direction with tributary 30' (9.14m). Calculate Cracking Moment M_{cr}

$$I_g = 15,360 \text{ in}^4 (6.40e+10^9 \text{ mm}^4)$$

$$y_t = 4 \text{ " (101.60 mm)}$$

$$f_r = 7.5 \sqrt{f'_c} = 7.5 \cdot \sqrt{5000} = 530.33 \text{ psi (3.66 MPa)}$$

$$M_{cr} = f_r I_g / y_t = 530.33 \cdot 15,360 / (4 \cdot 12000) = 169.70 \text{ k-ft (230 kNm)}$$

To determine the equivalent moment of inertia, the applied moment (M_a) for the “design strip” associated with the panel in question must be determined. Refer to Fig. 5.3-1, and use the relationship 5.3.1-1, reproduced below for ease of reference.

$$I_e = (M_{cr} / M_a)^3 * I_g + [1 - (M_{cr} / M_a)^3] * I_{cr} \leq I_g$$

The design strip associated with the panel under consideration is shown in Fig 5.3-1a. The solution obtained for distribution of moments is shown in Fig. EX5.4.1-1:

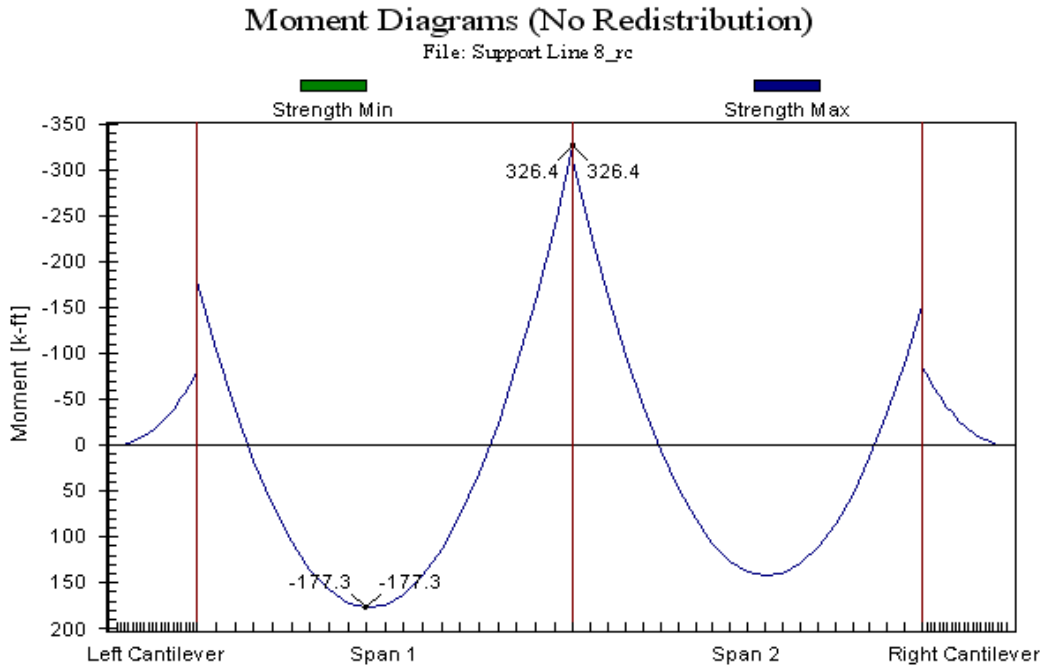


FIGURE EX5.4.1-1-1 Distribution of Moments Due to Dead plus Live Load

Consider the left face of support at span 1. From Fig. EX5.4.1-1-1

$$M_a = 326.4 \text{ k-ft (442.53 kNm)}$$

$$I_g = 15,360 \text{ in}^4 \text{ (6.40 e+9mm}^4\text{)}$$

$$M_{cr} = f_r I_g / y_t = 530.33 * 15,360 / (4 * 12000) = 169.70 \text{ k-ft (230 kNm)}$$

Where,

$$I_{cr} = (bk^3 d^3) / 3 + nA_s (d - kd)^2$$

$$kd = [(2dB + 1)^{1/2} - 1] / B$$

$$d = 6.81 \text{ in (173 mm)}$$

$$B = b/(nA_s)$$

$$n = E_s/E_c = 30000/4287 = 7.0$$

$$A_s = 10.12 \text{ in}^2 \text{ (6529 mm}^2\text{) (from design provided by computer program}^9\text{)}$$

$$B = 360/(7.0 * 10.12) = 5.08 \text{ /in}$$

$$k_d = [(2 * 6.81 * 5.08 + 1)^{1/2} - 1] / 5.08$$

$$= 1.45 \text{ in (36.83 mm)}$$

$$I_{cr} = (360 * 1.45^3) / 3 + 7.0 * 10.12 * (6.81 - 1.45)^2$$

$$= 2401 \text{ in}^4 \text{ (9.99e+8)}$$

$$I_e = (188.04 / 326.4)^3 * 17019 + [1 - (188.04 / 326.4)^3] * 2401$$

$$= 5196 \text{ in}^4 \text{ (2.16e+9)} = 0.31 I_g$$

Using the same procedure, the value of I_e at other locations required by are calculated and listed below:

Left cantilever:

$$I_e \text{ at face of support} = I_g = 1.536e+4 \text{ in}^4 \text{ (6.40e+9 mm}^4\text{)}$$

First Span:

$$I_e \text{ at left support centerline} = 1.70e+4 \text{ in}^4 \text{ (7.08e+9 mm}^4\text{)}$$

$$I_e \text{ at midspan} = 1.44e+4 \text{ in}^4 \text{ (5.99e+9 mm}^4\text{)}$$

$$I_e \text{ at right support centerline} = 5.20e+3 \text{ in}^4 \text{ (2.16e+9 mm}^4\text{)}$$

Second Span:

$$I_e \text{ at left support centerline} = 5.63e+3 \text{ in}^4 \text{ (2.34e+9 mm}^4\text{)}$$

$$I_e \text{ at midspan} = 1.536e+4 \text{ in}^4 \text{ (6.40e+9 mm}^4\text{)}$$

$$I_e \text{ at right support centerline} = 1.702e+4 \text{ in}^4 \text{ (7.08e+9 mm}^4\text{)}$$

Right cantilever:

$$I_e \text{ at face of support} = I_g = 1.536e+4 \text{ in}^4 \text{ (6.40e+9 mm}^4\text{)}$$

Using the averaging procedure suggested by ACI-318, the I_e values to be used for deflection calculation are:

Left and right cantilevers $I_e = I_g$

First span

$$\text{Average } I_e = [(1.70 * 10^4 + 5.20 * 10^3) / 2 + 1.44 * 10^4] / 2$$

$$= 12.755e+3 \text{ in}^4 \text{ (5.31e+9 mm}^4\text{)}$$

Second span

$$\text{Average } I_e = [(5.63 * 10^3 + 1.702 * 10^4) / 2 + 1.536 * 10^4] / 2$$

$$= 13.343e+3 \text{ in}^4 \text{ (5.55e+9 mm}^4\text{)}$$

⁹ ADAPT-RC

In order to use the same frame program for the calculation of deflected shape, the calculated equivalent moments of inertia are used to determine an equivalent thickness (h_e) for each of the spans. The equivalent thickness is determined as follows:

$$I_e = b \cdot h_e^3 / 12$$

Where, b is the width of the tributary of the design strip.

Left cantilever:	$h_e = 8$ in (203 mm)
First span	$h_e = 7.52$ in (191 mm)
Second span	$h_e = 7.63$ in (194 mm)
Right cantilever	$h_e = 8$ in (203 mm)

Using the same computer program, material values, boundary conditions and loads, but with the reduced slab thickness based on the modified moment of inertia a new solution is obtained.

