Analysis and Behavior of Microsound Surface Waveguides

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Abstract—An analysis is presented for the evaluation of frequency spectra and mode configurations of orthotropic microsound waveguides of arbitrary cross section. The analysis is based on the extended Rayleigh-Ritz energy method together with the concept of discretization of the guide cross section. The formulation satisfies all the boundary conditions and extracts with high accuracy a predetermined number of lowest frequencies and their modes. The method is employed to study the behavior of several different types of microsound surface waveguides of general interest. The results and their discussions reveal the influences of cross-sectional geometry and material properties on guide's performance. The solutions offered are general and provide a useful basis for the design of microsound surface waveguides.

INTRODUCTION

The advantages of acoustic waveguides over electromagnetic guides, for use in delay lines are discussed and emphasized in a number of recent publications [1-3]. Because acoustic waves are typically five orders of magnitude slower than electromagnetic waves, components whose size are of the order of wavelength can be realized in a volume which, in principle, could be 15 orders of magnitude smaller than required by their electromagnetic counterparts [2]. This permits the use of guides of very small cross section, and as a result the guide structures are, in most cases, not capable of self-supporting.

In the early stages a number of widely different types of microsound guides in conjunction with a supporting medium were proposed, of which surface guides were preferred for use in micro-electronic circuits due to a direct accessability of their free surface for input and output operations. Of the surface waveguides, thin film overlay guides are fast and consist of a thin layer of material of about one wavelength width deposited on a substrate such as strip of gold on quartz [4, 5]. Topographical guides being slower and larger in cross section are suggested by Ash *et al.* [2]. In this category the guide is cut in one of a number of possible shapes on the substrate surface, thus eliminating the use of a second material overlay. These waveguides are not the most suitable kind for microsound circuits because the components that can be made with them are limited in variety.

An overlay guide consists of an overlay of substantial thickness, of a suitably chosen cross section and material, on a substrate of normally much stiffer material [6]. Guides of this category confine the wave propagation essentially to the relatively flexible overlay and reduce the excitation of substrate [7]. Several different geometries of overlay guides are shown in Fig. 1(a) to (d).

In order to avoid mode coupling problems, it is desired to operate the microsound guides over a range of single-mode frequencies, i.e., between the lowest and the next cutoff frequencies of the guide. Therefore the evaluation of the lower cutoff frequencies of a guide are of particular importance in its design. Another critical question posed to the designer is over what range of wavelengths a single mode operation is possible for a particular guide, and in addition how to raise or lower the range of single mode operation frequencies of a guide by changing its geometry and by selection of its material.

Much work has been done in the general field of elastic wave propagation in solids. A comprehensive recent review of the works of interest to the behavior of microsound acoustic waveguides is given by White in [1]. However, due to the complexity of analysis the previous investigations have all dealt with unbounded media, semi-infinite solids or uniform plates. To date, except for some limited studies of rectangular waveguides of low-aspect ratio ($b/a < 0.4$) [3, 7, 8], virtually no work has appeared in the important area of waves in microsound guides of practical cross sections.

This work outlines in brief a general and powerful method for evaluating frequency spectra and mode configurations of orthotropic waveguides of arbitrary cross sections. The orthotropy refers to different elastic properties in the three orthogonal directions expressed by nine elastic moduli, with the direction of propagation coinciding with one of the elastic axes. The method is then employed to investigate the behavior of several of the more interesting overlay waveguides proposed by the previous investigators as potentially suitable designs, as well as an inlay guide (Fig. 1). The influence of geometry and material properties of the guides are demonstrated. The solutions offered and their discussions serve toward providing the needed knowledge for the general understanding of the behavior of microsound waveguides and their study leads to a more rational and effective design.

The method is based on the concept of discretization together with Rayleigh-Ritz energy method, which as finite elements has been used extensively in the analysis of structural engineering problems [9]. In the present analysis, the cross section is divided into discrete mathematical regions called elements. The components of displacement forms representing the waves in the bar are taken as a product of harmonic function in the propagation direction and an unknown function for each spatial direction. By using many elements to model the cross section, a large number of generalized coordinates result.

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The Lagrangian L of a representative length λ of the bar is given by the summation over the cross section of the appropriate energies of each element

$$L = \sum (T - V) = \frac{1}{2} \frac{\rho l^3}{\xi} \{\dot{r}\}^T [M] \{\dot{r}\} - \frac{1}{2} \frac{C_{ij} l}{\xi} \{r\}^T [K] \{r\} \quad (8)$$

where $[M]$ and $[K]$ are respectively the mass and stiffness matrices of the assemblage formed through summation of appropriate terms of elements' stiffness and mass matrices.

Application of Hamilton's principle to the Lagrangian of (8) gives the following equations of motion [10]

$$C_{ij} [K] \{r\} + \rho l^2 [M] \{\ddot{r}\} = 0. \quad (9)$$

For a SHM with $\{r_0\}$ being the amplitudes of nodal values of functions w', u', and v', equation (10) may be written.

$$\{r\} = \{r_0\} e^{i\omega t}. \quad (10)$$

Subsequently on substitution for $\{r\}$ from above into the equations of motion (9), the analysis of the steady state wave propagation reduces to the following algebraic eigenvalue problem

$$([K] - \Omega^2 [M]) \{r_0\} = 0 \quad (11)$$

where Ω is the normalized frequency given by

$$\Omega^2 = \frac{\omega^2}{C_{ij}/\rho l^2}. \quad (12)$$

Any of the standard eigenvalue and eigenvector extraction procedures may now be used for the evaluation of frequencies and their corresponding modes from (11). The present analysis employs an efficient iterative method as outlined in [11]. The eigenvalue and eigenvector extraction procedure employed evaluates a predetermined number of lowest eigenvalues and their eigenvectors without *a priori* knowledge of mode configurations or imposing of mode restraints. Prior to obtaining a solution, however, the complete displacement and force boundary conditions of the problem are implemented in $[K]$ and $[M]$ without difficulty [9].

