DESIGN FOR CRACK WIDTH[1]

Bijan O Aalami[2]

This preliminary draft is the complication of a number of references and numerical works regarding design for crack control. It covers both conventionally and pre- or post-tensioned members. Work is in progress in editing and organizing the material presented in this Technical Work

STEPS FOR CRACK WIDTH DESIGN
1 - Select the exposure classification of member to corrosive elements
2 - Using the exposure classification, and the type of prestressing, select the recommended design crack width
3 – Calculate the probable crack width of the member, using the member's forces and particulars
4 – If the computed probable exceeds the design value, use one of the following options to conclude the design
 4.1 Change the parameters of design, such as member thickness, loads, reinforcement and re-calculate the probable crack width; or
 4.2 Detail the location using the amount and size of the reinforcement to contain the probable crack widths within the acceptable range.
5 – Numerical Example

The following outlines the details.

1 – SELECT EXPOSURE CLASSIFICATION

Use the table below from the European building code (EC2)

[2] Professor Emeritus, San Francisco State University; Principal, ADAPT Corporation; bijan@adaptsoft.com

EN 1992-1-1:2004 (E)

Table 4.1: Exposure classes related to environmental conditions in accordance with EN 206-1

Class designation	Description of the environment	Informative examples where exposure classes may occur
1 No risk of corrosion or attack		
X0	For concrete without reinforcement or embedded metal: all exposures except where there is freeze/thaw, abrasion or chemical attack For concrete with reinforcement or embedded metal: very dry	Concrete inside buildings with very low air humidity
2 Corrosion induced by carbonation		
XC1	Dry or permanently wet	Concrete inside buildings with low air humidity Concrete permanently submerged in water
XC2	Wet, rarely dry	Concrete surfaces subject to long-term water contact Many foundations
XC3	Moderate humidity	Concrete inside buildings with moderate or high air humidity External concrete sheltered from rain
XC4	Cyclic wet and dry	Concrete surfaces subject to water contact, not within exposure class XC2
3 Corrosion induced by chlorides		
XD1	Moderate humidity	Concrete surfaces exposed to airborne chlorides
XD2	Wet, rarely dry	Swimming pools Concrete components exposed to industrial waters containing chlorides
XD3	Cyclic wet and dry	Parts of bridges exposed to spray containing chlorides Pavements Car park slabs
4 Corrosion induced by chlorides from sea water		
XS1	Exposed to airborne salt but not in direct contact with sea water	Structures near to or on the coast
XS2	Permanently submerged	Parts of marine structures
XS3	Tidal, splash and spray zones	Parts of marine structures
5. Freeze/Thaw Attack		
XF1	Moderate water saturation, without de-icing agent	Vertical concrete surfaces exposed to rain and freezing
XF2	Moderate water saturation, with de-icing agent	Vertical concrete surfaces of road structures exposed to freezing and airborne de-icing agents
XF3	High water saturation, without de-icing agents	Horizontal concrete surfaces exposed to rain and freezing
XF4	High water saturation with de-icing agents or sea water	Road and bridge decks exposed to de-icing agents Concrete surfaces exposed to direct spray containing de-icing agents and freezing Splash zone of marine structures exposed to freezing
6. Chemical attack		
XA1	Slightly aggressive chemical environment according to EN 206-1, Table 2	Natural soils and ground water
XA2	Moderately aggressive chemical environment according to EN 206-1, Table 2	Natural soils and ground water
XA3	Highly aggressive chemical environment according to EN 206-1, Table 2	Natural soils and ground water

48

2 – SELECT THE APPLICABLE DESIGN CRACK WIDTH
Use the following table from EC2

TABLE 4.4.1.2-1 Exposure Classification and Suggested Design Crack Width EC2 (mm) (T127)

Exposure Class	Reinforced members and prestressed members with **unbonded** tendons	Prestressed members with **bonded** tendons
	Quasi-permanent load combination (Sustained load combination)	Frequent load combination (Total load combination)
X0, XC1	0.4[1]	0.2
XC2, XC3, XC4	0.3	0.2[2]
XD1, XD2, XS1, XS2, XS3		Decompression

Note 1: For X0, XC1 exposure classes, crack width has no influence on durability and this limit is set to guarantee acceptable appearance. In the absence of appearance conditions this limit may be relaxed.
Note 2: For these exposure classes, in addition, decompression should be checked under the quasi-permanent combination of loads.