The maximum value of deflection for span 1 is 0.264 in (6.71 mm), compared to 0.231 in (5.87 mm), without allowing for reduction of stiffness due to cracking. Note that the above deflections do not account for the flexure of the slab in the orthogonal direction, as indicated in Example EX5.2-1. For engineering design, where panels are fairly square, the calculated values are commonly multiplied by 2 to represent the deflection at the middle of panel. Hence, mid-panel deflections would be $2 \times 0.264 = 0.528$ in. (13.42 mm).

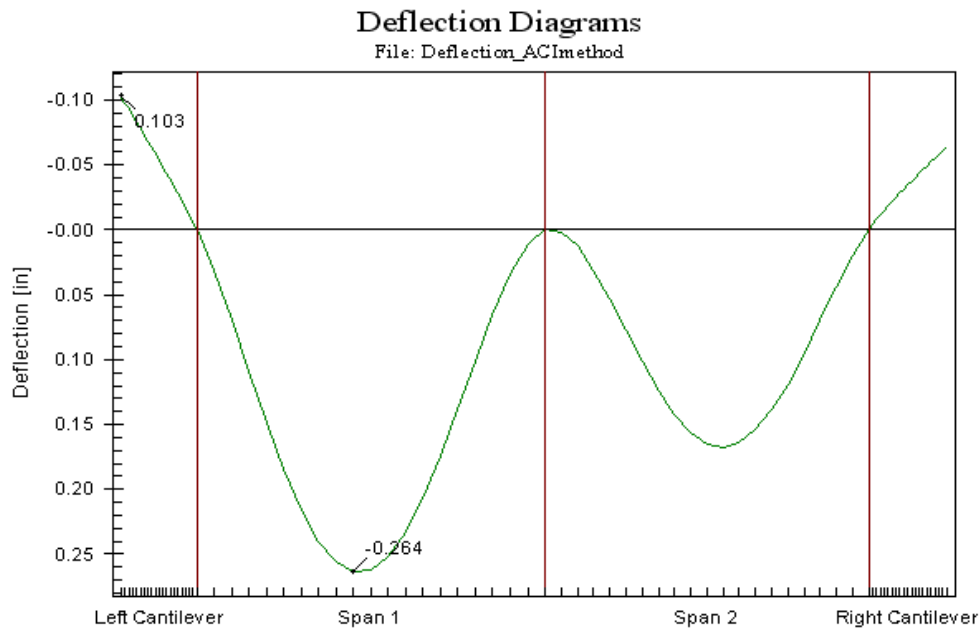


FIGURE EX5.4.1-1-2 Deflection Shape with Allowance for Cracking, Using Simplified Method

5.5 METHOD 4 : Strip method, using (i) linear elastic response; (ii) allowance for cracking, and (iii) local equivalent moment of inertia (I_e) combined with numerical integration;

The next step in increased accuracy of deflection calculation is (i) the use of equivalent moment of inertia I_e , (ii) the strip method as outlined in the preceding example, and (iii) numerical integration. In this scheme each span will be subdivided in a number of segments, typically 10 to 20 divisions. The equivalent moment of inertia for each division will be calculated separately, and a solution is obtained with recognition of a variable moment of inertia along the length of each span. This procedure along with a detailed numerical example is described in detail in reference [ADAPT TN294, 2008].

Figure 5.5-1 is an example showing the variation of moment along the first span of a two-span member, subdivision of the span into smaller segments, and the equivalent moment of inertia for each segment due to cracking.

Using a computer program¹⁰ this procedure is employed to determine the deflection of the design strip shown in Fig. 5.3-1a, with due consideration for cracking. The calculated deflection by the program is 0.235 in (5.97 mm), compared to 0.264 in (6.71 mm) where the simplified averaging of effective moment of inertia was used in calculation of cracked deflection.

From this method, the total deflection is estimated as:

$$\text{Total deflection} = 2 * 0.235 = 0.470 \text{ in. (12.0 mm)}$$

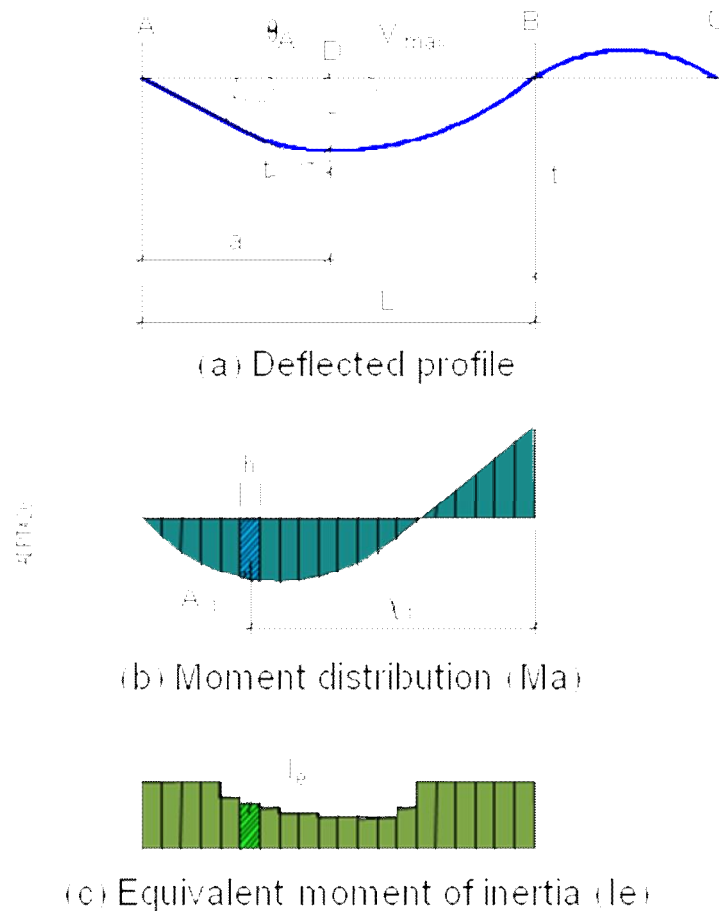


FIGURE 5.5-1 Variable Moment of Inertia along a Cracked Member

¹⁰ ADAPT-RC

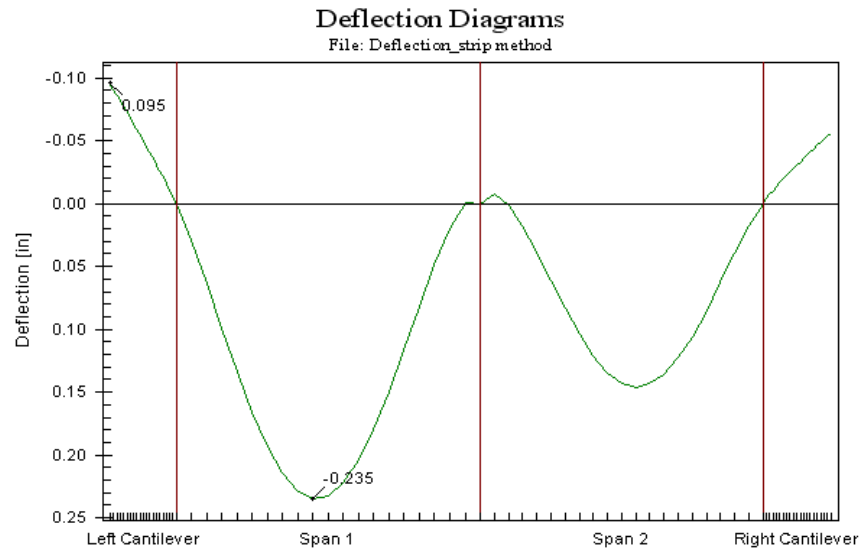


FIGURE 5.5-2 Deflected Profile of the Design Strip with Allowance for Cracking and Numerical Integration

5.6 METHOD 5 Modeling (i) the entire floor system, using Finite Element Method (FEM) , (ii) linear elastic response; and (iii) no cracking; and

Using the Finite Element Method (FEM), the salient features of the geometry and loading that are generally approximated in the aforementioned methods through idealizations are can be faithfully modeled. This leads to a more representative estimate of slab deflection. Figure 5.6-1 shows the discretization of the floor system used in the previous examples into finite element cells.