The effectiveness of this general method of analysis and its scope is demonstrated in [12]. In order to illustrate its accuracy, solutions obtained from the present analysis for the case of wave propagation through a homogeneous solid cylinder—a case for which exact solutions are available—are compared with results of [13] in Table I. The close agreement between the corresponding frequencies confirms the correctness of formulation. It should be noted that the slight differences between the two solutions in Table I may be partly attributed to the fact that the circular boundary of the cylinder cross section is, in the present analysis, modeled by 24 straight lines forming the sides of the 96 elements used. The computer time required for the evaluation of the first ten modes of each wavelength ratio is nine seconds on the IBM 360 computer.

BEHAVIOR

Several surface microsound waveguides of general interest are analyzed and their behavior discussed. The behavior illustrates the influence of a number of important factors in design, such as geometry and material properties. The guides selected represent the majority of the surface microsound geometries considered by previous investigators as potentially good designs. For the first type Fig. 1(a) limited approximate solutions are available. For the remainder, however, no solutions have been presented so far.

TABLE I
HOMOGENEOUS ISOTROPIC SOLID CYLINDER. ACCURACY COMPARISON OF FIVE LOWEST FREQUENCIES
[Frequency $\Omega = \omega/\omega_s$, where $\omega_s^2 = (\pi/l^2)\, G/\rho$[a]]

	$l/\lambda = 1$			
Mode	AGH[b]	Present Analysis	% Difference	Description
1	0.8766	0.8819	0.6	Flexure first mode
2	1.0000	1.0000	0.0	Torsion first mode
3	1.0607	1.0832	2.2	$n = 2$[c] first mode
4	1.1096	1.1290	1.8	Longitudinal first mode
5	1.2618	1.2861	2.1	Flexure second mode

[a] l is the radius of cylinder.
[b] From Armenakas *et al.* [13].
[c] n is the number of waves along the circumference.

For each wave guide the mode spectrum is given graphically for a range of wave length ratios from the cutoff frequencies to wave lengths of the order of guide dimensions. The spectra each include a given number of lowest modes throughout this range. A valuable feature of the results lies in that the analysis extracts all the possible modes and hence the graphs are comprehensive in containing all the modes. The magnitudes of the first four cutoff frequencies of the cases analyzed are summarized in Table II for comparison.

For ease of comparison and cross reference the mode displacement configurations of all the guides analyzed are presented in a unified manner. The deformation of a guide cross section under the wave motion is designated by a mode configuration code. The mode designation is necessary for identification. In general, at cutoff frequencies ($\lambda = \infty$), the cross sectional deformations are characterized by simple displacement forms which may be readily categorized into longitudinal L, shear S, dilatational D or their combinations. However, with reduction in wavelength, the initially relatively smaller displacement components of the cross section gain in magnitude, such that at wavelengths of the order of guide cross sectional sides the deformation configurations are frequently not distinguishable on the basis of their respective cutoff frequency forms. The mode designations are based on deformations immediately after the cutoff frequencies, when the nature of secondary displacement components of the cross section become evident. The mode designations are subsequently applied for increasing values of wave length on the basis of continuity of dispersion curves as well as shape of deformations. The mode designation procedure used is summarized in Fig. 3, and is found to be capable of describing the displacement configurations of all the guides used in the present analysis.

In the basic modes, the cross sectional deformation is primarily due to one of the three displacement components in its simplest form. The other displacement components being very small. However, for clarity these are shown exaggerated in Fig. 3. For higher modes, the letter codes denote the dominating component (or components) of the cross section; the

TABLE II
FIRST FOUR CUTOFF FREQUENCIES OF MICROSOUND SURFACE WAVEGUIDES
[Frequency $\Omega = \omega/\omega_s$, $\omega_s^2 = \pi C_{ij}^{a}/\rho l^2$]

Material property	Type of Guide		First Four Cutoff Frequencies and Their Mode Designation				
			Ω_1	Ω_2	Ω_3	Ω_4	$\Omega_2 - \Omega_1$
Isotropic	Rectangular overlay ($l = b$)	$b/a =$ 0.5	0.4347 (S_0)	0.5018 (L_0)	0.7139 (L_{21})	0.8151 (D_0)	0.0671
		1	0.3569 (S_0)	0.5018 (L_0)	0.8609 (D_0)	0.9594 (D_{21})	0.1449
		2	0.2417 (S_0)	0.5046 (L_0)	0.8569 (D_0)	0.9051 (D_{21})	0.2629
	Trapezoidal ($l = b$)	$b = a$ $\theta = 75°$	0.1987 (S_0)	0.4268 (L_0)	0.7250 (D_0)	0.7928 (D_{21})	0.2281
	Triangular (l = side)	$\theta = 60°$	0.7174 (S_0)	0.8539 (L_0)	1.4247 (D_0)	1.4608 (S_{12})	0.1365
	Rectangular inlay ($l = a$)	$b/a = 1.5$	1.0649 (L_0)	1.1498 (D_0)	1.4584 (L_{12})	1.8147 (S_0)	0.0849
Orthotropic	Square overlay ($l = b = a$)	Material No. 2	0.3306 (L_{21})	0.3481 (S_0)	0.5019 (L_0)	0.5165 (S_{21})	0.0175
		Material No. 3	0.0497 (S_0)	0.1509 (S_{12})	0.2596 (S_{13})	0.3771 (S_{14})	0.1012
		Material No. 4	0.1494 (S_0)	0.4532 (S_{12})	0.5018 (L_0)	0.6001 (L_{21})	0.3038