3 – CALCULATE THE PROBABLE CRACK WIDTH
Use the text below which is an excerpt from a forthcoming book. The information is from EC2

(15) Calculate "Computed" Crack Width
Using the procedure described below[3], and the provision of bonded non-prestressed reinforcement from the previous steps, the serviceability check continues with the computation of the probable crack width (w_k) for each design section. The calculation of the probable crack width (w_k) is explained below. A numerical example of it is given in Chapter 6.

Computed probable crack width,

$$w_k = s_{r,max}(\epsilon_{sm} - \epsilon_{cm}) \qquad \text{(Exp 4.10.3(15)-1)}$$

Where,

$s_{r,max}$ = maximum crack spacing;
ε_{sm} = mean strain in the reinforcement under the relevant combination of loads, including the effect of imposed deformations and taking into account the effects of tension stiffening; and
ε_{cm} = mean strain in the concrete between cracks.

$$\varepsilon_{sm} - \varepsilon_{cm} = \frac{\sigma_s - k_t \dfrac{f_{ct,eff}}{\rho_{p,eff}}\left(1 + \alpha_e \rho_{p,eff}\right)}{E_s} \geq 0.6\,\frac{\sigma_s}{E_s} \qquad \text{(Exp 4.10.3(15)-2)}$$

Where,

σ_s = the stress in the tension reinforcement calculated on the basis of a cracked section [f_{yk}];
α_e = E_s/E_{cm}

[3] EN 1992-1-1:2004(E), Section 7.3.4.

$$\rho_{p,eff} = \frac{(A_s + \xi_1{}^2 \times A_p')}{A_{c,eff}} \qquad \text{(Exp 4.10.3(15(−3)}$$

$A_{p'}$ = area of tendons within $A_{c,eff}$
$A_{c,eff}$ = effective area of concrete in tension surrounding the reinforcement or prestressing tendons of depth $h_{c,ef}$;
$h_{c,ef}$ = lesser of *2.5(h-d), (h-x)/3* or *h/2;*
h = depth of the member;
x = depth of neutral axis from the compression fiber;

$$\xi_1 = \sqrt{\xi \frac{\phi_s}{\phi_p}} \qquad \text{(Exp 4.10.3(12)-5)}$$

ξ = ratio of bond strength of prestressing and steel according to Table 6.2;
Φ_s = largest bar diameter of reinforcing steel;
Φ_p = equivalent diameter of tendon according to 6.8.2;
k_t = factor dependent on the duration of the load;
= 0.6 for short term loading
= 0.4 for long term loading
E_s = modulus of elasticity of steel; and

$$s_{r,max} = 1.3(h-x) \qquad \text{(Exp 4.10.3(15)-4)}$$

(16) Limit Crack Width
If the computed probable crack width from the previous step exceeds the value selected for design, the code provides two remedial options. Either add reinforcement using the following relationship to limit the probable crack width (w_k), or select non-prestressed bar diameter and spacing according to Table 7.2 N or 7.3 N. Adding reinforcement will increase $\rho_{p,eff}$ in the following.

$$w_k = s_{r,max}\left(\frac{\sigma_s - k_t \frac{f_{ct,eff}}{\rho_{p,eff}}\left(1 + \alpha_e \rho_{p,eff}\right)}{E_s}\right) \qquad \text{(Exp 4.10.3(16) − 1)}$$

Where,

$$\rho_{p,eff} = \frac{(A_s + \xi_1{}^2 \times A_p')}{A_{c,eff}} \qquad \text{(Exp 4.10.3(15)-3)}$$

4 – COMPARE THE CALCULATED CRACK WIDTH (w_k) WITH THE DESIGN VALUE
If the calculated value exceeds the design value, either change the parameters of the member, or detail as shown below, by selecting the recommended bar size and spacing.