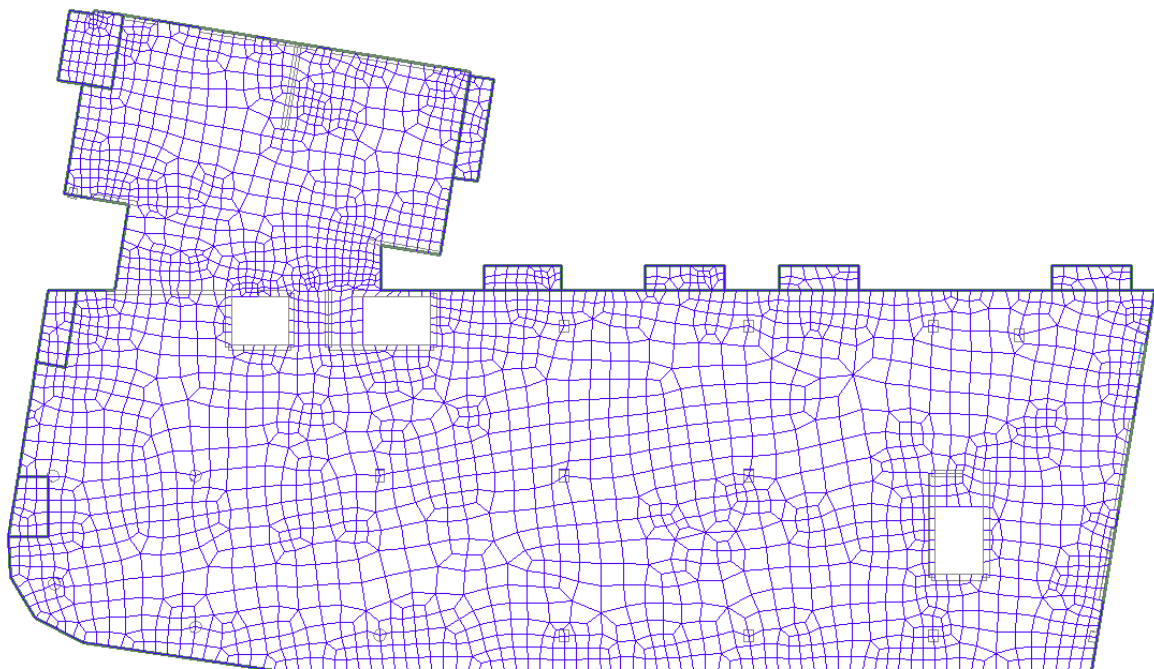


FIGURE 5.6-1 Discretization of the Typical Floor Slab for Finite Element Analysis (Floor Pro)

EXAMPLE 5.6-1

For the same geometry and parameters of examples EX5.2-1, using a finite element procedure determine the deflection at center of the panel identified in Fig. EX5.2-1-1.

ADAPT-FLOOR Pro¹¹ program was used to model the slab and obtain a solution. The distribution of deflection for the given load is shown in Fig. EX5.6-1-1. The maximum deflection at the center of the panel under consideration is reported as 0.54 in.

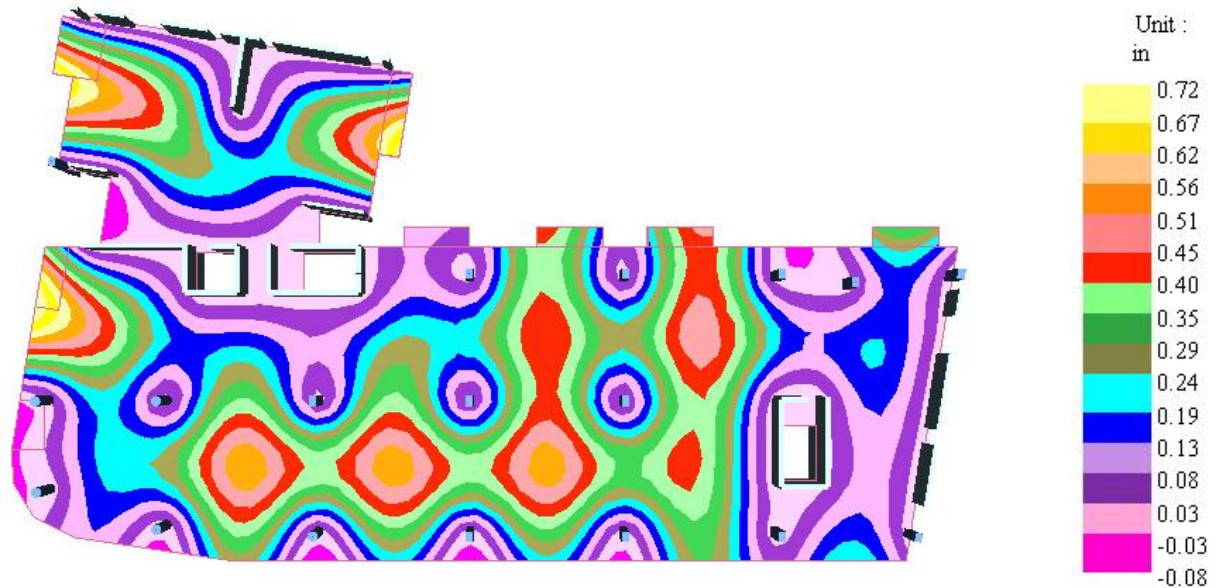


FIGURE EX5.6-1-1 Deflection Contour of the Floor System under Combined Actions of Dead and Live Loads

5.7 METHOD 6: Modeling the (i) entire floor system, using FEM; (ii) linear elastic response, and (iii) allowance for cracking.

Formulation of finite elements with allowance for cracking is somewhat complex. The complexity arises from the fact that cracking and reduction in stiffness depend on the presence, amount and orientation of reinforcement, including prestressing tendons, if any. Before a solution is obtained, the reinforcement detailing of a floor system must be fully known, since the loss of stiffness in each finite element cell depends on the availability and exact location of the reinforcement in that cell.

The following briefly describes the steps for a finite element deflection calculation, with allowance for cracking, featuring a commercially available software¹².

1. Using the geometry, boundary conditions, material properties, and the load combination for which the deflection is sought, the program discretizes the structure, sets up the system stiffness matrix of the structure based on gross moment of inertia (I_g), and obtains the distribution of moments (M_a) over the entire structure.

¹¹ ADAPT-Floor Pro is a finite element program for analysis and design of conventionally reinforced or post-tensioned floor systems. www.adaptsoft.com

¹² ADAPT-Floor Pro

2. Using the applicable building code, and the design values from computation, at each location of the floor the reinforcement necessary is determined and added. The added reinforcement is the amount in excess of what a user may have defined as base reinforcement. Thus, at any location in the floor system, the available reinforcement to be used for crack evaluation can consist of:

- a - User defined top and bottom reinforcement mesh;
- b - User defined grouped or distributed reinforcement bars at top and bottom of slab and beams;
- c - Reinforcement calculated and reported by the program for minimum requirements of the code, strength check, initial condition, or other code related criteria; and
- d - Post-tensioning tendons defined by the user, each with its own location and force.

3. Next, the algorithm matches the calculated moment (M_a) of each finite element cell with the Cracking moment (M_{cr}) of the same cell. Where the applied moment (M_a) exceeds the cracking moment (M_{cr}), the parameters along with the steps below are followed to determine the reduction in moment of inertia at the affected location.:

- a. Finite element cell thickness (to obtain uncracked second moment of area);
- b. Available reinforcement associated with each cell (nonprestressed and prestressed), with recognition of orientation and height of each individual reinforcement;
- c. Cracking moment of inertia associated with each cell in each direction (I_{cr});
- d. Cracking moment associated with each cell (M_{cr}); and
- e. Applied moment (M_a).