[a] For isotropic guides $C_{ij} = G$, $\nu = 0.3$. For orthotropic guides: for material No. 2 $C_{ij} = C_{44}$, for materials No. 3 and No. 4 $C_{ij} = C_{55}$.

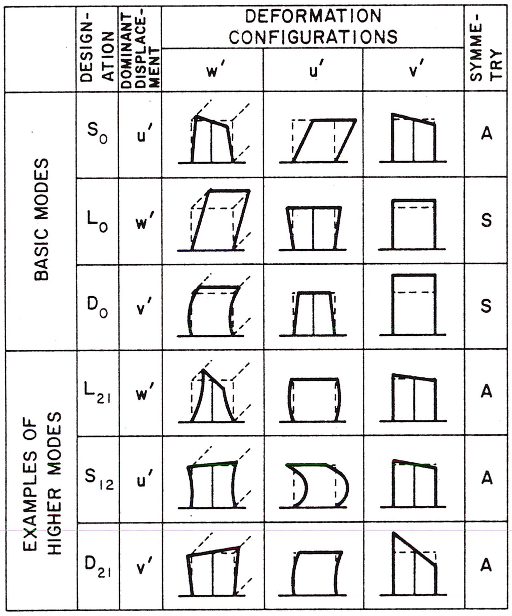


Fig. 3. Mode designations and displacement configurations for overlay microsound waveguides. Broken lines show undeformed transverse section; heavy lines depict maximum deformation of section due to wave motion. In each case these is dominant component of deformation (given in third column) accompanied by slight deformations due to other displacement components, latter are shown exaggerated for clearance. Deformations are either symmetrical S or antisymmetrical A about y–y center line of the section.

first numerical subscript describes the higher mode of deformation of this component along the crest of the guide, and the second number refers to the order of mode deformation in direction perpendicular to the crest. This procedure of mode designation is somewhat similar to the manner used by Waldron [8].

It should be noted that, as indicated on the last column of Fig. 3, the possible lower mode configurations given by the analysis for the types of guides considered are such that the three displacement components are either symmetrical or antisymmetrical about the y-y center line of the guide.

The guides analyzed are all assumed to have been mounted on, or inlaid in a rigid substrate. This condition, though desirable because it confines the wave energy to the overlay, is not strictly met in practice. However, Waldron [7] has shown that provided the density and stiffness of the guide and substrate are about 1 : 10, virtually all the energy is transferred through the guide and the influence of the substrate bulk vibrations may be disregarded. While the present analysis is capable of accounting for the leakage due to the finite stiffness and mass density of the substrate, in light of the above conclusions it is considered more proper at this stage to emphasize the geometry variation and material orthotropy, where a shortcoming is evident.

A. Rectangular Overlay Guides

Three guides with b/a equal to 0.5, 1, and 2 are analyzed, the mode spectra of which are shown in Fig. 4(a) to (c), respectively. The magnitudes of the corresponding first four cutoff frequencies are given in the first three rows of Table II.

The normalized frequencies of the basic longitudinal L_0 and dilatational D_0 modes at cutoff values (given in Table II) are not significantly affected by the geometry of the guide. This is in agreement with the general theories of wave propagation in elastic solids. For, the basic longitudinal mode L_0 of the rectangular overlay guide is in fact analogous to the first antisymmetric thickness shear mode of an infinite elastic plate of depth $2b$, for which the circular frequency is given [14, 15] as

$$\Omega = \frac{\omega b}{\pi} \sqrt{\frac{\rho}{G}} = 0.500.$$

Results given by the present analysis deviate at most 1 percent from this value. The corresponding values from the approximate analysis proposed by Waldron [8] (twice the horizontal ordinates of Figs. 6 and 7 for L_{10}) are for $b/a = 0.25$ about 0.60 and for $b/a = 0.4$ about 0.86. However, the author [8] warns that these values, for the larger aspect ratio should not be regarded quantitatively as accurate.

The maximum [illegible] has been [illegible] considered as a rigid [illegible] along the [illegible] de- [illegible] corrected [illegible]. It has shown [illegible] provided [illegible] stiffness of the groove [illegible] are about 1:10, [illegible] all the [illegible] is transferred through the [illegible] and the [illegible] of the lateral wall [illegible] may be disregarded. While the present analysis is capable of accounting for [illegible] leakage [illegible] finite [illegible] dependence of the substrate, in light of the above [illegible] considered [illegible] at this stage [illegible] orthotropy, [illegible] have [illegible] evident.