For pre-tensioned concrete, where crack control is mainly provided by tendons with direct bond, Tables 7.2N and 7.3N may be used with a stress equal to the total stress minus prestress. For post-tensioned concrete, where crack control is provided mainly by ordinary reinforcement, the tables may be used with the stress in this reinforcement calculated with the effect of prestressing forces included.

Table 7.2N Maximum bar diameters ϕ^*_s for crack control[1]

Steel stress[2] [MPa]	Maximum bar size [mm] w_k= 0,4 mm	w_k= 0,3 mm	w_k= 0,2 mm
160	40	32	25
200	32	25	16
240	20	16	12
280	16	12	8
320	12	10	6
360	10	8	5
400	8	6	4
450	6	5	-

Notes: 1. The values in the table are based on the following assumptions:
c = 25mm; $f_{ct,eff}$ = 2,9MPa; h_{cr} = 0,5; $(h\text{-}d)$ = 0,1h; k_1 = 0,8; k_2 = 0,5; k_c = 0,4; k = 1,0; k_t = 0,4 and k' = 1,0
2. Under the relevant combinations of actions

Table 7.3N Maximum bar spacing for crack control[1]

Steel stress[2] [MPa]	Maximum bar spacing [mm] w_k=0,4 mm	w_k=0,3 mm	w_k=0,2 mm
160	300	300	200
200	300	250	150
240	250	200	100
280	200	150	50
320	150	100	-
360	100	50	-

For Notes see Table 7.2N

The maximum bar diameter should be modified as follows:

Bending (at least part of section in compression):

$$\phi_s = \phi^*_s\,(f_{ct,eff}/2,9)\,\frac{k_c h_{cr}}{2\,(h\text{-}d)} \qquad (7.6N)$$

Tension (uniform axial tension)

$$\phi_s = \phi^*_s(f_{ct,eff}/2,9)h_{cr}/(8(h\text{-}d)) \qquad (7.7N)$$

where:

- ϕ_s is the adjusted maximum bar diameter
- ϕ^*_s is the maximum bar size given in the Table 7.2N
- h is the overall depth of the section
- h_{cr} is the depth of the tensile zone immediately prior to cracking, considering the characteristic values of prestress and axial forces under the quasi-permanent combination of actions
- d is the effective depth to the centroid of the outer layer of reinforcement

Where all the section is under tension h - d is the minimum distance from the centroid of the layer of reinforcement to the face of the concrete (consider each face where the bar is not placed symmetrically).

5 – NUMERICAL EXAMPLE
The following is an excerpt from the contents of a book being prepared for publication.

Example 1 – crack width calculation

A. Based on EC2[4]:

The allowable crack width for members reinforced with unbonded tendon (quasi-permanent load combination) is 0.3 mm, and for bonded tendon (frequent load combination) is 0.2 mm. Since in this example the maximum computed tensile stress for frequent load exceeds the threshold limit, crack width calculation is required based on section 7.3.4. If the calculated crack width exceeds the threshold, EC2 recommends to limit the bar diameter and bar spacing to the values given in Table 7.2N or 7.3N of EC2 to limit the width of cracks. The following example illustrates the point.

Point A:

Crack width, $W_k = S_{r,\,max}\,(\varepsilon_{sm} - \varepsilon_{cm})$ [5]

$\varepsilon_{sm} - \varepsilon_{cm} = [\sigma_s - k_t *(f_{ct,eff}/\rho_{p,eff})(1 + \alpha_e\,\rho_{p,eff})] / E_s \geq 0.6\,\sigma_s / E_s$

Where,

α_e = E_s / E_{cm} = 199,949.20/ 32194.71 = 6.21

$\rho_{p,eff}$ = $(A_s + \xi_1^2\,A'_p)/ A_{c,eff}$

A'_p = area of tendons within Ac,eff =12 *99 =1188 mm^2 (1.84 in^2)

A_s = 0 mm^2

ξ_1 = $\sqrt{(\xi * \varphi_s / \varphi_p)}$

ξ = 0.5 (From Table 6.2)