4. Having determined the effective second moment of area of each finite element cell in each of the principal directions, the stiffness matrix of each cell is re-constructed..

6. The system stiffness matrix is re-assembled for a new deflection calculation. The solution obtained at this stage is a conservative estimate for the floor deflection with cracking. For practical engineering design, it is recommended to terminate the computations at this point.

To be more rigorous, the newly calculated moments and axial forces can be used to re-determine the cracked locations, update the loss in stiffness and re-calculate deflections. Continue the iterations, until the solutions converge to within a pre-defined tolerance.

EXAMPLE 5.7-.1

Using finite elements, determine the deflection of the panel identified in Fig. EX5.2-1 for the loads and conditions described in same example. Calculate the deflection for the combination of dead and live loads. Use ACI318-11 to determine the reinforcement necessary for both the in-service and strength requirements of the code. Use the calculated reinforcement to determine the cracked deflection.

Using the above procedure, the cracked deflection of the floor is calculated and illustrated in. EX5.7-1-1. The deflection at the center of the panel under consideration is 0.70 in. (17.8 mm) compared to 0.54 in. (13.7 mm) for the uncracked slab. The cracked deflection can be reduced by adding reinforcement at the locations of crack formation, in addition to the minimum requirements of the code already included in the analysis.

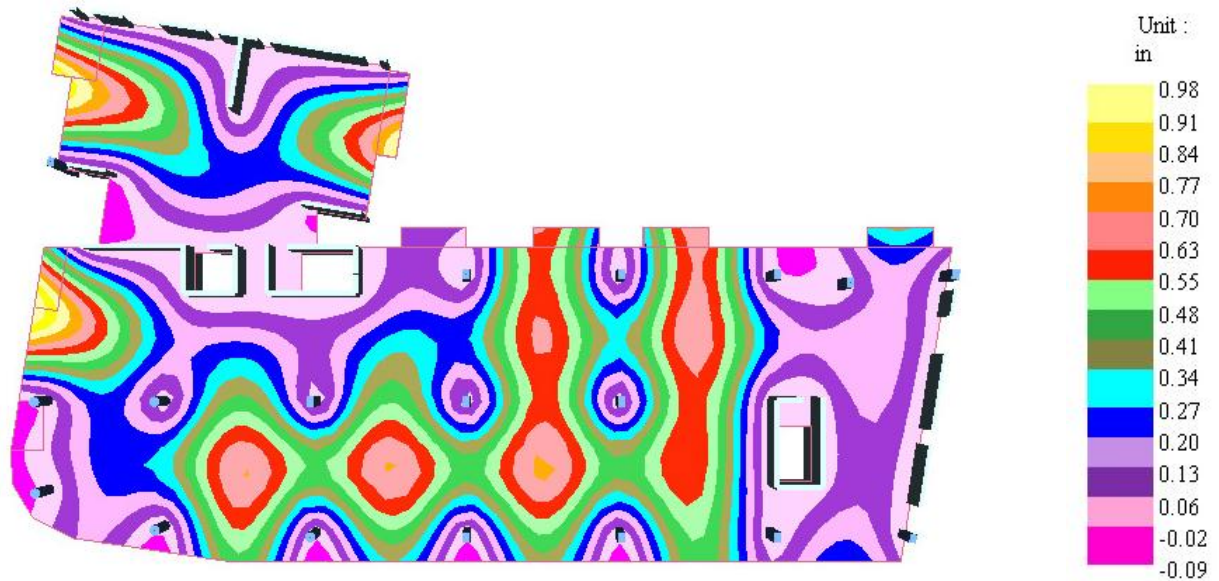


FIGURE EX5.7-1-1 Deflection Contour of Slab with Cracking

The locations of crack formation and the extent of cracking are illustrated in Figs. EX5.7-1-2 and EX5.7-1-3. At each location, the reduction in effective moment of inertia is based on the calculated moment at that location and the amount, position and orientation of reinforcement at the same location. The largest loss of stiffness occurs around the columns and the support lines joining the columns. The maximum loss of stiffness is 69% reducing the effective moment of inertia to 31% of its uncracked value.

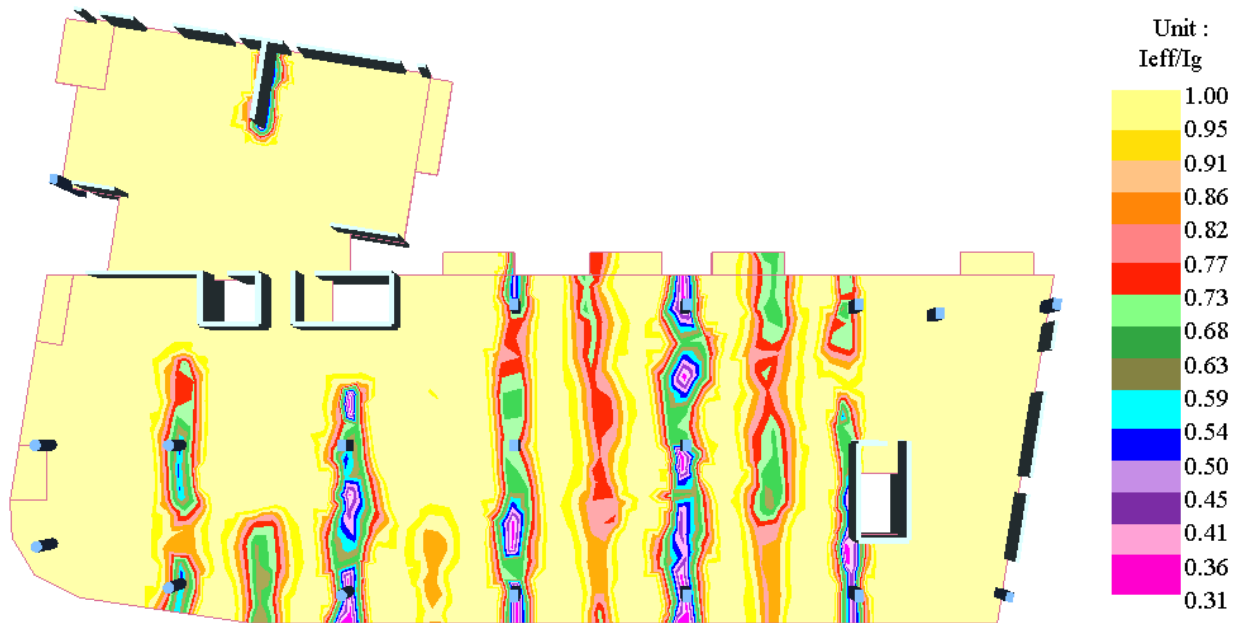


FIGURE EX5.7-1-2 Extent of Cracking Shown Through Reduction in Effective Moment of Inertia I_e About Y-Y Axis