4. [illegible]

Three guides with b/h equal to 0.5, 1, and 2 are analyzed, [illegible] of which [illegible] shown in Fig. [illegible] respectively. The [illegible] values of the corresponding [illegible] frequencies are given in [illegible] the first three [illegible] Table [illegible]. The normalized frequencies of the [illegible] longitudinal [illegible] different [illegible] cutoff values shown in Table [illegible] not significantly affected by the geometry of the guide. [illegible] with the [illegible] of wave propagation in an elastic [illegible] the [illegible] longitudinal mode [illegible] of the rectangular wave guide [illegible] first [illegible] increments [illegible] of an infinite elastic [illegible] depth [illegible] which the [illegible] frequency [illegible] [14]

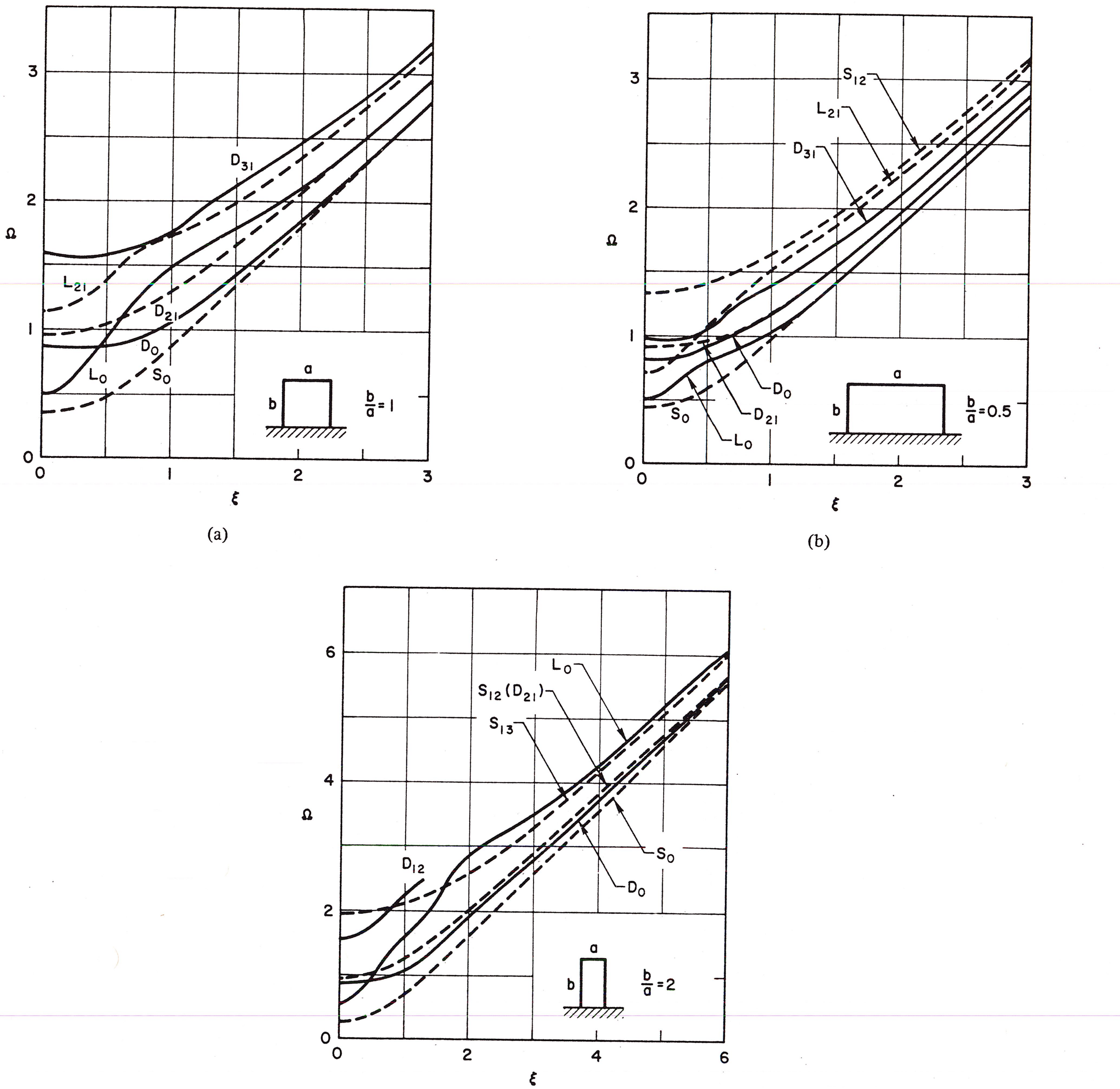
$$\frac{2b}{[illegible]}\sqrt{[illegible]} = 0.5[illegible]$$

[illegible] the [illegible] of [illegible] displacement [illegible] (twice [illegible]) [illegible] is 6 and [illegible] about 0.8. However, [illegible] that these values [illegible] [illegible].

Fig. 3 Mode [illegible] and displacement configurations for [illegible] and [illegible] wave guides. Broken lines show undeformed [illegible]; [illegible] lines depict maximum deformation of [illegible] wave motion [illegible]. In both [illegible] these dominant component [illegible] (given in [illegible]) accompanied by [illegible] deformations [illegible] displacement components, latter are shown exaggerated [illegible] [illegible].

The [illegible] subscript describes the [illegible] along the [illegible] of the [illegible] configuration along the [illegible] of the guide, and the second number refers to the order of mode [illegible] direction perpendicular to the [illegible]. The [illegible] designation is somewhat similar to the manner used [illegible] [8].