φ_s = largest diameter of bar =22 mm (# 7)

φ_p = 1.75* 13 = 23 mm (0.90")

ξ_1 = $\sqrt{(0.5* 22 / 23)}$ = 0.70

$A_{c,eff}$ = $h_{c,eff}$ * bw

$h_{c,eff}$ = lesser of (2.5*(h-d), (h-x)/3 , (h/2))

x = 2.99* 762/(2.99+3.66) = 343 mm (13.49 in)

d = 762- 51-22/2 = 700 mm (27.56 in)

$h_{c,eff}$ = lesser of (2.5*(762-700), (762-343)/3 , (762/2))
= 140 mm (5.51")

$A_{c,eff}$ = 140 * 457 =63980 mm^2 (99.17 in^2)

$\rho_{p,eff}$ = $(0 + 0.70^2*1188)/ 63980$ =0.0091

σ_s = (f/Ec)*Es

f = tensile stress due to DL+0.5LL = $(M_D+0.5\,M_L)/s$
= $(621.8+0.5*186.6)*1000^2/6.23*10^7$
= 11.48 MPa (1665 psi)

σ_s = (11.48/32194.71)* 199,949.20 =71.30 MPa (10341 psi)

k_t = 0.4 (coefficient for long-term loading)

$f_{ct,eff}$ = $f_{ctm} = 0.3 *(27.58)^{(2/3)}$ = 2.74 MPa (397psi)

$\varepsilon_{sm} - \varepsilon_{cm}$ = $[\sigma_s - k_t *(f_{ct,eff}/\rho_{p,eff})(1 + \alpha_e\,\rho_{p,eff})]/Es$
= [71.30– 0.4 *(2.74/0.0091)(1 + 6.21*0.0091)]/ 199,949.20
= -0.0003 ≤ 0.6 * 71.30 / 199,949.20 =0.000214, use 0.000214

[4] EN 1992-1-1:2004(E) , Section 7.3.3
[5] EN 1992-1-1:2004(E) , Section 7.3.4

$s_{r,max}$ = 1.3*(h-x) =1.3* 419 =545 mm
Crack width, W_k = 545 * 0.000214
= 0.12 mm < 0.2 mm OK
Provide minimum reinforcement for cracking.

EXAMPLE 2 Detailing for Crack Width Control

For demonstration of EC2[6] procedure for crack control, let the maximum hypothetical tensile stress in concrete exceed the threshold set in the code (3.21 MPa). Determine the required crack control reinforcement for the section reinforced with unbonded tendons.[7]

Given:
f_b = 3.5 MPa (508 psi) (tension at bottom)
f_t = 5.2 MPa (754psi) (compression at top)
Depth of section = 241 mm (9.5 in)
Width of section = 10,360 mm (34 ft)
Required: Reinforcement for crack control
$\sigma_s = f_{yk}$ = 413.69 MPa (60 ksi)
k = 1
Depth of tension zone at bottom, using Fig. 7.4-1 = 3.5*241/(3.5+ 5.2) = 97 mm (3.82 in)
A_{ct} = 97 *10360 = 1.004e+6 mm^2
$k_c = 0.4* [1 -(\sigma_c /(k1 (h/h^*) f_{ct,eff})]$
$\sigma_c = N_{ED}$ /bh = 1.10 MPa (average precompression)
h* = 241 mm (9.5 in)
k_1 = 1.5 (since section is in compression)
$f_{ct,eff} = f_{ctm} = 0.3 *(34.47)^{(2/3)}$ = 3.18MPa (461 psi)
k_c = 0.4* [1 - (1.10 /(1.5 (241/241) 3.18)] = 0.31

$A_{smin} = k_c\ k\ f_{ct,eff}\ A_{ct} /\sigma_s$
A_{smin} = 0.31*1*3.18* 1.004e+6 /413.69 = 2375 mm^2 (3.68 in^2)

[6] EN 1992-1-1:2004(E), Section 7.3.2(3)
[7] For members reinforced with grouted tendons, the cross-sectional area of grouted tendons can be used to contribute to the minimum required area for crack control.