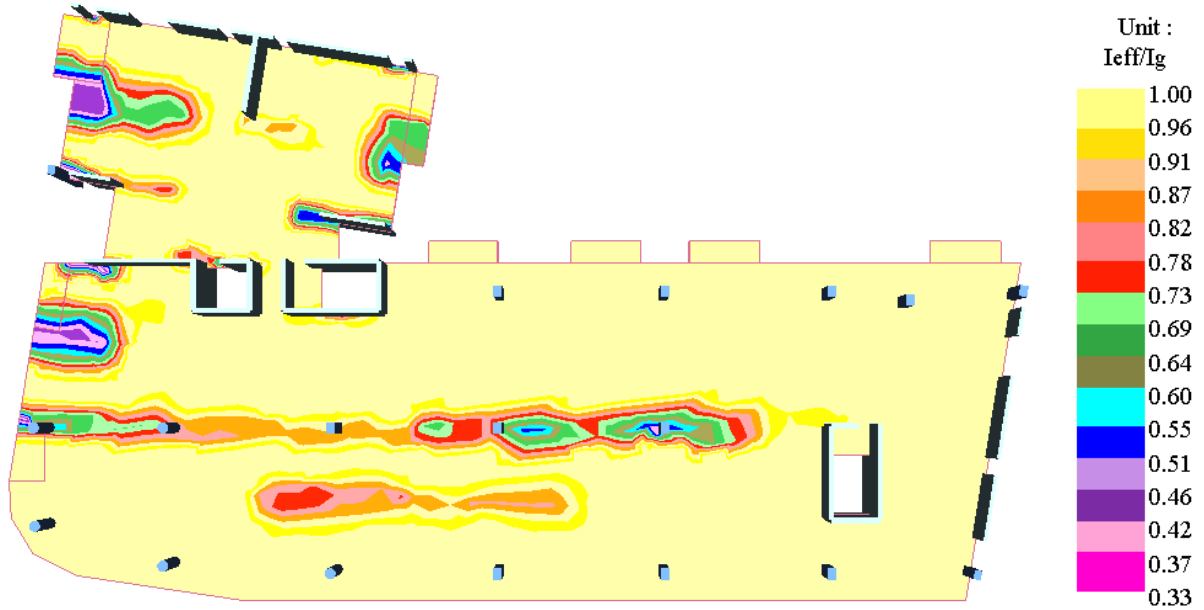


FIGURE EX5.7-1-3 Extent of Cracking Shown Through Reduction in Effective Moment of Inertia I_e About X-X Axis

6 - COMPARISON OF METHODS OF DEFLECTION CALCULATION

Table 6-1 lists the outcome of the various methods commonly in use for estimating the deflection of conventionally reinforced floor systems. Note that for the typical floor system selected, the difference between the various methods can be as much as 30%. Finite Element Method with due allowance for crack formation gives the largest deflection. The strip method with no allowance for cracking produces the smallest value.

TABLE 6-1 DEFLECTION VALUES AT CENTER OF PANEL OF THE NUMERICAL EXAMPLE

	Calculation Method	Deflection in(mm)	Normalized Deflection
1	Closed form formulas	0.480 (12.2)	69 %
2	Strip method (uncracked) ACI 318	0.462 (11.7)	66 %
3	Strip method (cracked) ACI 318 – averaging I_e	0.528 (13.4)	75 %
4	Strip method (cracked) ACI 318 – numerical integration	0.470 (11.9)	67 %
5	Finite Element Method (FEM) No allowance for cracking	0.540 (13.7)	77 %
6	Finite Element Method (FEM) With allowance for cracking	0.700 (17.8)	100 %

7 – DEFLECTION OF POST-TENSIONED FLOORS

Two-way post-tensioned floor systems designed to ACI-318 provisions and the PTI-recommended slab-to-depth ratios (Table 3.3-1), either do not crack under service condition, or crack to an extent that does not invalidate calculations based on gross cross-sectional geometry and linear elastic theory. This is because, unlike other major non-US codes, the allowable tensile stresses in ACI-318 are relatively low.

The preceding observation does not hold true for post-tensioned one-way slab and beams, where ACI-318 permits designs based on post-cracking regime. For such post-tensioned floor systems, designers must include allowance for cracking in their designs.

8 – LIVE LOAD DEFLECTION

For members that are not cracked, using the commonly predominant assumption of linear elastic response, the principle of superposition applies. Hence, deflection to live load can simply be obtained by prorating deflections from similar loads, such as selfweight. However, as indicated in Fig. 8-1, for cracked members, the increment of deflection due to live load depends on the existing loads on the member and its extent of cracking.

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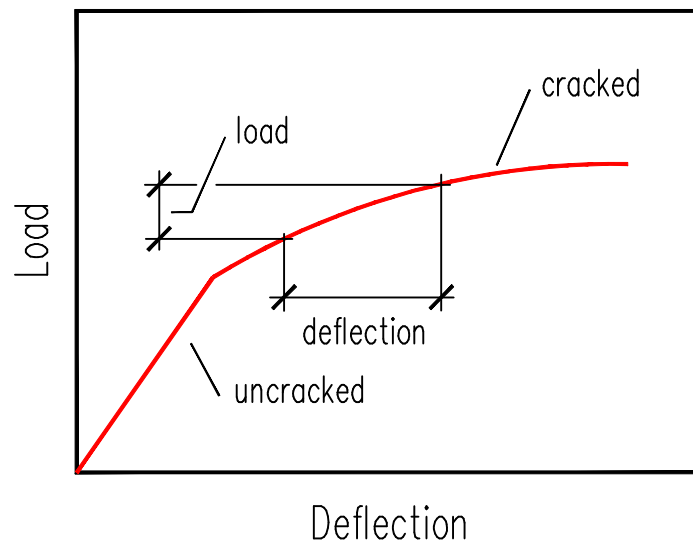


FIGURE 8-1 Load-Deflection with Allowance for Cracking

The load combination application for live load deflection can be summarized as follows:

- ❖ **Uncracked member**
A single solution with the following load combination
 $1.0*LL$
- ❖ **Cracked member**
Two solutions are required. The net live load deflection being the difference between the two, as indicated below

First solution indicated by U1 with load combination (DL + LL)
Second solution U2 using load combination (DL)

$$\text{Live load deflection} = U1 - U2$$

9 - LONG-TERM DEFLECTIONS

A concrete member's deformation increases with time, due to shrinkage and creep. Shrinkage is caused by loss of moisture, and hydration. Creep is increase in displacement under stress. Under constant loading, such as selfweight, the effect of creep diminishes with time. Likewise, under normal conditions, with loss of moisture, the effect of deformation due to shrinkage diminishes. Restraint of supports to free shortening of a slab due to shrinkage or creep can lead to cracking of slabs and thereby an increase in deflection due to gravity loads.

While the methodology and tools are readily available to determine the increase in instantaneous deflection of a floor system due to creep and shrinkage over any time interval, the common practice for residential and commercial buildings is to estimate the long-term deflection through a magnification of the instantaneous values.

9.1 Multiplier Factors for Long-Term Deflections

For design purposes, the long-term deflection of a floor system due to creep and shrinkage can be approximated as a multiplier to its instantaneous deflection.

Long-term deflection due to sustained load:

$$\Delta_l = C * \Delta_i \quad (9.1-1)$$

Where

- Δ_l = long-term deflection;
- Δ_i = instantaneous deflection; and
- C = multiplier.

ACI-318 suggests the multiplier factor shown in Fig. 9.1-1 to estimate long term deflections due to sustained loads

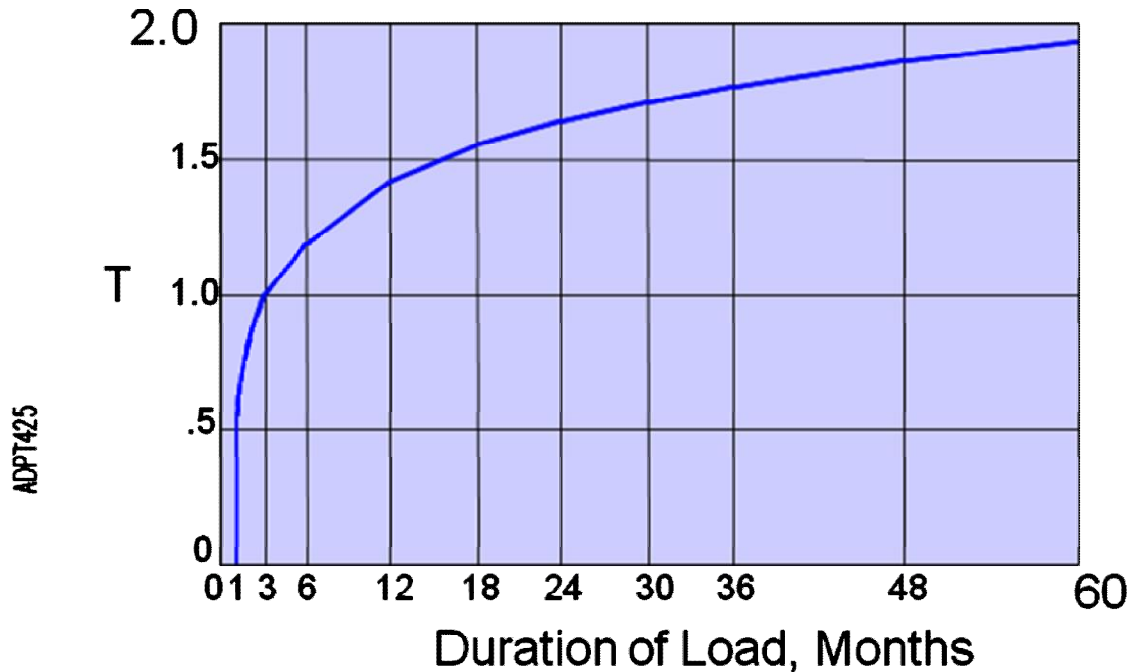


FIGURE 9.1-1 Multiplier Factor for Long-Term Deflection

The multiplier can be reduced, if compression reinforcement is present. The factor (λ) for the reduction of the multiplier is given by:

$$\lambda = C / (1 + 50\rho') \quad (9)$$

Where ρ' is the value of percentage of compression rebar at mid-span for simple and continuous members and at support for cantilevers.