It should be noted that [illegible] is included on [illegible] the [illegible] lower mode configuration [illegible] analysis [illegible] guide considered [illegible] displacement components are [illegible] symmetric [illegible].



(a)

(b)

(c)

Fig. 4. Rectangular isotropic overlay guides on rigid substrate. Frequency spectrum of lowest modes for different wave lengths (Frequency ratio $\Omega = \omega/\omega_s$, Wavelength ratio $\xi = l/\lambda$, $l = b$, $\omega_s^2 = \pi G/\rho l^2$, $\nu = 0.3$, heavy lines refer to symmetrical cases, and broken lines refer to antisymmetrical configurations). (a) Square overlay guide. (b) Rectangular overlay guide $b/a = 0.5$. (c) Rectangular overlay guide $b/a = 2$.

The basic dilalational mode D_0 corresponds to the first symmetric thickness stretch mode of an infinite plate of depth $2b$, with the difference that the rectangular guide being free on the sides accommodates part of the volume change by its transverse deformation in x direction (u component). The normalized circular frequency of a plate for this mode is [15].

$$\Omega = \frac{\omega b}{\pi}\sqrt{\frac{\rho}{G}} = \frac{1}{2}\sqrt{\frac{2(1-\nu)}{(1-2\nu)}} = 0.9354, \qquad \text{for } \nu = 0.3.$$

The D_0 frequencies of the rectangular guides discussed here vary between 0.815 and 0.861, which in view of the reduced stiffness of the analyzed guides for this type of deformation is a reasonable agreement. In [8], the corresponding frequency given for $b/a = 0.2$ is about 0.98 (twice value of D_{20}, Fig. 6). A solution obtained from the present analysis for $b/a = 0.205$ gave $\Omega(D_0)$ equal to 0.936.

As may be expected intuitively, an increase in b/a ratio results in guides with more flexibility in transverse direction (x direction) and consequently lowers the values of shear cutoff frequencies (S_0). From the rectangular guides considered $b/a = 2$ has the least (S_0) cutoff frequency and hence offers the widest range of single mode frequency operation (4 times the case of $b/a = 0.5$) as given on last column of Table II.

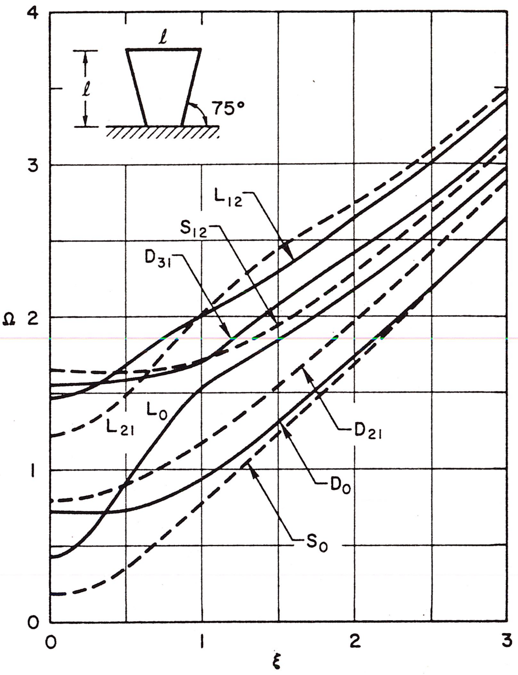


Fig. 5. Trapezoidal isotropic overlay guide on rigid substrate. Frequency spectrum of the lowest eight modes for different wavelengths (Frequency ratio $\Omega = \omega/\omega_S$, Wavelength ratio $\xi = l/\lambda$, l = height = length of crest, $\omega_S^2 = \pi G/\rho l^2$, $\nu = 0.3$, heavy lines refer to symmetrical cases and broken lines to antisymmetrical configurations).

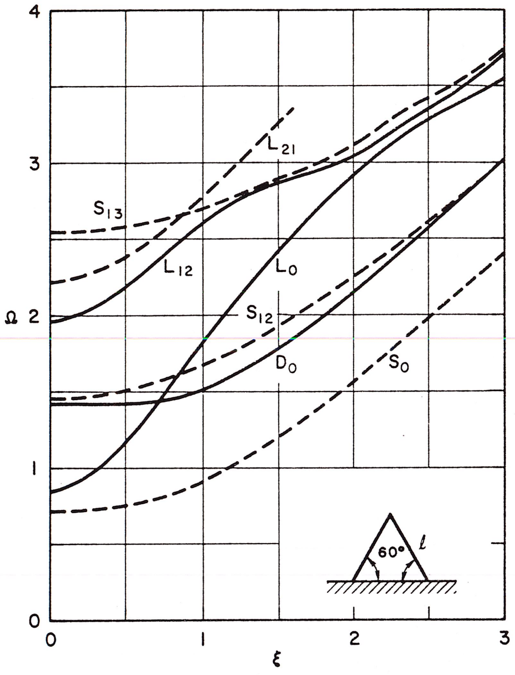


Fig. 6. Triangular isotropic overlay guide on rigid substrate. Frequency spectrum of lowest six modes for different wavelengths (Frequency ratio $\Omega = \omega/\omega_S$, Wavelength ratio $\xi = l/\lambda$, l = height = length of crest, $\omega_S^2 = \pi G/\rho l^2$, $\nu = 0.3$, heavy lines refer to symmetrical cases and broken lines to antisymmetrical configurations).

B. Trapezoidal Overlay Guide

An inverted trapezoidal guide on a rigid substrate with crest equal to height and angle $\theta = 75°$ as shown in Fig. 1(b) is considered. Ash *et al.* [2] have proposed the same geometry in conjunction with a topographical guide. The mode spectrum of this guide is shown in Fig. 5, with the values of its first cutoff frequencies in Table II.

This guide is comparable to the square overlay case of Fig. 4(a) in that its geometry is contained in the same format. However, the tapering at base has had the marked effect of widening the range of single-mode frequencies of the trapezoidal geometry, and in addition lowering the corresponding magnitudes of the cutoff frequencies. Should practical considerations not favor the choice of a thin upright rectangular overlay guide such as the case of $b/a = 2$, this trapezoidal guide of one-half depth serves as a good alternative by offering almost the same range of single-mode operation ($\Omega_2 - \Omega_1 = 0.2629$).

C. Triangular Overlay Guide

This type of guide is also proposed in [2], but no solutions regarding its behavior are presented. To study the general characteristics of its performance, the case of an equilateral triangular guide mounted on a rigid substrate is considered. The dispersion curves for the lowest six modes are shown in Fig. 6. The geometry of this guide too is dimensionally contained in the square format of Fig. 4(a). The results indicate that this triangular shape offers a single mode operation range almost as wide as the square overlay guide ($\Omega_2 - \Omega_1 = 0.1365$, last column of Table II), but for frequencies of the order of twice the former. In addition, in the range of double mode operation wavelengths corresponding to modes (S_0) and (L_0) are widely apart which contributes in suppressing mode coupling and interference while making it possible to operate at wavelengths of the order of base width of guide. It is worth mentioning that the mode configurations of this guide are found to preserve their characteristic differences throughout the spectrum range. It is anticipated that this guide would exhibit less mode coupling in multimode ranges of operation too.

D. Inlay Guides

A layer of relatively flexible material inserted between two rigid blocks is termed a sandwich guide [2] and is considered to possess such wave propagation characteristics that would make it suitable to be employed as a microsound surface wave guide. In order to gain an insight into the behavior of sandwich guides a rectangular guide ($b/a = 1.5$) inlaid in a rigid substrate with its width on the free surface is analyzed. The fact that the inlay guide treated does not penetrate deep into the substrate—as a sandwich guide would do, is believed not to lead into a significantly different general behavior, because the energy transfer is localized near the flexible free surface.

The mode spectrum for the first six modes are given in Fig. 7. As may be anticipated, the basic shear mode S_0 is no longer the lowest form of wave propagation due to the lateral support provided by the rigid sides. The magnitudes of the cutoff frequencies are significantly higher than the corresponding frequencies of an overlay guide, which is due to its overall increase in rigidity.

E. Orthotropic Overlay Guides

Materials employed for making microsound waveguides are often not isotropic. They may possess orthotropic properties,

TABLE III
PROPERTIES OF MATERIALS USED FOR GUIDES

Material	No.	Normalized Elastic Moduli[a]								
		C_{11}	C_{12}	C_{13}	C_{22}	C_{23}	C_{33}	C_{44}	C_{55}	C_{66}
Isotropic	1	3.50	1.50	1.50	3.50	1.50	3.50	1.0	1.0	1.0
Very weak transverse normal modulus	2	0.035	0.015	0.015	3.50	1.50	3.50	1.0	1.0	1.0
Very weak transverse shear modulus	3	3.50	1.50	1.50	3.50	1.50	3.50	0.01	1	0.01
Weak transverse shear modulus	4	3.50	1.50	1.50	3.50	1.50	3.50	0.1	1	0.1

[a] Isotropic material ($\nu = 0.3$) normalized with respect to $C_{44} = G$.
Material No. 2 normalized with respect to C_{44}.
Materials Nos. 3 and 4 normalized with respect to C_{55}.

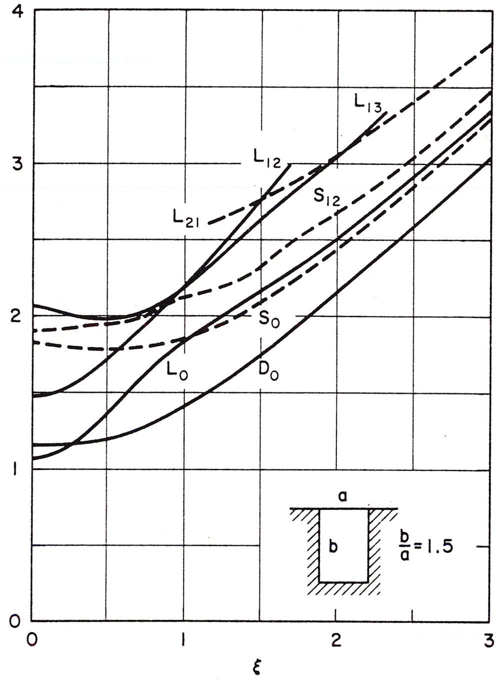


Fig. 7. Rectangular isotropic guide inlaid in rigid substrate. Frequency spectrum of the lowest six modes for different wavelengths (Frequency ratio $\Omega = \omega/\omega_s$, Wavelength ratio $\xi = l/\lambda$, $l = a$, $\omega_s^2 = \pi G/\rho l^2$, $\nu = 0.3$, heavy lines refer to symmetrical cases and broken lines to antisymmetrical configurations).

and besides they may be cut and placed such that the direction of propagation does not coincide with one of the elastic axes. In this case the material exhibits anisotropic properties in direction of wave propagation. For orthotropic materials with only slight variations of elastic moduli, the guide performance is basically the same as in the isotropic case. However, materials with markedly different relative elastic moduli from the corresponding isotropic values can lead to significantly different behavior, and if effectively designed can increase the bandwidth of single mode operation; they can result in the lowering of certain modes and the elimination of others from the operational range.