The ACI-318 recommended multiplier factors are intended for practical structures. In practical structures there is typically a delay between the time the forms are removed, the superimposed load, such as floor cover is applied, and the structure is placed in service for application of live loads. The time-delay, as it is expounded henceforth, reduces the net deflection that is the objective of design check and is likely to impact the nonstructural members likely to be damaged from long-term deflection.

Several Investigators [Sbarounis, 1984, Branson, 1977, Graham and Scanlon, 1986b] have recommend long-term multiplier coefficients larger than ACI-318 recommendations for deflections from the time of removal of forms, using gross cross-sectional area in computation.

The load combination to be used for the long-term deflection of a floor system depends on the objective of the evaluation. ACI-318 recommends a load combination that reflects the “sustained” live load on the member. However, the code leaves the fraction of the design live load to be considered as “sustained” to the design engineer’s judgment. The European code EC2 offers the following recommendation for sustained (quasi-permanent) load case.

TABLE 9.1-1 RECOMMENDED “SUSTAINED LOAD” AS FRACTION OF DESIGN LIVE LOAD¹³

Occupancy	Fraction of design live load
Dwellings and offices	0.3
Shopping; congested areas	0.6
Storage	0.8
Parking	0.6

The following describe several common scenarios.

9.2 -Total Long-Term Displacement Subsequent to Removal of Forms

The simple, conservative first estimate load combination for long-term deflection is

$$(1.0*SW + 1.0*SDL + 1.0*PT + 0.3*LL)* C$$

Where

- SW = selfweight;
- SDL = superimposed dead load, (floor cover and partitions);
- PT = post-tensioning;
- LL = design live load; and.
- C = long-term multiplier.

The above load combination is conservative as it assumes the application of superimposed loads as well as the application of sustained live load of the structure to take place at the time of removal of the supports below the cast floor. The factor 0.3 suggested for live load is for “sustained” load combination. The significance of the above load combination is that it provides an upper-bound measure for the total deflection from the position of the forms at the time of concrete casting. The values are more applicable to parking structures and roofs, where a floor is placed in service in essentially in as-cast condition.

9.3 Detailed Combination of Long-Term Deflections

Detailed calculations generally apply, when the objective of estimating deflection is to determine its impact on non-structural members that are likely to be damaged. Graphs, such as the example reproduced in Fig. 9.3-1 can be used to determine the increment of long-term deflection with respect to time for specific applications.

¹³ EC2 - 2004

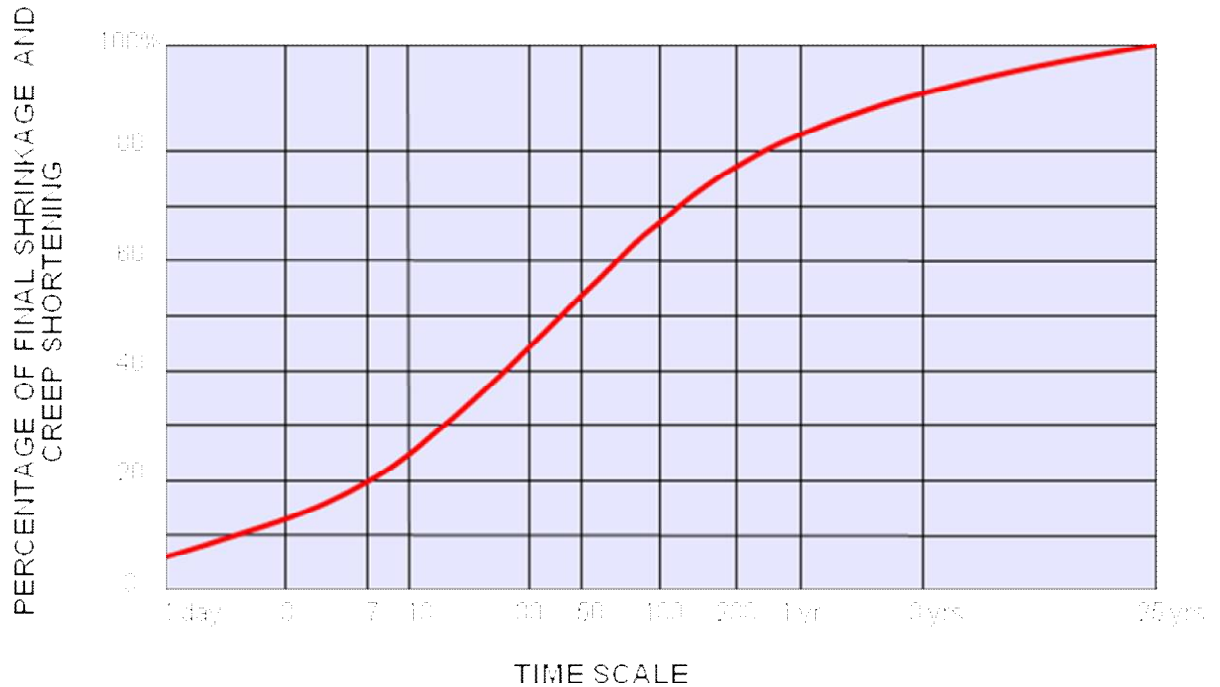


FIGURE 9.3-1 Long-Term Shortening of Concrete Members with Time Due to Creep and Shrinkage

The application of detailed computation of long-term deflections is illustrated through the following numerical example.

9.4 Example

For the floor system shown in Fig. EX5.2-1, determine the deflection at the center of panel identified for the following cases:

Condition 1 – No cracking; simplified method

Assume loads to have been applied all at removal of concrete supports; assume a magnification factor C=2.2

Condition 2 – No cracking; detailed method

- ❖ Superimposed load applied 45 days after removal of forms
- ❖ Structure placed in service 180 days after removal of forms

Condition 3 – Allow for cracking; detailed method

- ❖ Superimposed load applied 45 days after removal of forms
- ❖ Structure placed in service 180 days after removal of forms

The applicable deflection criteria for the center of the panel selected are:

$L = 39.86 \text{ ft (18.83 m)}$

- ❖ For visual impact and sense of comfort

$d \leq L/240 = 39.86 * 12 / 240 = 2.0 \text{ in. (50.8 mm)}$

- ❖ For damage mitigation

$$d \leq L/480 = 39.86 * 12 / 480 = 1.0 \text{ in. (25.4 mm)}$$

9.4 Condition 1 – No cracking; simplified method

The method conservatively assumes that all applied at removal of forms. With the assumption of no cracking, the principle of superposition applies. Solutions obtained from the analysis of the design strip are:

Load	Deflection
Selfweight (SW)	0.33 in. (8.4 mm)
Superimposed dead load (SDL)	0.08 in. (2.0 mm)
Live load (LL)	0.13 in. (3.3 mm)