In order to demonstrate this influence, three orthotropic square overlay guides on rigid supports are analyzed and their cutoff frequencies compared with the isotropic case already discussed Fig. 4(a). The elastic moduli of the materials considered are listed in Table III together with the normalized moduli of the isotropic case for comparison. The elastic properties are selected such as to reveal certain characteristic features of orthotropic behavior.

Material No. 2 has a very weak transverse normal modulus (in x direction). Material No. 3 has a very weak transverse shear modulus and material No. 4 is weak in transverse shear. The cutoff frequencies of the first four modes are given in Table II for each material used. Material No. 4 having the appropriate weak transverse shear moduli offers the widest range of single mode operation compared to all the guides analyzed. Material No. 3 is too weak in transverse shear and results in the first four shear modes all being lowered below its basic longitudinal mode L_0 at the cutoff frequencies (Ω for L_0 equals 0.5017, not included in Table II).

A material of the type discussed here may not be readily available. But new materials are being constantly developed to meet predetermined specifications. The results given provide a clear quantitative indication of the influence of the magnitudes of a number of leading elastic moduli, and serve as a directional guide toward the selection and development of materials for use in microsound surface waveguides.

F. Composite Overlay Guides

One other method of influencing the spectrum characteristics of a guide is by constructing the guide of several layers of different materials as shown in Fig. 1(d). Several groups of two layer square overlay guides were studied, with the thickness of upper layer being $b/4$.

An increase in mass density and elastic moduli of the upper layer results in a general reduction of cutoff frequencies (normalized with respect to material properties of the lower part), and lowering of the mode spectrum. For the case of material properties of upper part being 10 times the material properties of the lower region this reduction is about 50 percent. Whereas if mass densities are kept constant, but only elastic moduli of the upper layer are increased, the changes in the initial cutoff frequencies are insignificant.

CONCLUSIONS

An analysis is given for evaluating frequency spectra of orthotropic waveguides of arbitrary cross section. The analysis is based on the extended Rayleigh-Ritz energy method and the concept of discretization of the cross section into mathematical subregions (better known as finite elements in the field of structural engineering). The main features of this

[illegible]

[illegible] material [illegible]

[illegible] in Material No. 5 [illegible] is very weak [illegible] weak [illegible] The cutoff frequencies [illegible] Table II for each material used. Material No. 4 having the appropriate weak transverse shear coupling [illegible] the cutoff frequencies [illegible] compared to all the [illegible] of Material No. [illegible] in the first four [illegible] modes [illegible] lowered below the basic longitudinal mode [illegible] frequencies [illegible] for L equals 0.301 [illegible] (Table II).

A [illegible] of the [illegible] may not be readily [illegible], but new materials are being constantly developed to meet predetermined specifications. The results given provide a quantitative indication of the influence of the magnitudes of a number of leading elastic moduli, and serve as a directional guide toward the selection and development of materials for use in [illegible] acoustic surface waveguides.

[illegible] Composite Overlay Guides

One other method of influencing the spectrum characteristics of [illegible] guides is by constructing the guide of several layers of different materials as shown in Fig. 1(f). [illegible] two layers [illegible] with the thickness of upper layer being h.

[illegible] increase in mass density and elastic moduli of the upper layer results in a general reduction of cutoff frequencies [illegible] with respect to material properties of the lower layer, and lowering of the mode spectrum. For the case of [illegible] properties of upper layer being 10 times the material properties of the lower region, this reduction is about 50 percent. Whereas if mass densities are kept constant but only [illegible] moduli of the upper layer are increased, the changes in the [illegible] cutoff frequencies are insignificant.

CONCLUSION

An analysis is given for evaluating the mode spectrum of orthotropic waveguides of arbitrary cross section. The [illegible] extended Rayleigh–Ritz energy method and the [illegible] of the cross section [illegible] mathematical [illegible] (better known as finite elements) [illegible] The [illegible] of this

Fig. [illegible] Rectangular anisotropic guide inlaid in [illegible] substrate. [illegible] spectrum of the lowest six modes for different wavelengths. Frequency ratio $\Omega = \omega/\omega$ [illegible]. Wavelength ratio [illegible]; [illegible] $= 0$ [illegible] lines [illegible] symmetrical cases and broken lines to antisymmetrical configurations.

and hexagonal they may be cut and placed such that the direction of propagation does not coincide with one of the elastic axes. In this case the material exhibits anisotropic properties in directions of wave propagation. For orthotropic materials with only slight variations [illegible] overall, the guide performance [illegible] the same as in the isotropic case. However, materials with [illegible] different relative [illegible] from the corresponding isotropic values can lead to significantly different behavior, and if effectively designed [illegible] the bandwidth of single mode operation; they can result in the lowering of certain modes and the elimination of others from the operational range.