- ❖ For visual and sense of comfort

On the assumption that the slab was not cambered at time of construction, the visual effect is governed by the total of immediate and long-term deflection, subsequent to removal of concrete support. The load combination includes a fraction of the live load as sustained (0.3LL) and the balance of it applied instantaneously, as given below:

$$(1 + C) * (SW + SDL + 0.3*LL) + 0.7*LL$$

Where, C is the magnification factor, assume 2.2.

$$\text{Deflection } d = (1 + 2.2)(0.33 + 0.08 + 0.30*0.13) + 0.7 * 0.13 = 1.53 \text{ in. (38.9 mm)}$$

$$< d/L - 1/250 = 2.0 \text{ in (50.8 mm)}$$

- ❖ For damage control

Under the assumption that the loads are applied immediately after removal of forms, the long-term effect of the applied loads increased by the immediate deflection of non-sustained fraction of live load will apply. The associated load combination is:

$$C*(SW + SDL + 0.3LL) + 0.7LL$$

$$\text{Deflection } d = 2.2 (0.33 + 0.08 + 0.3*0.13) + 0.7*0.13 = 1.08 \text{ in (27.4 mm)}$$

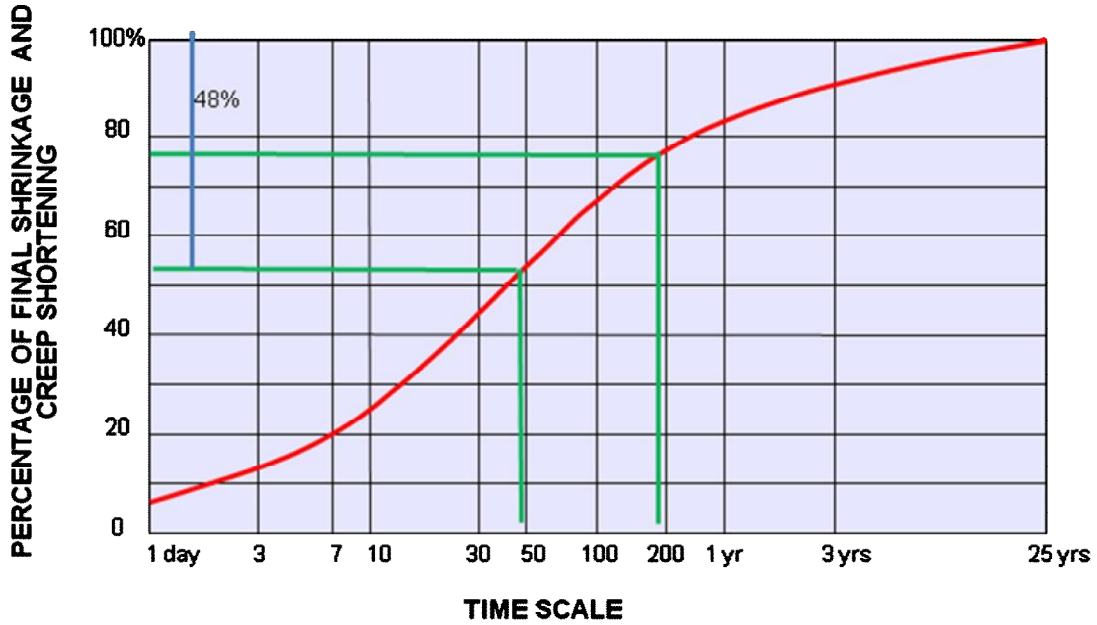
$$> L/480 = 1.00 \text{ in. (25 mm)}$$

Recognizing that there is a significant degree of latitude in the accuracy of the values obtained from this method of calculation, based on engineering judgment the result is considered acceptable.

9.5 Condition 2 – No cracking; detailed method

- ❖ Superimposed load applied 45 days after removal of forms
- ❖ Structure placed in service 180 days after removal of forms

From Fig. 9.5-1, the balance of deflection likely to take place subsequent to application of loads on days 45 is 48% .

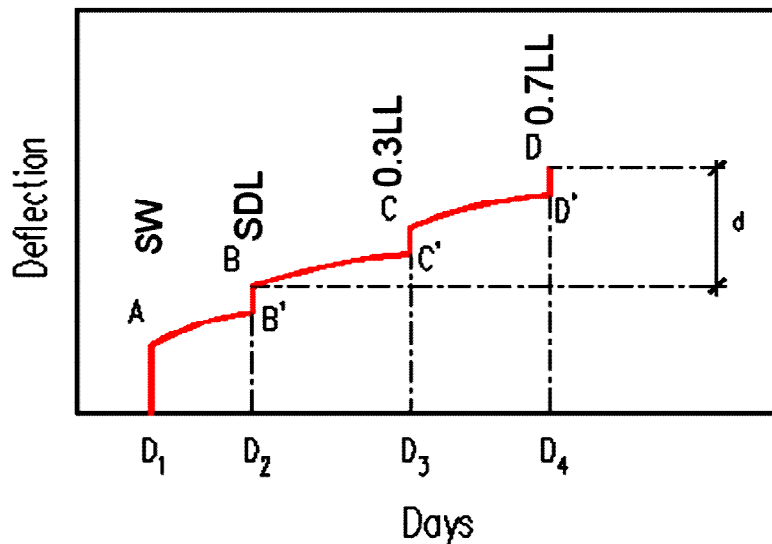


LONG-TERM SHORTENING OF CONCRETE

FIGURE 9.5-1

The time-line for application of the load and its consequence is shown in Fig. 9.5-2. Note that at each application of load, such as sustained live load of 0.3LL, there will be an immediate deflection, followed by long-term deflection from the latest applied load, as well as residual of long-term deflection from loads applied earlier. For example, in the diagram, point B' signifies the total deflection of the panel due to selfweight on day 45. Once the superimposed load is applied, the structure deflects to point B. Finally, once the balance of the design live load (0.7) is applied, the structure deflects from D' to D'. It is assumed that the design live load is unlikely to be applied frequently and be sustained. Hence, there is no allowance for its long-term effect.

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Long-term deflection with multiple loading times

FIGURE 9.5-2

- ❖ For visual and sense of comfort

Using the aforementioned concept, the load combination for the visual effect will encompass the entire displacement as given below. It is similar to the condition;

$$(1 + C) * (SW + SDL + 0.3*LL) + 0.7*LL$$

Where, C is the magnification factor, assume 2.2.

$$\begin{aligned} \text{Deflection } d &= (1 + 2.2)(0.33 + 0.08 + 0.30*0.13) + 0.7 * 0.13 = 1.53 \text{ in. (38.9 mm)} \\ &< d/L = 1/250 = 2.0 \text{ in (50.8 mm)} \end{aligned}$$

From the above load combination, the delayed application of load does not appear to impact the total deflection. Strictly speaking, however, due to aging of concrete and increase its modulus of elasticity with time, the instantaneous response of the structure to loads that are applied at concrete with greater age will result in less deflection. However, while the implementation is straight forward, it is not considered in routine design, where concrete is assumed to be 28 days old with respect to its modulus of elasticity.

- ❖ For damage control

For damage control, the fraction of displacement subsequent to application of member likely to be damaged applies. From Fig. 9.5-1, given that the superimposed dead load and elements likely to be damaged are installed on day 45, the balance of deflection likely to cause damage is marked by offset “d” in Fig 9.5-2:

- From selfweight the balance of long-term deflection is 48%
- From superimposed dead load (including the weight of material likely to be damaged), only the long-term displacement is applicable – hence the long-term multiplier “C” times the instantaneous deflection due to added load,
- From sustained live load the instantaneous and its long-term contribution apply
- From the balance of design live load (0.7LL) the instantaneous contribution applies.

$$\begin{aligned} d &= 0.48*C*SW + C*SDL + (1 + C)*.30LL + 0.70*LL = \\ &= 0.48*2.2*0.33 + 2.2*0.08 + (1 + 2.2)*0.3*0.13 + 0.7*0.13 = 0.74 \text{ in (18.8 mm)} \\ &< d/L = L/480 = 1.00 \text{ in (25.4 mm) OK} \end{aligned}$$

9.6 Condition 3 – Allow for cracking; detailed method

- ❖ Superimposed load and elements likely to be damaged installed 45 days after removal of forms
- ❖ Structure is placed in service 180 days after removal of forms

The post-cracking displacement of a member falls in the non-linear range of the load-deflection curve. Hence, increments of deflection shall be obtained with due consideration for possible loss of stiffness.

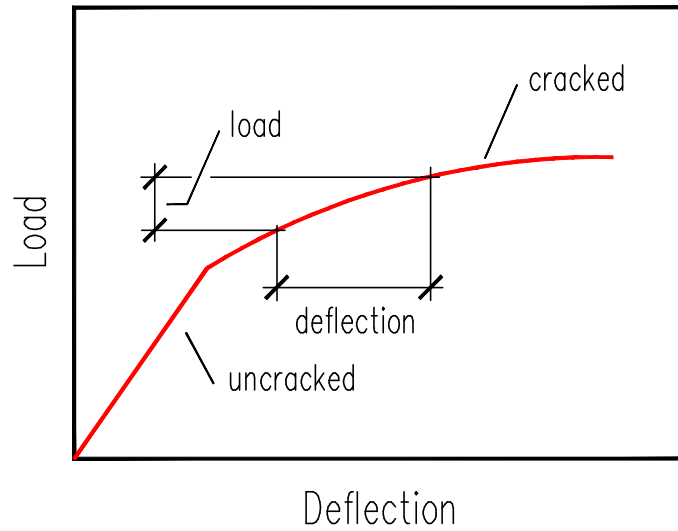


FIGURE 9.6-1 Load-Deflection Relationship with Cracking

❖ For visual and sense of comfort

For visual effects, the instantaneous, and long-term effects of selfweight, superimposed dead load, and sustained live load will be the base deflection, to which the instantaneous deflection due to balance of design live load (0.70LL) will be applied. The steps to be followed by are detailed below:

- Applicable load combination

$$d = (1 + C) * (SW + SDL + 0.3*LL) + 0.7*LL$$

- Instantaneous effects of SW; SDL and 0.3LL denoted by $\Delta_{sw,SDL,0.3LL}$

$$\Delta_{sw,SDL,0.3LL} = (SW + SDL + 0.3LL) = 0.53 \text{ in (13.5 mm)}$$

- Long-term effects of the above is $C * \Delta_{sw,SDL,0.3LL}$

- The instantaneous deflection due to application of balance of live load (0.70LL) denoted by $\Delta_{0.7LL}$ is:

$$\Delta_{0.7LL} = \Delta_{sw,SDL,LL} - \Delta_{sw,SDL,0.3LL} = 0.12 \text{ in. (3.0 mm)}$$

Hence, the total deflection for visual impact and sense of comfort becomes:

$$(1 + C) * \Delta_{sw,SDL,0.3LL} + \Delta_{0.7LL} = (1 + 2.2) * 0.53 + 0.12$$

$$= 1.82 \text{ in (46.2 mm)} < L/240 = 2.0 \text{ in. (50.1 mm) O K}$$

❖ For damage control

For damage control the instantaneous displacements from the selfweight, superimposed dead load, and elements likely to be damaged will not contribute. The contributory components of the deflection (offset d in Fig. 9.5-2) are:

- Long-term effects of selfweight

$$\Delta_{sw,LT} = 0.48 * C * \Delta_{sw}$$

The associated load case is 1.0*SW

$$\Delta_{sw,LT} = 0.48 * C * \Delta_{sw} = 0.48 * 2.2 * 0.38 = 0.40 \text{ in. (10.2 mm)}$$

- Long-term effects of superimposed dead load

The immediate deflection from SDL does not impact the results, since installation of members subject to damage is deemed to be in progress. But, the long-term effects of superimposed dead load apply

$$\Delta_{SDL,LT} = C * \Delta_{SDL}$$

This is obtained from two independent solutions, one including and the other excluding the superimposed dead load as given below

$$\Delta_{SDL} = \Delta_{sw,SDL} - \Delta_{sw} = 0.1 \text{ in. (2.5 mm)}$$

$$\Delta_{SDL,LT} = C * \Delta_{SDL} = 2.2 * 0.1 = 0.22 \text{ in. (5.6 mm)}$$

The structure is placed in service after six months, when (0.3 LL) applies. This results in immediate displacement to point C, followed by long-term deflection to D'. The immediate displacement $\Delta_{0.3LL}$ of the structure (CC') contributes to the total displacement (d), likely to damage the structure. Its value is given by subtracting the deflection values $\Delta_{sw,SDL,0.3LL}$ from $\Delta_{sw,SDL}$ to be obtained from two different solutions as indicated below:

- Solution for immediate response to sustained live load $\Delta_{0.3LL}$ is

$$\Delta_{0.3LL} = \Delta_{sw,SDL,0.3LL} - \Delta_{sw,SDL}$$

For $\Delta_{sw,SDL}$ the load case is;

$$(1.0*SW + 1.0*SDL) = 0.48 \text{ in. (12.2 mm)}$$

For $\Delta_{sw,SDL,0.3LL}$ the load case is

$$(1.0*SW + 1.0*SDL + 0.30*LL) = 0.53 \text{ in. (13.5 mm)}$$

- Immediate deflection for application of sustained live load (segment CC') in Fig. 9.5-2.

$$\Delta_{0.3LL} = \Delta_{sw,SDL,0.3LL} - \Delta_{sw,SDL} = 0.53 - 0.48 = 0.05 \text{ in. (1.3 mm)}$$

- The long-term effect of sustained live load is given by:

$$\Delta_{0.3LL,LT} = C * \Delta_{0.3LL} = 2.2 * 0.05 = 0.15 \text{ in. (3.8 mm)}$$

At a later date , the structure is checked for (0.7 LL), when it deflects to point D.

$$0.7 \Delta_{LL} = \Delta_{sw,SDL,LL} - \Delta_{sw,SDL,0.3LL} = 0.69 - 0.53 = 0.16 \text{ in (4.1 mm)}$$

The **total deflection** experienced by the members likely to be damaged is deflection due:

- Long-term effects of selfweight after day 45; $\Delta_{sw,LT}$ 0.40 in (10.2 mm)
- Long-term effects of superimposed dead load after day 45; $\Delta_{SDL,LT}$ 0.22 in (5.6 mm)
- Immediate deflection due to sustained LL at day 180; $\Delta_{0.3LL}$ 0.05 in (1.3 mm)
- Long-term effects of sustained live load after day 180; $\Delta_{0.3LL,LT}$ 0.15 in (3.8 mm)
- Immediate deflection due to the balance of design live load $0.7 \Delta_{LL}$ 0.16 in (4.1 mm)

Total 0.98 in (25 mm)

< span/480 = 1.00 in (25.4 mm) OK

9.7 Summary of Long-Term Deflection Calculation

TABLE 9.7-1 SUMMARY OF LONG-TERM DEFLECTION CALCULATIONS

Description	Visual effect	Potential of damage
Simple method no cracking	1.53 in (38.9 mm)	1.08 in (27.4 mm)
Detailed method no cracking	1.53 in (38.9 mm)	0.74 in. (18.8 mm)
Detailed method allow for cracking	1.82 in. (46.2 mm)	0.98 in. (25.0 mm)

APPENDIX A

CHARACTERISTICS OF PARISSA APARTMENTS TYPICAL FLOOR

Geometry

Slab thickness and support dimensions (see plan)

Concrete

f'c (28 day cylinder strength) = 5000 psi (34.47 MPa)
 Wc (unit weight) = 150 pcf (2403 kg/m³)
 Ec (modulus of elasticity at 28 days) = 4,287 ksi (29558 MPa)

Non-Prestressed Reinforcement

Yield stress = 60 ksi (400 MPa)

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