In order to demonstrate this, [illegible] three orthotropic [illegible] overlay guides [illegible] are analyzed and their cutoff frequencies compared with the isotropic case already discussed (Fig. 6(a)). The elastic moduli of the materials considered are listed in Table III together with the normalized moduli of the isotropic case for comparison.

analysis are that, with equal ease, it accounts for variations in cross sectional geometry of the guides; and it satisfies all the boundary conditions. With no necessity for *a priori* knowledge of modal configurations, and no imposition of preferential behavior on deformations, the analysis extracts, for any given wave length ratio ξ, a predetermined number of lowest frequencies together with their relating configurations.

The accuracy of the results obtained increases with the fineness of discretization used in modeling the guide. The order of magnitude of the accuracy of the solutions offered is assessed by comparing numerical results obtained from this analysis with available exact solutions for solid cylinders (Table I). It is shown that the accuracy of these frequencies is about 2 percent. The solutions given for the microsound surface waveguides are obtained using a finer degree of discrete division (96 elements for cylinder, 100–154 elements for the guides). Hence, it is expected that the surface guide results given are more accurate than the above quoted value. In addition, where possible, the guide's cutoff frequencies are compared with exact theories. The agreements are very good.

A number of microsound surface waveguides with the more interesting cross sections are case studied. The mode spectra of the guides are given graphically together with the magnitude of their first four cutoff frequencies.

To avoid mode coupling problems, it is desirable to use the microsound wave guides within the range of first and second cutoff frequencies. The results discussed reiterate that, among other factors, the spectrum characteristics of a guide depend markedly on its geometry. The solutions provide a basis for the estimation of this range of frequencies for the guides considered. In addition a study of the spectra of the cases analyzed and their discussion shows that through a proper choice of dimensions and geometry, it is possible to materialize the designers' requirement of single mode operation at a predetermined level of frequencies. For example for same material and overall dimensions triangular overlay guides and inlay guides operate at higher single mode frequencies compared to rectangular overlay guide. The corresponding range of frequencies is lower for the inverted trapezoidal overlay guides.

The selection of the right orthotropic material properties provides another effective means of implementing the required design criteria. It is shown that by appropriate selection of material properties the single mode operation range of a square overlay guide may be increased several times (material no. 4, last column of Table II).

In light of the availability of this general analysis, the influence of substrate flexibility on the guide performance need be further examined in future works. Furthermore, as an alternative to development of, and search for new materials with special characteristics for making microsound wave guides, the exploration of composite multilayer overlay guides may prove rewarding.

It is clear that for each guide geometry considered, there will be variations in behavior due to changes in its geometry parameters. This aspect of the problem, not being within the scope of this work, is not treated here.

A useful and timely extension of the analysis presented is to consider the problem of general anisotropic materials, in which case two sets of displacement functions, with a phase difference, need be assumed in place of (1), in order to admit waves that are physically possible in a fully anisotropic structure.

CONCLUDING NOTE

After the first version of this paper was completed, the author's attention was drawn to several related recent works in [16–23], some of which are pending publication at the time of preparation of this note. Mason *et al.* [16] have conducted some experimental work on topographic guides. Finite element analysis is employed in [17–20] for the theoretical treatment of acoustic surface wave guides. Rectangular, trapezoidal and triangular guides are discussed in [17–23].

APPENDIX

The matrices $[M(X, Y)]$ (a 3×9) and $[\mathrm{trig}_1]$ (a 3×3), used in the present analysis are as given (only nonzero terms are given) in the following:

$$[M(X, Y)] = \begin{bmatrix} 1 & X & Y & & & & & & \\ & & & 1 & X & Y & & & \\ & & & & & & 1 & X & Y \end{bmatrix}$$

$$\begin{matrix}[\mathrm{trig}_1] \\ \text{(diagonal)}\end{matrix} = \begin{bmatrix} \sin(\pi\xi Z) & & \\ & \cos(\pi\xi Z) & \\ & & \cos(\pi\xi Z) \end{bmatrix}.$$

NOMENCLATURE

a	Maximum width of guide in x direction.
b	Depth of guide in y direction.
C_{ij}	Elastic constants.
G	Shear modulus of elasticity.
$[k]$	Element stiffness matrix (9×9).
$[K]$	Assemblage stiffness matrix.
l	Any representative length.
$[m]$	Element mass matrix (9×9).
$[M]$	Assemblage mass matrix.
$[M(X, Y)]$	Matrix of displacement functions in terms of X and Y, (3×9, see Appendix).
$\{r\}$	Nodal values of functions w', u', v'.
$\{r_0\}$	Nodal amplitudes of functions w', u', v'.
t	Time.
$[\mathrm{trig}_1]$	Diagonal matrix in terms of trigonometric functions of (1), (3×3, see Appendix).
T	Kinetic energy.
u, v, w	Displacements in x, y, and z directions, respectively.
u', v', w'	Displacement functions in terms of x, y, and t.
V	Potential energy.
x, y, z	Cartesian coordinates axes.
X, Y, Z	Nondimensionalized coordinates.
α	Time dependent coefficients.
ϵ	Direct strains.
θ	Angle describing cross sectional geometry.
λ	Half-wavelength.
ν	Poisson's ratio.

ξ	Wavelength ratio = l/λ.
ρ	Density.
σ	Stress.
ω	Circular velocity.
Ω	Normalized circular velocity = $\omega/(C_{ij}/\rho l^2)^{1/2}$.
$(\dot{\cdot})$	Differentiation with respect to time.

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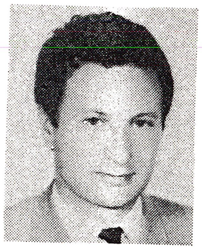

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